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SPEAKERS

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Welcome. In this lecture, we're going to talk about the fundamental theorems of calculus. There are two of these, the famous fundamental theorems that people talk about. There are other theorems in calculus, but these they usually call the fundamental theorems of calculus. So there's kind of some reasons why they're interesting and important. One of them is that they tell you how what we mean when we say that differentiation and integration undo each other. Right? We've kind of been talking about that when we talked about antiderivatives. And then this is going to show the relationship of those in integrals. And the other thing is that they kind of show you how to actually compute an integral, you're probably not going to be doing a Riemann sum, you're going to be doing some antiderivatives. Okay, so let's get started. So what's kind of our setup. So we want to suppose that we have, suppose we have some function F . So F is going to be continuous, and then on some kind of interval A, B . So on this interval A, B . Okay, so those are the things that must be true in order to apply the rest of the theorem. So here's the rest of the theorem, then, kind of two parts. So there are kind of two theorems. The first one is a fundamental theorem of calculus one or the first fundamental theorem of calculus.

Okay, so what does this one say? It says that if we have the F of X . So if we have that F of X equals the integral from A to X . And this right, this X and this X , these are both variables. So this X and this X are the same, and they're both variables that vary, okay? Because this is a function, right, so I'm saying that this is a function here, right? Because I get different values if I put in different F 's, X 's, sorry, of F of T , DT . Okay, so if this holds, then I know that, so what do I know? I know that taking the derivative of this function with respect to X , so this is I'm taking this derivative with respect to X of this function is going to equal right we do this, we're using the other notation for a reason, I so much prefer the $D DX$, because it says we're taking the derivative with we're taking the derivative with respect to. But in this case, it's going to get a little confusing. So let's, let's do it the other way. So I'm going to do F of prime of X . So I'm taking the derivative of this with respect to X , then this is actually going to equal F of X , F of X . And this is going to be true for each X in this interval, so each X in interval A, B . Okay, so we started out with this interval. And we know that if it's true that this is the function that gives you, right, so what is its value at X ? It's the integral from A to X of F of T DT , so this is a function. We can take its derivative, and we're saying that these two are the same as functions on this interval. So no matter what X in this interval I stick in, then I'm going to get out the same output. Okay? So here's an example.

Is that if I'm going to take, so let's take $F(X)$ equals the integral from 1 to X of $\sin(T) dT$. Okay? So looking at this, this is like my green function, and this is my red function. Right? So if I define F this way, so I'm defining the function F of X , I'm defining it like that. Then I have that this function, so F , okay, so then I have F and then I take the derivative of that, this is going to equal the function $\sin$ of X . Okay, so what do I mean by this? I just replaced T with X in the integrand. Okay, so all I did was I just replaced T with X in the integrand.

That's all I did. But sometimes people expect that you kind of know this and know how to do this. The real point are that these functions undo each other. Okay, but let's look at the, the second one. So here's the second fundamental theorem of calculus. And we're under the same assumptions, right? We're still under the same assumptions. So what does this one tell me? It tells me that the integral from A to B of $F(X) dX$, so $F(X) dX$, and this is the one that's really good for computing, okay? How do I actually do this? This is really kind of cool, is that I'm going to take the antiderivative. And first, I'm going to plug in the top one. And then I'm going to plug in the bottom one. That's it. That's actually how you compute that integral, right? So that's very, very useful. So let's show an example.

Oh, so this is where, we so much have been using the color coding that I didn't, I knew you would know without my saying, but let's explicitly say, where F is any antiderivative of, so it's any antiderivative of, so F is any antiderivative of f , right, i.e., if I take the derivative of this, so if I take the derivative with respect to X of F , I'm going to get a f . Okay, great. So let's look at an example of this one.

So if I take the integral from, and let's go from 2 to 3 of $X^2 dX$, $X^2 dX$. And this is going to equal, right, we know that what do we have to do? We have to add 1 and then we have to divide by that 2. So let's kind of do that to get the antiderivative oops, and then we just, right, we put notice, okay, this is how I want to write it, this is good, I like this. Okay. So I'm going to do, but I'm going to do this in red to make it clear I'm doing the antiderivative, okay? So I'm going to do X^3 divided by 3, and then I'm going to evaluate it at, I'm telling myself now I'm going to plug in X equals 3. And then I'm going to subtract off, and this is again the antiderivative. And I'm going to evaluate this at X equals 2.

So 2 was top, so that came first. 2 was bottom, so that came next. And then we have so this is equal to, right, so this is where I like to put the parentheses replace X , or I make sure that I put in what I need to there. So let's just kind of go ahead and write it like this before we put in the numbers, right? And in the first one, I need to put in 3. And then the second one, I need to put in 2. And what do I get out? I got out 19 divided by 3. Okay, so 3 seems a little difficult to see. There we go. Okay, so what kind of went on here? Right, we have these two theorems that are going to help us actually compute antiderivatives. This one's a little more, you're not going to use it as much. It's a cool fact, and people often use it in higher mathematics. This one is how you're going to compute integrals. So this is one to like know. So what do you do? You take the antiderivative, and then you take, apply the antiderivative to the top, minus applying the antiderivative to the bottom. It works for any antiderivative. Why does it work for any antiderivative? Because you end up with the constant minus the constant and that cancels out to zero. Okay? So now we do that in a particular example, we know

that the antiderivative of X^2 is X^3 divided by 3. So I did kind of an intermediary step to make sure I did it right. So I did the antiderivatives. And I told myself I was going to evaluate this one at 3, which was the top, this one at 2, which was the bottom. So and then I you know, I do this trick where I just replace the X with parentheses and then I know that I'm putting things in where they're supposed to go. And so that the 3 goes first because it's the top, 2 goes next because it's the bottom and I get out 19 thirds. Okay, so, now you know how to actually compute integrals. So I hope that made some sense and I will see you in the next lecture.