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SUMMARY KEYWORDS

antiderivative, equals, generalized, constant, $3x$, add, derivative, function, vertical, anti derivatives, precisely, translates, important, parallel, unique, picture, facts, draw, differ, theorem

SPEAKERS

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Welcome. In this lecture, we're going to answer the question from last lecture about how the antiderivative is related and then introduce the generalized antiderivative. Okay, so let's start with what I might call, like important facts or a theorem. So here are some important facts. This is a theorem. Okay? The first one is that anti derivatives are never unique. I'm going to write this in green because we're talking about antiderivatives. So anti, oh I was making them red before, let's do them red, okay?

Okay, so here we go. So antiderivatives are never unique. So antiderivatives are never unique. And right, because we could add a constant, and that's all you can do. Otherwise, they're the same, okay? So they precisely differ by all constants, okay? So they meaning antiderivatives precisely differ, like for the same function. They differ by all constants. So I could add any constant to my antiderivative, and it's still an antiderivative. And then the other ways that that's, the only way to get between antiderivatives for the same function, okay? So this kind of spikes or prompts us to create a definition. So we call, if I take this F , so I take any antiderivative, and then I add C . Okay, um, that we have, we call this a generalized antiderivative. Okay, so call this a generalized antiderivative generalized.

This is a little too light, so I generalized antiderivative. Okay, so this is kind of a new word. I'm going to make it, box in red anyway. We're going to use red for both of them. Okay, so let's kind of draw the picture. So let's, and go through kind of an example. So here's my example. So if I have F of X equals X plus 3, then, right, so what should my generalized antiderivative be? Okay, then F of X equals, right, oh, that's not X plus 3. Oh, I thought I took the antiderivative wrong in my notes, but it was that I wrote this down wrong. That should be 3, then F of X equals $3X$ plus C , okay, is the, is a generalized antiderivative. This is a generalized antiderivative. Okay, and what like, what is the picture look like? So it's like I have a X axis, and I have my Y axis. And one of those would be the one that goes like, right through the origin, right? Like this, right? So this would be one of them. So this is like Y equals $3X$. But I mean, here's another one. Right? If I add a constant, these are supposed to be parallel. I think I failed a little bit at that. So I could do Y equals $3X$ minus π , for example. Or I could do, right, I could do Y equals $3X$ minus E squared. Okay? I could also do something like I can add, right, I could

do Y equals $3X$ plus E . Okay? And so this is what we call, this is like a family of antiderivatives, right? So this is a family of antiderivatives. So it's an entire set. Right? When I'm talking about the generalized antiderivative, I'm actually talking about an entire set.

Okay, so it's a family of antiderivatives. I tried to, in an attempt to get it all squeezed on here I made it way too difficult to read. So derivatives, okay? And so the kind of important thing is if I had drawn this correctly, so note that these are all vertical. So note these are all vertical translates of each other because you're adding a constant. So they're all vertical translates of each other. So there's infinitely many. It's all of the translates, it's precisely the vertical translates. Because that's precisely what happens when you add constants, right? Or another way to think, they're all parallel. Okay? So just to kind of recap, we talked about one antiderivative, we talked about the fact that like, you could add any constant and get the same, you know, and you'd get another antiderivative, because when you take the derivative of a constant, you get zero, right? So that part just kind of drops off. And, and we can kind of think about those all at once like as a set or the family of functions. And that's the generalized antiderivative. It's kind of cool in the picture that they all are vertical translates of each other because they're all adding constants to each other, and it's precisely all the vertical translates. These would be parallel if I could draw correctly. So I hope that made some sense and I will see you in the next lecture.