

PfaffModule8L01

Mon, 2/7 7:51PM 10:27

SUMMARY KEYWORDS

derivative, antiderivative, chain rule, function, $3x$, sine, trickier, $3e$, undo, constant, interval, cosine, differentiation, denoted, important, talk, matter, mind, test, circumstance

SPEAKERS

Catherine Pfaff

Welcome. In this lecture, we're going to talk about antiderivatives, which are in some sense kind of the opposite or undoing of derivatives. And they're actually going to lead us to how we're actually, in the ultimately, in the end how we're going to be computing integrals. So let's kind of do our setup first. So here's our setup. Setup, we're going to have, there we go. So we're going to have that F is going to be a function, and we're going to have that I is going to be an interval, right? So an interval A, B , okay, so like, A, B , so this is I , it's also going to be written, right, like this A, B . It's like inclusive, right? So we go like this, okay? so this is we have these, so this is, we have this, so F is a function, and I is an interval. So this is kind of our setup throughout what we're doing here. And then by definition, so this is definition, we're going to have that the antiderivative of F , okay, so this is going to be denoted red. Okay, so the antiderivative.

This is the new word, so the antiderivative. And it's specific to what we're talking about. So the antiderivative of our function F on this interval I is going to be, it's going to be the function, it's going to be a function. So a function that is going to be denoted kind of, we'll usually denoted by big F , when that's little F . It's just kind of how we like to do it. So that we end up having that, so that, so what does this function need to satisfy? So it's going to be a function F satisfying that if I were to take its derivative, so if I took the derivative of this at some X , okay, then I , this would actually give me this function F of X , right, so this would work for all X . So right, this is the X , this is the X . And this would work for all X inner interval. So for all X that are in our interval I okay? So as long as I 'm between A and B , so as long as I 'm in this interval, then if I take the derivative of this function, I get the same thing as the original function, which is why it's the antiderivative. Okay, so let's do a little bit of practice. So let's try the antiderivative. So here's one practice.

Okay, so I want to find this antiderivative, antiderivative of, and my function is going to be cosine of X . Okay? So I want to function so that when I take its derivative, I get cosine of X . The first one that probably comes to your mind is sine of X . So that's an okay answer. But here's kind of the fun part, right? I could also get sine of X plus 1, by which I mean like the plus 1 is on the outside. I could take a sine of X minus π , sine of X plus $3E$ squared, right? I could add any constant and it would work that if I took the derivative of that, I would get cosine of X . Okay? And this is why we say that, like when I talk about an antiderivative, I 'm talking about an antiderivative. Let's try one more. So let's try the,

we want to find the antiderivative of, this is one more, it's also a little bit trickier. So actually, we're going to build up to it. So I'm going to do the antiderivative. This one won't be as tricky yet. Right, so if I take the antiderivative of E to the X , right, what function when I take its derivative gives me E to the X ? Well, E to the X , or E to the X plus anything, okay? So this one's not bad at all. So this is just E to the X plus the square root of π , doesn't matter. Okay, so now we'll do the slightly trickier one. It's good to warm up to trickier things, make sure we have all the pieces in our mind. Okay, so what is the antiderivative of? So the antiderivative of, and then we make this slightly trickier because we go E to the $3X$. Okay.

Now you're first going to want to say, E to the $3X$, right, because of that one that you just saw. And then you have to think for a moment and you have to be like, wait, but when I take the derivative of E to the $3X$, I have to apply the chain rule, which means I end up with a 3 in front. Okay, so I'm going to show you what the answer is. And then we'll go back through and check, you can kind of see why that's what it has to be. So maybe $1/3$, E to the $3X$. Or, right, $1/3$ E to the $3X$ minus $1/2$, doesn't matter. Okay. So let's give this a try. So what happens when I take the derivative with respect to X ? So let's test. Right? So if I take the derivative with respect to X of this E to the $3X$, what happens? Well, and if I have a minus $1/2$, then constant just becomes zero, right? So that one doesn't matter. Well, this is going to equal, right, I have, it's like, this is E to the, so I have the derivative with respect to U of E to the U times the derivative with respect to X of $3X$.

Okay? And if I keep going with this, this is just going to give me EU times, well what's that derivative, it's 3. So I've got, moving the 3 to the front. Oops, okay, so I have $3E$ to the $3X$. Okay? So that would have meant that I've got to put the $1/3$ in there. Okay? So if I actually put the $1/3$ in there, let's do this kind of artificially, so you can see. So if I put the $1/3$ in here, right? $1/3$, actually, let's not put it inside here, put it on the outside here, $1/3$. Right, and then I put a $1/3$ here, well, all of a sudden, the $1/3$ and the 3 cancel, and so I get out what I wanted, which is E to the $3X$. Okay, so you can kind of see why in such a circumstance, so this is like something to be careful about, like if there's a chain rule or something like that, when you undo like, to undo differentiation, you're probably going to end up with something in front or something like that. And this is why I'm going to teach at some point substitution. Because the, I'm not teaching too many integration techniques, but this one in particular, I think, is really important. Because it shows you like the importance of undoing the chain rule that these really are opposites of each other. And you really need to undo them properly. Okay? And I kind of want to highlight, right, you might have seen, have some guesses here. But I want to highlight something that's going to happen next time. Which is this question of, like, we've shown there can be a whole bunch of antiderivatives and it works to add any constant. So you might start to get hints about this is how are the different antiderivatives?

So the different antiderivatives for a function related? Okay, with this having in mind, we can at least add a constant, okay? So what is this this antiderivative, this is this new thing that kind of undoes differentiation. And so, and it's usually good to test for yourself that if I took the derivative of sine of X , I will get cosine of X . So this is an antiderivative. That one's easy to see. If I took the derivative of this, I would get E to the X . So you can test that something actually is an antiderivative. And then the last thing that's important is that if there was a chain rule in there, you're going to have to undo that. So watch out for that. Okay, so I hope that made some sense, and I'll see you in the next lecture where we actually address this question.

