

OPTICAL POLARIZATION STUDIES OF LATEX BEADS IN
AQUEOUS SOLUTION: AN ANALOG FOR RADAR
SCATTERING IN A WATER-ICE MEDIUM

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Abstract

This study presents low phase angle $0^\circ - 5^\circ$ measurements of polarimetric properties of icy planetary regolith analog materials acquired using the custom-built Multi-Axis-Goniometer-Instrument (MAGI). We present same sense (SC), and opposite sense (OC) backscatter circular polarization coefficients, circular polarization ratio (CPR), and degree of linear polarization (DLP) of spherical latex (non-dye) beads of varying sizes and volume concentrations (v/v) in aqueous solutions ($\lambda = 0.8 \mu m$) in water. We leverage Mie scattering calculations to accurately simulate the polarization behavior of light in an aqueous solution of latex beads. We also present measurements of alumina powder in air at $\lambda = 1.064 \mu m$. Measurements showed that at a low incidence angle ($i = 0^\circ$), backscatter is dominated by surface specular single-bounce scattering, which hides other scattering processes. At high ($i = 15^\circ$) incidence angle, surface single-bounce surface scattering becomes negligible, allowing for the detection of diffuse, dihedral (multiple bounces) scattering. We find that classical Mie alumina particles ($2.1 \mu m, 4.0 \mu m$) enhance subsurface scattering due to a larger void space relative to larger Mie particles ($30 \mu m$), which cause the radar signal to scatter forward off small imperfections, maintaining the polarization properties of the signal and generating high > 1 CPR . Latex beads, representing impurities, demonstrate the impact of isotropic and anisotropic scattering on radar signatures. Experiment and model found that the scattering medium's anisotropy correlates to the size of the beads, while the void space of the medium inversely correlates with the bead size and the volume concentration (v/v) of the beads. Model and measurements show that Rayleigh-sized beads (impurities), due to isotropic scattering from the reduced scattering cross-section and higher transparency relative to larger impurities, generate subsurface single bounce scattering, producing $OC \gg SC$ and a low (< 0.5) CPR and across all v/v with SC , and CPR proportional to v/v , but with OC , and DLP inversely proportional to v/v . Model of Rayleigh-sized beads (impurities) has increased modeled transparency that results in more simulated single-bounce scattering relative to experiment. Conversely, model and measurements of classical and large Mie beads show anisotropic scattering that intensifies scattering in the forward direction with high CPR , inversely proportional to the volume concentration with discrepancies between $4^\circ - 5^\circ$ relative to model.

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Related publications

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Antoine Bourget, Parvathy Prem, David Bleweet, Mike Daly. *[Bourget et al., 2023]* 54th Lunar and Planetary Science Conference, page 2823, the Woodland, Texas.

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Chapter 1

Introduction

1.1 Ices in the Solar System

This work will be primarily focused on water-ice, more specifically on water-ice analogs. The objective of the proposed work is to increase understanding of the factors that give rise to the unusual radar characteristics of icy regoliths. We did conduct a series of optical analog measurements designed to explore key variables that control the backscattering of electromagnetic radiation from such icy deposits 6. We aim at understanding the surface and subsurface scattering properties of suspended latex beads in aqueous solution (water) by varying the size and the volume concentration of the beads and, measuring the same sense (*SC*), opposite sense (*OC*), circular polarization ratio (*CPR*) and the degree of linear polarization (*DLP*) at low $0 - 5^\circ$ phase angle in the context of understanding radar scattering in water-ice mediums. By varying the size 6.2 and the volume concentration 6.3 of the bead individually, we can understand how they affect surface and subsurface scattering independently. This study can help us understand radar scattering in water ice, water-ice is transparent at radar wavelength ($1.25 - 2.5\text{ m}$) in air, but impurities (silicate, metals, void space, liquid water) are not and will scatter the radar signal. Liquid water under $1.064\mu\text{m}$ (NIR) is transparent has a an index of refraction of $m_r = 1.33 + 10^{-6}$ [Segelstein, 1981], resulting in a relatively low loss tangent ($10^{-6} - 10^{-5}$) that is similar to pure water-ice under ice penetrating radar. The latex beads are not transparent at visible and NIR wavelengths and will act as scatterers in a similar manner to impurities in

a water-ice medium that scatter the radar signal. By working with aqueous suspensions of micro-beads, illuminated with a right circularly polarized (*RCP*) beam, we are able to conveniently isolate and analyze important aspects of the phenomenon. We will examine the following variables: size 6.2, 6.2.2, the number density of scatterers (C) $\frac{\text{beads}}{\text{m}^3}$ 6.3, the absorption properties of the scatters themselves 6.4. Using a unique laboratory facility capable of making reflectance measurements at phase angles at and around zero (i.e., direct backscatter conditions, also known as opposition) in 5, we will map out the effects of these variables (bead size and volume concentration) on the strength and polarization of the received signal as a function of phase angle. Our comprehensive exploration of the parameter space will provide a framework for interpreting planetary radar observations of ice-bearing and potentially ice-bearing deposits 7. This will help to constrain factors such as the purity of the ice and the kind, abundance, and size distribution of scatterers responsible for coherent effects 7.1.3, 7.1.4, 7.1.5. We combined and compared the results of our laboratory analog investigation with an electric field Monte-Carlo (EMC) simulations from Sawicki et al. [2008] and written by Parvathy et al. The EMC has been integrated into a Python workflow, employing Mie scattering outputs in 3.1.1, 3.1.2 based on Mätzler [2002] to simulate the propagation of right circularly polarized (*RCP*) light at $0.8\ \mu\text{m}$ through suspended latex beads in an aqueous solution in 6.5. We test the EMC simulation in 3.2.7. The EMC simulation has been modified to account for the photon reflecting off our sample platform in 3.2.3.

For more information about the acronyms, refer to Table 9 from .9. *SC*, *OC*, *CPR*, and *DLP* are derived from Stokes parameters in 2.3 with their interpretation in 2.4.1, 2.4.2, and 2.5. Stokes parameters can be expressed as a vector that represents the polarization state of light that arises from polarized electromagnetic wave 2.2-2.3. Previous studies (Nelson and Hapke et al.) have 2 major limitations: first, the apparatus used in Nelson et al. [2000, 2002, 2018], and Hapke [1990] does not permit 0° phase angle measurements due to obstruction between the source and detector. Second, Nelson et al. and Hapke et al. did not study the effect of varying volume concentration in Nelson et al. [2000, 2002, 2018] and varying bead size in Hapke [1990]. Moreover, a similar experimental investigation at radar frequencies would be prohibitively cumbersome and expensive.

In the solar system, Mercury, the Moon, Ceres, have small axial tilts (0.03° , 1.5° , and 4°), and large variations in their topographic relief near the poles. This makes the presence of permanently shadowed regions (PSR) possible at high latitudes [McGovern et al., 2013]. The presence of water-ice deposits within the polar cold traps of the Moon has been a contentious issue since the mid-1990s due to the lack of direct ground observations and measurements [Campbell et al., 2006, Stacy et al., 1997, Simpson and Tyler, 1999]. The icy Galilean satellites of Jupiter and the permanently shadowed polar craters on Mercury, and the Moon have strong and highly polarized radar returns that are attributed to a coherent backscatter effect caused by scatterers or void spaces embedded within ice-bearing deposits. Controversy exists regarding interpretation of radar data for the Moon and the possible presence of icy deposits in lunar polar craters. On Mars, water-ice can be found in polar ice caps, subsurface deposits, and surface frost [Laskar et al., 2002]. It can be transparent, especially if it's pure and recently deposited. However, the ice's transparency is reduced when mixed with dust. Factors like low atmospheric pressure, lower temperatures, and different ambient radiation conditions on Mars can also affect the ice's crystal structure and optical properties. Europa, Ganymede, Callisto and Enceladus are icy satellites, and are also known to have surface water-ice. The distribution and abundance of ice on terrestrial bodies can provide valuable insights into the solar system history, and has implication for potential in situ resource utilization. Nelson et al. [2000, 2002, 2018] conducted a series of experiments to understand the reflectance and polarization characteristics of aluminum oxide powders with particle sizes across different scales relative to the wavelength of incident light. These powders served as analogues for planetary regoliths found on atmosphere-less solar system bodies (ASSBs), such as icy satellites and certain asteroids. In experiments spanning from 1998 to 2018, they used a goniometric photopolarimeter to measure how the reflectance and circular polarization ratio of these powders changes with the phase angle, specifically within a small range of $0.05 - 15^\circ$, where the effects of coherent backscattering are presumed to be significant. The measurements, conducted at laser wavelengths of $0.633\ \mu\text{m}$ and $0.543\ \mu\text{m}$, revealed that the reflectance depended on particle size and that the reflectance and circular polarization ratio (*CPR*) increased at low phase angles, which is consistent with the coherent backscattering opposition effect

(CBOE in Figure 2.2). In 1991, to explain the unusual radar characteristics of icy satellites of Jupiter, Hapke and Blewett [1991] proposed a coherent backscatter model for high CPR based on radar waves traveling in opposite directions in a weakly absorbing medium (such as water ice) that add coherently, resulting in high CPR . Laboratory experiments using polystyrene beads suspended in water and laser light validated this model, replicating the icy moons' radar signal characteristics - suggesting that the icy moons' surfaces are made of a transparent matrix with embedded scatterers (void spaces) roughly the size of the radar wavelength. To predict the reflectance phase behavior of outer solar system bodies, Mishchenko and Dlugach [1992] used a gamma distribution for the number density (n) as a function of the particle radius (r). Formally,

$$n(r) = Kr^{\frac{1-3b}{b}} e^{-\frac{r}{ab}} \quad (1.1)$$

such that K is a normalization constant, $a = R_0$ (central particle radius), and has domain $[0.5R_0, 2R_0]$ and $b = 0.04$. This distribution was later used by Nelson et al. [2000, 2002, 2018], Hapke and Blewett [1991] to provide a better understanding of the icy moons' subsurface composition and the scattering processes affecting radar and optical observations in the solar system. The dielectric properties of water-ice play a crucial role in how ice-penetrating radar (IPR) signals behave. These properties determine how much of the radar signal is reflected, absorbed, or transmitted through the water-ice. The index of refraction m can be written as $m = m_r + i\kappa$, where m_r is the real part, and κ is the imaginary part of the index of refraction associated the amount of attenuation when the electromagnetic wave propagates through the scattering medium [Hecht, 2002]. The loss tangent is $\tan(\delta) = \frac{\kappa}{m_r}$ and represents the amount of attenuation (loss) within scattering medium [Hecht, 2002]. Pure water-ice has a relatively low loss tangent ($10^{-4} - 10^{-5}$) at radar frequencies, that can be affected by the temperature of the ice and the frequency of the radar instrument. This low loss tangent value means that radar waves can penetrate deeply into pure ice. Under ice penetrating radar wavelength, liquid water has a much higher dielectric constant, around 80 at 273K [Callister Jr and Rethwisch, 2020]. This high value causes a strong reflection of the radar signal at the interface between ice and water, which is useful for detecting subglacial lakes or the bottom of an ice sheet, or

cracks (void spaces). When ice contains liquid water (as in temperate glaciers or the bottom layers of polar ice sheets) or cracks (void spaces), its dielectric constant increases resulting in greater loss tangent than ice with no liquid water or void spaces, reducing the penetration depth of the radar. Generally, pure ice has low tangent loss, allowing the radar waves to travel through it with minimal attenuation. However, impurities in the ice (like volcanic ash or dust) and void spaces can increase dielectric losses, scattering and absorbing more of the radar signal and reducing penetration depth due to the interfaces (boundaries) between the two mediums. The polarization of the radar wave can also be affected by the ice structure. For example, anisotropic ice crystal structures can lead to anisotropic dielectric properties, which will affect the polarization of the reflected radar signal. Moreover, the contrast in dielectric properties at interfaces or boundaries (like ice-water, ice-rock boundaries or void spaces) leads to reflections of the radar signal. These reflections are crucial for determining the structure and depth of the ice.

1.2 Water-Ice on Mercury

Mercury is the closest planet to the sun in our solar system, with an average density of $5.43g/cm^3$, and an equatorial radius of $2439.7\ km$. It orbits at about $4.9 \times 10^7\ Km$ ($0.4\ AU$) away from the sun, and it is in a 2 : 3 resonance orbit with a year lasting 88 earth days [Phillips W. Davis, October 19, 2021]. Previous observations of the poles of Mercury with Earth-based radar revealed a signature characteristic of thick, pure ice deposits. Later, MESSENGER, the Mercury Surface, Space Environment, Geochemistry and Ranging spacecraft, imaged these ice deposits. We showed Mercury’s polar deposits to be dominantly composed of water ice and extensively distributed in both Mercury’s north and south polar regions [Rubanenko et al., 2019]. Furthermore, Mercury has a small axial tilt (0.03°) and large variations in its topographic relief near the poles, making the presence of permanently shadowed regions (PSR) possible at high latitudes (polar regions) [McGovern et al., 2013]. Permanently shadowed polar craters on Mercury have strong and highly polarized radar returns that are attributed to a coherent backscatter effect (CBOE in Figure 2.2) caused by scatters (cracks or void spaces) embedded within ice-bearing deposits [Vasavada et al., 1999]. Moreover, Mercury, like Venus, the Earth, and Mars, was

heavily bombarded with water-ice rich comets in the late heavy bombardment (LHB). In addition, Mercury has a magnetic field and is much closer to the sun. Consequently, Mercury is on average much hotter than the moon, at typical temperatures prevailing on Mercury’s dayside surface, H_2O that can be produced from reactions involving OH groups on or within the H-saturated regolith grain interfaces [Jones et al., 2020]. Once produced, H_2O is released into the exosphere and then transported to the poles and processed via Jeans escape, photodissociation, dissociative adsorption, or condensation. Water reaching cold traps will be bound over geological periods [Jones et al., 2020]. Permanently shadowed craters are very cold, only receiving energy from high-standing topography, background stars, and from interior geothermal flux. Therefore, water ice mixed within the regolith may be stable for billions of years [Watson et al., 1961]. Water is of scientific interest because it is an essential ingredient for life and can be extracted to make fuel through hydrolysis.

1.3 Water-Ice on the Moon

The Moon is earth’s biggest natural satellite, with a density of $3.34g/cm^3$, and an equatorial radius of $1.737.5 \times 10^3 km$, orbiting the earth at about $3.85 \times 10^5 Km$, it takes on average 27 earth days for the Moon to orbit the earth [Phillips W. Davis, August 9, 2021]. It is also tidally locked, meaning that the period of rotation matches the orbital period of the Moon, thus the same side faces the earth. Radar observations of polar craters of the Moon reveals unusually high CPR , the possible existence of water-ice deposits in the polar cold traps of the Moon has been a controversial issue since the mid-1990s [Campbell et al., 2006]. First, as a result of the combination of the large variation in surface topography and the small inclination angle between the lunar equator and the ecliptic plane (1.5°), crater floors at the lunar polar regions are permanently shaded from the Sun [Bussey et al., 2005, Mazarico et al., 2011]. Receiving heat only from space and the interior of the Moon. This forms cold traps with temperature as low as $40 K$ [Vasavada et al., 1999, Paige et al., 2010]. Second, over the long geological history of the Moon, significant quantities of water could accumulate in these cold traps due to water-bearing cometary debris and meteorites striking the Moon [Fa and Cai, 2013]. Radar is regarded as one of the most

effective tools for detecting water ice, because as a low dielectric loss medium, water ice could produce coherent backscatter opposition effect (CBOE in Figure 2.2) under certain conditions [Hapke, 1990, Peters, 1992]. Those conditions are related to the size, absorption and concentration of the scatterers embedded within ice-bearing deposits (icy regoliths). Our current understanding of PSRs on the moon come from 3 missions, the first one being Clementine spacecraft that conducted a bistatic radar experiment in 1994. Where the spacecraft transmitted a 13.2 *Cm* signal from its high-gain antenna toward the lunar south pole [Nozette et al., 1996, Weidenschilling, 1997], and received the signal on the Earth at bistatic angles ranging between 0° and 5° . Targets that exhibit the coherent backscatter opposition effect, such as water ice, will have an opposition surge at $\beta = 0^\circ$, with the signal decreasing sharply at angles greater than 1° . A return of this nature was reported over Shackleton crater, but the inference of ice was not supported by other workers in their interpretation of subsequent, higher resolution polarimetric imaging of shadowed crater floors. The controversy is reinforced further due to the lack of direct ground observation and measurement (ground-truth) [Stacy et al., 1997, Simpson and Tyler, 1999], and there is still disagreement whether the observed polarization anomalies are due to ice [Campbell et al., 2006]. The second and third missions were Mini-SAR instrument on Chandrayaan-1 in 2008 and Mini-RF on LRO in 2009 allowed for high-resolution, monostatic imaging of the lunar poles [Neish and Carter, 2014]. The VIPER rover built by NASA is scheduled to launch by September 2025 for a 100-Earth-day mission [Chen, 2020]. It should be capable of traversing an incline of $\sim 15^\circ$ and be telerobotically driven at about 0.5 *mph* (0.8 *Km* per hour). The rover will carry a 1 *m* drill, The Regolith and Ice Drill for Exploring New Terrains (TRIDENT), and three science instruments: a Neutron Spectrometer System (NSS), a Near-Infrared Volatiles Spectrometer System (NIRVSS), and a Mass Spectrometer Observing Lunar Operations (MSolo) [Lemelin et al., 2021]. The VIPER rover and Lunar Reconnaissance Orbiter (LRO) would provide both radar and ground-direct measurements of the lunar surface, thus making our understanding of lunar bright radar deposits more complete. Of the permanently shadowed regions identified there, some do appear to have enhanced *CPRs*, but many others do not. Complicating matters, the presence of high *CPRs* does not necessarily coincide with the presence of water-ice and, can be explained

through other mechanisms, such as dihedral scattering, that do not require the presence of water ice [Neish and Carter, 2014]. Where the results from these 3 missions seem to indicate that CPR greater than unity is not a unique signature for diagnostic of water ice. Geologic targets, such as lunar crater deposits, lava flow, and a rocky surface, can have CPR as high as 2 to 4 [Campbell, 2012].

1.4 Water-Ice on Mars

Mars is the forth planet in our solar system, with a density of $3.93gm/cm^3$, and an equatorial radius of $3.390 \times 10^3 km$, orbiting at about $2.299 \times 10^8 Km$ ($1.5 AU$) away from the sun, Mars takes on average 687 days to complete a full orbit. Moreover, a martian day is 24.6 hours almost the same as on earth, Mars has a 25° tilt axis which is slightly greater than Earth's 23.4° tilt [Phillips W. Davis, March 17, 2022]. Due to the absence of a substantial stabilizing moon, the axial tilt of Mars has varied greatly over the course of its history. Mars undergoes oscillations in obliquity, perihelion, and eccentricity that strongly affect the planet's climate [Laskar et al., 2002]. This variability determines the amount of sunlight that reaches the surface and in turn the stability of ice at all latitudes, creating pronounced climate cycles as on Earth [Levrard et al., 2007, Head et al., 2003]. Mars had flowing water, a thick atmosphere and a strong magnetic field 4.5 – 3.5 billion years ago, however being only $\frac{1}{10}$ the mass of earth and, half half the diameter of earth, the martian outer core cooled down much quicker than earth's core therefore shutting down the dynamo effect and causing the magnetic field to collapse. Furthermore, during the late heavy bombardment 4.1 – 3.8 billion years ago, a lot of giant impacts occurred forming large basins such as Hellas and Argyre Planitia. Those large impact would have demagnetized surface rocks and heat up the mantle, thus diminishing the temperature gradient between the outer core and mantle, hence shutting down convection between the outer core and mantle [Rochette et al., 2003]. After 500 million years, the martian atmosphere is substantially destroyed, all due to the disappearance of Mars global magnetic field [Shi, 2016]. Because of the loss of its magnetic field Mars' atmosphere is extremely thin (150 times less dense than earth's atmosphere) making the presence of vast bodies of liquid water impossible. However, not all the water and atmosphere has been lost to

space, a chunk of that atmosphere is now stored as CO_2 ice at the north and south poles of Mars, but only represents 1% of the mass, the rest being pure water-ice and water-ice mixed with impurities (dust). Unlike the south pole where CO_2 ice is a permanent feature, CO_2 ice only exists transiently at the north pole due to seasonal variation and is not a permanent feature. Moreover, CO_2 ice, an atmospheric constituent of Mars that seasonally covers latitudes from $\sim 50^\circ$ to the poles in each hemisphere. The water-ice distribution on Mars is especially high at the north pole, this because of the basin Vastitas Borealis which might have been home to a large ancient ocean. The Vastitas Borealis formation, located inside a putative shoreline of the ancient ocean, has a low dielectric constant (low $\tan(\delta)$) compared with that of typical volcanic materials [Mouginot et al., 2012]. The north pole of Mars is part of an immense sedimentary basin that records over 3 billion years of deposition and erosion of ice, dust, lava, and other lithic material of varying composition and includes the plateau of Planum Boreum, covering an area of $\sim 800,000 km^2$ [Phillips et al., 2008]. The north polar layered deposits (NPLD) of Mars contain the second largest surface reservoir of water ice on the planet, covering a quasi-circular region $\sim 1,000 Km$ in diameter and up to $2.0 km$ thick [Phillips et al., 2008, Tanaka, 2005]. Where beneath a thin residual cap of bright, nearly pure water ice, the north polar layered deposits (NPLD) [Blasius et al., 1982] make up the bulk of Planum Boreum and are probably the result of cyclical variations in the orbit and rotation of Mars that affected insolation at the poles and elsewhere. In addition, it is established that water-ice mixed with dust particles and containing cracks or void spaces (scatterers) generate high and strong shadow radar returns found by the SHARD instrument (shadow radar) on the MRO (Mars Reconnaissance orbiter). Analysis of SHARD data at 10 different study sites, found that some layers are a lot dustier than we previously believed, meaning our previous understanding of the Mars ice and dust cycles is incomplete. It was also found that the composition of layers was very different from place to place, which means local conditions are playing a large role in ice and dust accumulation [Lalich et al., 2019]. Additionally, Mars has a South Polar Residual Cap (SPRC) that persists throughout the warmest months on the planet [Isen and Smith, 2022]. The south pole of Mars, is among the most dynamic regions on Mars, hosting a large number of sublimation features which are flat floored, circular depressions embedded

in the ice and dubbed "Swiss Cheese Terrain" [Thomas et al., 2000, Malin and Edgett, 2001, Milkovich and Plaut, 2008]. It occurs in the southern highland and $\sim 4\text{ km}$ above the surrounding cratered terrain [Barlow, 2014, Byrne and Wolch, 2009]. The south polar area is notably higher in elevation than its northern counterpart which rests on topographically low plains [Carr, 1996, Zuber et al., 1998]. Due to the large eccentricity of the martian orbit 0.1 (compared to the Earth 0.02) and the coincidence of its apocenter and southern winter, winters in the southern hemisphere are longer and colder which possibly lead to the south seasonal CO_2 ice cap being more extensive $\sim 45^\circ S$ than that of the north pole $\sim 55^\circ N$ [Kieffer, 1979, Titus et al., 2003, Byrne and Wolch, 2009, Carr, 1996, Langevin et al., 2007]. The south polar area is basically composed of H_2O ice and CO_2 ice with dusts and cracks (void spaces) that are embedded and act as scatterers [Leighton and Murray, 1966, Kieffer, 1979, Bibring et al., 2004, Barlow Nadine, 2008]. Interaction of these components with the atmosphere causes the polar area to undergo distinct changes over martian seasons. While the south polar area is a result of conditions and changes of Mars climates, it plays an essential role in the global energy budget and balance of Mars [Hao, 2020]. Hence Mars, especially at north north and south poles contains scatterers (dusts, cracks or void spaces) embedded within water ice-bearing deposits, even if it does not feature permanently shadowed craters as extensively due to its large axial tilt relative to Mercury, the Moon, and Ceres.

1.5 Water-Ice in the Asteroid Belt

The asteroid belt spans a region $1.40 \times 10^8\text{ Km}$ across (1.5 Au) and, has a mass of just $\sim 4\%$ that of our Moon. Where Ceres, Pallas, Vesta and Hygie are the largest known body by mass and radius, with a equatorial radius of 476 Km , 272.5 Km , 262.5 Km , 222 Km respectively. Furthermore, the asteroid belt has been heavily influenced by the gravitational influence of Jupiter and Saturn, which cleared the belt of most of its mass, hence preventing the formation of any substantial terrestrial planets. It is thought that the Jupiter-Saturn grand tackle prevented the formation of any super-earth in the solar system. Ceres is a dwarf planet, with a density of 2.08 g/cm^3 , having a radius of 476 Km and orbiting the sun at a distance of $4.05 \times 10^8\text{ Km}$ (2.8 AU), Ceres taken on average 4.6 earth

years to complete a full orbit. Ceres is more similar to the terrestrial planets (Mercury, Venus, Earth, and Mars) due to its differentiated interior than its asteroid neighbors, but it is much less dense. One of the similarities is a layered interior, but Ceres' layers aren't as clearly defined. Ceres probably has a solid core and a mantle made of water ice. In fact, Ceres could be composed of as much as 25% water. If that is correct, Ceres has more water than Earth does. Ceres' crust is rocky and dusty with large salt deposits. The salts on Ceres aren't like table salt (sodium chloride), but instead are made of different minerals like magnesium sulfate. Ceres is covered in countless small, young craters, but none are larger than 175 miles (280 *Km*) in diameter. This is surprising, given that the dwarf planet must have been hit by numerous large asteroids during its 4.5 billion-year lifetime. The lack of craters might be due to layers of ice just below the surface. The surface features could smooth out over time if ice or another lower-density material, such as salt, is just below the surface. It's also possible that past hydrothermal activity, such as ice volcanoes, erased some large craters. Within some of Ceres' craters, there are regions that are always in shadow. It's possible that without direct sunlight, these "cold traps" could have water ice in them for long periods of time [Phillips W. Davis, August 5, 2021]. Ceres is of particular interest to scientist due its radar bright properties. Ceres has only a small spin axis tilt 4° higher than the Moon and Mercury, and craters near its rotational poles can experience permanent shadow and trap volatiles (cold traps), as is the case on Mercury and on Earth's Moon. Thus allowing the accumulation of water-ice over geological time scale. It is estimated that in the northern hemisphere PSRs cover $\sim 1800 \text{ km}^2$ or 0.13% of the region, oscillating seasonally. Current modeling shows that water molecules seasonally reside around the winter pole and ultimately an estimated 0.14% of molecules get trapped. Even for the lowest estimates of the amount of available water, this predicts accumulation rates in excess of loss rates, and hence, there should be fresh ice deposits in the cold traps. The relative PSR area is similar to Mercury, but unlike Mercury most of these areas are cold enough to serve as efficient cold traps for water ice [Schorghofer et al., 2016]. The water-ice and other volatiles could be mixed with embedded scatteters of various sizes and absorptions and thus generating highly polarized radar return attributed to a coherent backscatter effect (CBOE in Figure 2.2). Moreover the studies show that

Ceres’s axial title varies greatly over time due to the gravitational influence of Jupiter. Indeed, the obliquity of Ceres is shown to vary between $\sim 2^\circ$ and $\sim 20^\circ$ with a period of $\sim 2.5 \times 10^3$ yr, the pattern is oscillatory and does not exhibit chaotic behavior, therefore ensuring a stable environment for the formation of ice deposits inside PSR area. At such high obliquity, most of the present-day PSRs receive direct sunlight. However, PSRs still exist at an obliquity of 20° . The current obliquity ($\sim 4^\circ$) thus leads to significantly more “PSR” area than what could be considered typical [Ermakov et al., 2017].

1.6 Water-Ice on the Jovian Moons

The Galilean moons are the 4 biggest Jovian satellites located at $5.2 AU$ away from the sun; they are: Io, Europa, Ganymede, and Callisto with a density of $3.53g/cm^3$, $3.01g/cm^3$, $1.94g/cm^3$, $1.83g/cm^3$ respectively. Moreover, Io, Europa, Ganymede, and Callisto have an equatorial radius of $1.8216 \times 10^3 km$, $1.5608 \times 3 Km$, $2.6312 \times 10^3 km$, $2.4103 \times 10^3 Km$ respectively. They orbit Jupiter at a distance of $\sim 4.20 \times 10^5 Km$, $6.65 \times 10^5 Km$, $1 \times 10^6 Km$, $1.8 \times 10^6 Km$, with an orbital period of 42, 84, 172, and 408 hours respectively [Phillips W. Davis, October 30, 2021]. Galilean satellites, especially Europa, Ganymede, and Callisto, are of scientific interest because they are beyond the snow line of the solar system, and they have a small axial tilt ($0.1^\circ, 0.33^\circ, 0^\circ$), making the presence of icy regoliths very abundant. The snow line is $2.7 AU$ [Hayashi, 1981] away from the sun and is defined as the distance at which it is cold enough for water to condense into solid ice grains [Phillips W. Davis, October 30, 2021]. Current research suggests that icy satellites generally have high radar reflectivities and high circular polarization ratios (*CPRs*). This indicates that the radar scattering is dominated by subsurface volume scattering, not single scattering from the vacuum-surface interface, as is typical for rocky bodies. This scattering behavior has been modeled as a coherent backscatter effect resulting from scatterers (cracks or void spaces) embedded in the weakly absorbing water ice surfaces of these moons [Neish and Carter, 2014].

1.6.1 Europa

Europa is the second closest Galilean moon to Jupiter, at a distance of $6.71 \times 10^5 \text{ Km}$ away from Jupiter. Europa has, like our planet, an iron core, a rocky mantle, and an ocean of salty water. Unlike Earth, however, Europa's ocean lies below a shell of ice probably $15 - 25 \text{ Km}$ thick, and has an estimated depth of $60 - 150 \text{ Km}$. Despite the strong evidence, the presence of an internal ocean has been a controversial issue, but in the mid 1990s the magnetometer on the Galileo spacecraft confirmed the presence of an ocean through magnetic measurements of Europa induced magnetic field. One of the most important measurements made by the Galileo mission showed how Jupiter's magnetic field was disrupted in the space around Europa. This measurement strongly implied that a special type of magnetic field is being created (induced) within Europa by a deep layer of some electrically conductive fluid beneath the surface. Based on Europa's icy composition, scientists think the most likely material to create this magnetic signature is a global ocean of salty water [Phillips W. Davis, 2019]. Based on the small number of observable craters, the surface of this moon appears to be no more than 40 to 90 million years old, which is youthful in geologic terms (the surface of Callisto, another of Jupiter's moons, is estimated to be a few billion years old). In addition, Europa's water-ice surface is crisscrossed by long, linear fractures like cracks, making the presence of void spaces embedded within the water-ice very likely. The cracks and ridges on Europa's surface are primarily caused by the intense tidal forces exerted by Jupiter. As Europa orbits Jupiter, the gravitational pull from the giant planet causes the moon's ice shell to flex and stretch. This tidal flexing generates stress within the ice, leading to the formation of long, linear fractures, ridges, and bands [Phillips W. Davis, 2019]. Researchers have found that the parallel ridges on Europa's surface may be formed by the refreezing of shallow pockets of liquid water within the ice. These ridges suggest the presence of void spaces and cracks within the ice, which are crucial for understanding radar scattering [Culberg et al., 2022]. Cracks and void spaces within the ice can cause multiple scattering of radar waves. When radar signals penetrate the ice, they can reflect off various internal structures such as cracks and voids, leading to complex scattering patterns. This multiple scattering increases the radar cross-section and can produce strong backscatter signals, which are observed as abnormal radar echoes. The

presence of subsurface voids or cracks can create dielectric discontinuities at boundaries or interfaces between the two mediums 1.1. These discontinuities can enhance radar reflections because radar waves reflect strongly at boundaries (interfaces) where there is a significant change in the dielectric properties. The water-ice surrounding the voids or cracks would have different dielectric properties compared to the voids themselves, resulting in strong echoes [Culberg et al., 2022]. In a medium with many small scatterers, such as a fractured ice surface, radar waves can undergo coherent backscatter. This effect occurs when waves scattered in the backward direction by the scatterers interfere constructively, leading to enhanced radar returns. Cracks and voids within the ice can create conditions suitable for coherent backscatter, contributing to the abnormal radar echoes. A rough surface or a subsurface with irregularities such as cracks can depolarize the incident radar signal, increasing the observed polarization ratio. The depolarization occurs because the rough or fractured ice surface causes multiple reflections and scattering, altering the polarization state of the radar waves [Culberg et al., 2022]. All along Europa's many fractures, and in splotchy patterns across its surface, is a reddish-brown material whose composition is not known for certain, but likely contains salts and sulfur compounds that have been mixed with the water ice and modified by radiation. This surface composition may hold clues to the moon's potential as a habitable world. NASA's Galileo spacecraft explored the Jupiter system from 1995 to 2003 and made numerous flybys of Europa. Galileo revealed strange pits and domes that suggest Europa's ice layer could be slowly churning, or convecting (cooler, denser ice sinks, while warmer less-dense ice rises) due to heat from below. Long, linear fractures are often only $1 - 2 \text{ Km}$ wide but can extend for thousands of kilometers across Europa's surface. Some of these fractures have built up into ridges hundreds of meters tall, while others appear to have pulled apart into wide bands of multiple parallel fractures. Galileo also found regions called "chaos terrain," where broken, blocky landscapes were covered in mysterious reddish material. In 2011, scientists studying Galileo data proposed that chaos terrains could be places where the surface collapsed above lens-shaped lakes embedded within the ice Phillips W. Davis [November 4, 2021].

1.6.2 Ganymede

Ganymede is the third Galilean moon away from Jupiter, at a distance of $1.07 \times 10^6 \text{ Km}$ from Jupiter. Ganymede has three main layers: A metallic iron core at its center, a spherical shell of rock (mantle) surrounding the core, and a spherical shell of mostly ice surrounding the rock shell. The surface is the very top of the ice shell. Scientists have discovered irregular lumps beneath Ganymede's icy surface. These irregular masses may be rock formations, supported by Ganymede's icy shell for billions of years. Ganymede's interaction with Jupiter's strong gravitational field can also play a role in its geological features. Tidal forces can induce stress within the ice shell, potentially leading to the formation of cracks and voids. These tidal interactions can contribute to the complex radar scattering properties observed on Ganymede. Abnormal radar echoes from Ganymede, similar to those observed on Europa, can be attributed to a variety of factors related to its ice shell and subsurface features. Ganymede, like Europa, has an ice shell that is believed to contain a subsurface ocean. The presence of void spaces, cracks, and other subsurface structures within this ice can cause multiple scattering of radar waves. This scattering can lead to high radar backscatter and unusual radar signatures. Studies have indicated that the ice shell on Ganymede may be complex, with a mixture of ice and rocky material, similar to that of a club sandwich [Phillips W. Davis, November 10, 2021]. The interface between these materials can create dielectric discontinuities, which enhance radar reflections. Ganymede exhibits signs of geological activity, including tectonic features such as grooves and ridges. These features are likely the result of past and possibly ongoing tectonic processes, which can create fractures and voids within the ice. These fractures can influence radar signals by providing multiple reflective surfaces [Phillips W. Davis, 2019]. The Galileo spacecraft provided high-resolution images showing complex surface features, which suggest the presence of internal heat and geological activity that could maintain liquid water beneath the ice shell. This internal heat can contribute to the formation of cracks and voids, further affecting radar echoes [NASA, 1998]. Observations at different radar wavelengths (70 cm, 13 cm, and 3.5 cm) from the Arecibo telescope have shown high backscatter and polarization ratios for Ganymede. These ratios indicate that multiple scattering processes, possibly involving both surface and subsurface structures,

are responsible for the radar echoes [Black et al., 2001]. The high polarization ratios observed in radar data from Ganymede suggest the presence of rough surfaces and complex internal structures. These features can depolarize the radar signal and enhance backscatter, leading to the observed abnormal radar echoes [Phillips W. Davis, 2019, NASA, 1998]. Spacecraft images of Ganymede show its surface is a mix of two types of terrain. 40% of the surface is covered by highly cratered dark regions, and the remaining 60% is covered by a light grooved terrain, which forms intricate patterns across the moon. Some of the groove ridges are as high as $2 \times 10^3 \text{ ft}$ (700 m), and the grooves run for thousands of miles across Ganymede's surface. The large craters on Ganymede are mostly flat, probably due to settling in the soft icy surface. Both bright and dark rays of ejecta can be seen around Ganymede's craters. In 2004, scientists discovered irregular lumps beneath the icy surface of Ganymede. The irregular masses may be rock formations, supported by Ganymede's icy shell for billions of years. This tells scientists that the ice is probably strong enough, at least near the surface, to support such rock masses from sinking to the bottom of the ice. However, this anomaly could also be caused by piles of rock at the bottom of the ice [Phillips W. Davis, November 10, 2021].

1.6.3 Callisto

Callisto is the furthest of Galilean moon, at a distance of $1.88 \times 10^6 \text{ Km}$ away from Jupiter, it experiences much weaker tidal forces relative to Europa and Ganymede. Despite Callisto being icy, there are discrepancies in the observed radar echoes from Callisto, especially in the same sense *SC* circular polarization and circular polarization ratio *CPR* [Neish and Carter, 2014, Black et al., 2001]. Callisto's surface is heavily cratered and appears to be older and less geologically active than Ganymede and Europa. The radar echoes from Callisto are generally weaker, which can be attributed to its relatively uniform and less fractured ice surface. The lack of significant internal heat means fewer subsurface structures like voids or cracks that would enhance radar backscatter. In contrast, Europa and Ganymede exhibit more complex and dynamic surfaces with significant geological activity. This activity creates cracks, void spaces, and varied terrain, leading to stronger and more complex radar echoes due to multiple scattering mechanisms [Neish and Carter,

2014]. Callisto's geological activity is minimal compared to Europa and Ganymede. The limited tectonic activity results in fewer surface and subsurface features that would cause strong radar reflections. This is supported by the low radar cross-sections observed at 70 *cm* wavelength for Callisto, indicating less complex surface and subsurface scattering. Both Europa and Ganymede show signs of significant past and present geological activity. Europa's surface is characterized by a network of cracks and ridges, suggesting an active subsurface ocean interacting with the ice shell. Ganymede shows evidence of tectonic features and possible cryovolcanism, contributing to its stronger radar echoes. The radar albedo of Callisto is lower compared to Europa and Ganymede. This lower radar albedo reflects the smoother and more homogeneous nature of Callisto's ice surface. The polarization ratios are also lower, indicating less surface roughness and fewer internal scattering centers. Europa and Ganymede, unlike Callisto, have higher radar albedo and polarization ratios on these moons suggest rougher surfaces and more complex internal structures. The presence of subsurface oceans and geological activity enhances radar backscatter and polarization ratios, leading to the observed discrepancies. Moreover, Callisto's surface is heavily cratered likely by comets and asteroids, and is the oldest at 4 billion years old indicating a lack of recent geological resurfacing. The radar echoes are therefore dominated by surface reflections from craters and impact debris rather than subsurface structures. Europe and Ganymede on the contrary have younger surfaces with fewer craters on Europa and Ganymede indicate more recent geological processes that have created features affecting radar reflections, such as ice tectonics and possible cryovolcanic flows [Neish and Carter, 2014, Black et al., 2001]. Data gathered by the Galileo spacecraft indicate Callisto may have a subsurface ocean and scientists estimate it may be 155 miles (250 *Km*) below the surface. More recent research reveals that this ocean may be located deeper beneath the surface than previously thought, or may not exist at all. If there is an ocean, it may be interacting with rocks, giving Callisto a chance of supporting life. Callisto's interior may have layers of ice mixed with rock and metal, possibly extending to its center. Callisto looks like it's sprinkled with bright white dots that scientists think are the peaks of the craters capped with water ice [Phillips W. Davis, July 19, 2021].

1.7 Water-Ice on the Moons of Saturn

The Saturn system is comprised of Saturn, its rings, and its moons. Saturn orbits the sun at a distance of $\sim 1.5 \times 10^9 Km$, having an orbital period of ~ 29.5 Earth years. Saturn's ring system extends up to $2.82 \times 10^5 Km$ from the planet, yet the vertical height is typically about $30 ft$ ($10 m$) in the main rings. Saturn's rings are thought to be pieces of comets, asteroids, or shattered moons that broke up before they reached the planet, torn apart by Saturn's powerful gravity. They are made of billions of small chunks of ice and rock coated with other materials such as dust [Phillips W. Davis, August 18, 2021]. Early results suggest that Saturn's icy satellites also appear to be dominated by subsurface volume scattering, although their radar reflectivities are somewhat lower than those of the Galilean satellites at $13 cm$. This suggests that the subsurface scattering is operating less efficiently than on the Galilean satellites, most likely due to a difference in composition. A likely candidate is ammonia, since water ice's absorption length is extremely sensitive to ammonia concentration [Neish and Carter, 2014].

1.7.1 Enceladus

Enceladus is the sixth largest moon of Saturn, orbiting Saturn at an average distance of $2.38 \times 10^5 Km$, it has a density of $1.608 g/cm^3$ and an equatorial radius of $2.52 \times 10^2 km$ Phillips W. Davis [December 19, 2019]. For decades, scientists did not know why Enceladus was the brightest world in the solar system, or how it related to Saturn's E ring. Enceladus has an icy and dynamic surface that exhibit abnormal radar echoes, similar to Europa and Ganymede. Enceladus has a highly reflective surface in radar wavelengths, particularly in the southern polar region where the famous "tiger stripes" are located. These stripes are large fractures that emit jets of water vapor and ice particles, making the presence of crack (void spaces) very likely. The presence of these jets and the underlying warm ice, and the presence of cracks (void spaces) contribute to strong radar reflections. The rough and structured nature of the ice in these regions enhances the radar echoes, similar to surfaces known to have coarse-grain ice [Institute, 2011, NASA Science, 2011]. The geological activity on Enceladus, driven by internal heat, is a significant factor in the abnormal

radar echoes. The heat maintains the subsurface ocean and drives the plumes observed at the south pole. The transition from a high pressure-temperature environment to a low pressure-temperature environment sublimates or froze the expelled water, creating the plumes. Depending on the initial velocity of the plumes, parts of it escape into space that forms the outermost ring of Saturn (E-ring), the rest is deposited as snow on the surface Enceladus. This activity creates a dynamic and changing surface, which affects the radar signals. The grooves and fractures (void spaces) formed by the extension and pulling apart of the moon's crust provide multiple scattering surfaces for radar waves [Institute, 2011]. Data from the Cassini spacecraft's synthetic-aperture radar (SAR) provide high-resolution measurements of Enceladus's surface. These data reveal intricate patterns of grooves and fractures, indicating a rough surface on the scale of the radar wavelength ($\sim 2\text{ cm}$). The SAR images show bright central features, suggesting recent geological activity and subsurface heating, which further contribute to the strong radar echoes observed [NASA Science, 2011, Institute, 2011]. Similar to Europa and Ganymede, the radar echoes from Enceladus are influenced by the presence of subsurface structures and the roughness of the ice. However, Enceladus's active plumes and the heat-driven processes make its radar signatures particularly strong and distinctive. This is in contrast to Callisto, which has a more stable and less dynamic surface, resulting in weaker radar echoes [Institute, n.d.]. Cassini found that both the fresh coating on its surface, and icy material in the E ring originate from vents connected to a global subsurface saltwater ocean that might host hydrothermal vents. Enceladus is an active moon that hides a global ocean of liquid salty water beneath its crust. What's more, jets of icy particles from that ocean, laced with a brew of water and simple organic chemicals, gush out into space continuously from this fascinating ocean world. The material shoots out at about 800 mph (400 m/s) and forms a plume that extends hundreds of miles into space. Some of the material falls back onto Enceladus, and some escapes to form Saturn's vast E ring. The plume material that falls back into Enceladus is made out of icy water (snow) mixed with impurities and is a significant driver for the abnormal radar echoes from Enceladus. The E ring is mostly made of ice droplets, but among them are peculiar nanoparticles. Cassini detected these passing silica nanograins, which can only be generated where liquid water and rock interact

at temperatures above $90\text{ }C^{\circ}$ ($\sim 200\text{ }F^{\circ}$). This points to hydrothermal vents deep beneath Enceladus' icy shell, not unlike the hydrothermal vents that dot the ocean floor here on Earth. During a close flyby in 2008, Cassini's instruments sampled the plume directly and detected a surprising mix of volatile gases, water vapor, carbon dioxide and carbon monoxide, as well as organic materials. The density of organic materials was about 20 times denser than expected. The Fountains of Enceladus Cassini images of Saturn's moon Enceladus backlit by the sun show the fountain-like sources of the fine spray of material that towers over the south polar region. Over time, scientists determined that the cracked terrain stretching across Enceladus' active south pole is in constant motion, stretching in some places and buckling in others as the cracks pinch and pull from gravitational forces. It is possible Enceladus is heated by a tidal mechanism similar to Jupiter's moon Io [Phillips W. Davis, September 25, 2018].

Chapter 2

Theoretical Background

In this chapter, the Stokes parameters (2.3) are derived from the definition of the electric field (2.2). From the Stokes vector, the same sense (*SC*), the opposite sense (*OC*) circular polarization, and the circular polarization ratios, (*CPR*), and the degree of linear polarization (*DLP*) (2.5) that were introduced in 1 are derived.

2.1 Electromagnetic Wave Theory

Stokes parameters are derived from the equations of polarized electromagnetic (EM) waves that arise from the wave equation (2.8-2.9). The wave equation is a solution of Maxwell's equations from Pollack et al. [2005] that in differential form reads as follows:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \quad (2.1)$$

$$\nabla \cdot B = 0 \quad (2.2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (2.3)$$

$$\nabla \times B = \mu_0 j + \frac{1}{c^2} \frac{\partial E}{\partial t} \quad (2.4)$$

The solutions of 2.3-2.4 will be solved using a Cartesian coordinate system shown in Figure 2.1:

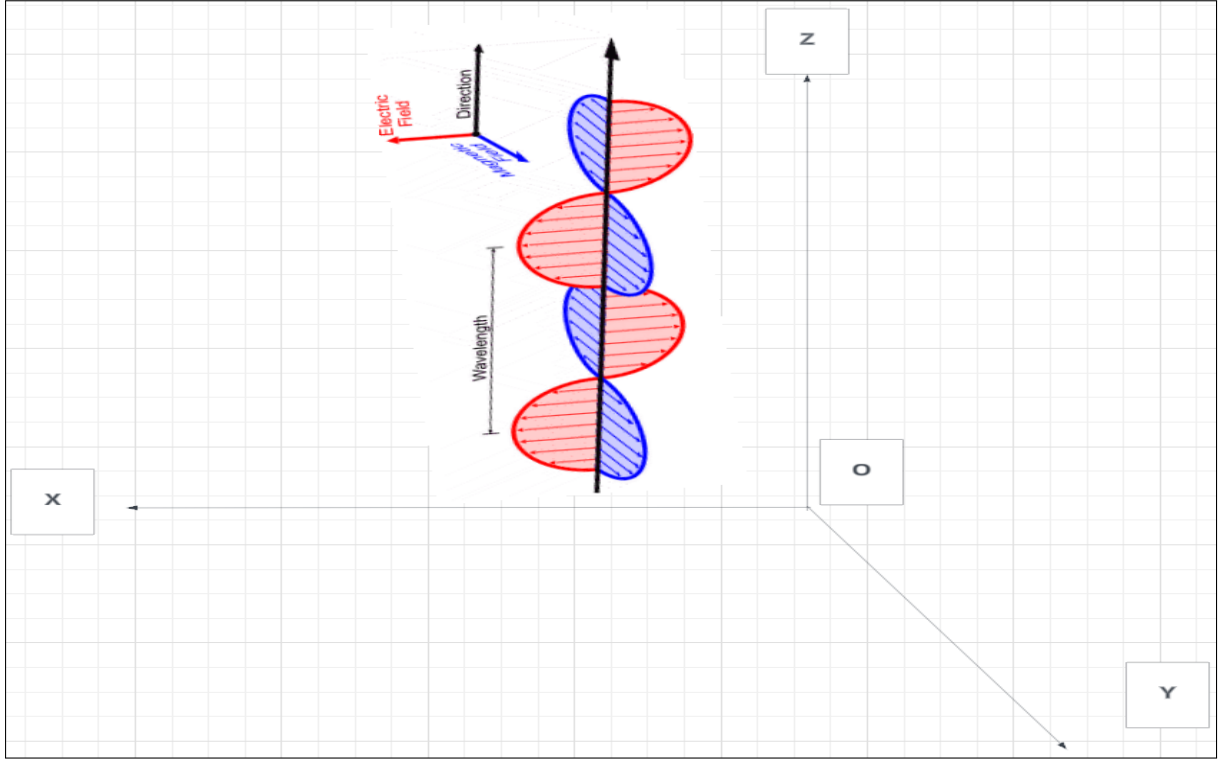


Figure 2.1: Cartesian coordinate system used for solving Maxwell's equations in 2.1, the EM wave is from National Oceanic and Atmospheric Administration (NOAA) [2024]

EM Propagation

To solve Equations 2.3 and 2.4 we have to decouple them using the curl identity from Pollack et al. [2005]:

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A \quad (2.5)$$

$$\nabla \times (\nabla \times E) = -\frac{\partial(\nabla \times B)}{\partial t} \quad (2.6)$$

$$\nabla \times (\nabla \times B) = \frac{1}{c^2} \frac{\partial(\nabla \times E)}{\partial t} \quad (2.7)$$

Thus the decoupled Equations 2.3-2.4 using Equations 2.6-2.7 read:

$$\frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = 0 \quad (2.8)$$

$$\frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} - \nabla^2 B = 0 \quad (2.9)$$

Solving Equations 2.8 and 2.9 yields:

$$\vec{E}(r, t) = E_0 \cos(k \cdot r - wt + \delta) \hat{n} \quad (2.10)$$

Where δ is the phase shift and is 0 for non-elliptically polarized waves.

$$\vec{B}(r, t) = B_0 \cos(k \cdot r - wt) \hat{z} \times \hat{n} \quad (2.11)$$

Where $\vec{E}$ is orthogonal to $\vec{B}$

Therefore the equations for $\vec{E}$ and $\vec{B}$ fields for non-polarized light using the Cartesian coordinate system from 2.1 Pollack et al. [2005]:

$$\vec{E}(z, t) = E_0 \cos(kz - wt) [\cos(\phi) \hat{x} + \sin(\phi) \hat{y}] \quad (2.12)$$

$$\vec{B}(z, t) = \frac{E_0}{C} \cos(kz - wt) [-\sin(\phi) \hat{x} + \cos(\phi) \hat{y}] \quad (2.13)$$

Such that $B_0 = \frac{E_0}{C}$

It is easier to express $\sin(x)$ and $\cos(x)$ in complex notation form using Euler's formula:

$$e^{+/-ix} = \cos(x) + / - i \sin(x) \quad (2.14)$$

$$\tilde{\vec{E}}(z, t) = E_0 e^{i(kz - wt)} [\cos(\phi) \hat{x} + \sin(\phi) \hat{y}] \quad (2.15)$$

$$\tilde{\vec{B}}(z, t) = \frac{E_0}{C} e^{i(kz - wt)} [-\sin(\phi) \hat{x} + \cos(\phi) \hat{y}] \quad (2.16)$$

The actual waveform of $\vec{E}(z, t)$ and $\vec{B}(z, t)$ is the real part of both $\tilde{\vec{E}}(z, t)$ and $\tilde{\vec{B}}(z, t)$.

Pollack et al. [2005]

2.2 Polarized EM Wave

In this section, we derive the polarized electric field using 2.15 as it will be useful to derive the Stokes parameters in section 2.3 that only require the electric field. Writing Equations 2.15 and 2.16 as a complex exponential using Equation 2.14 gives: Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = E_0 e^{i(kz - \omega t)} [\cos(\phi) e^{i\delta_x} \hat{x} + \sin(\phi) e^{i\delta_y} \hat{y}] \quad (2.17)$$

such that:

$$\tilde{E}_{0x} = E_0 e^{i\delta_x} \cos(\phi) \quad (2.18)$$

$$\tilde{E}_{0y} = E_0 e^{i\delta_y} \sin(\phi) \quad (2.19)$$

Where δ_x and δ_y in Equations 2.18 and 2.19 are the phase shifts with respect to the x-axis and y-axis, respectively. Thus, the polarization is:

$$\delta = \delta_y - \delta_x \quad (2.20)$$

Hence the complex forms of Equations 2.17 and 2.16:

$$\tilde{\vec{E}}(z, t) = e^{i(kz - \omega t)} [\tilde{E}_{0x} \hat{x} + \tilde{E}_{0y} \hat{y}] \quad (2.21)$$

The horizontal polarization ($\delta = 0$):Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = E_0 e^{i(kz - \omega t)} \hat{x} \quad (2.22)$$

The vertical polarization ($\delta = \frac{\pi}{2}$):Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = E_0 e^{i(kz - \omega t)} \hat{y} \quad (2.23)$$

+45^{deg} polarization ($\delta = \frac{\pi}{4}$):Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = \frac{\sqrt{2}}{2} E_0 e^{i(kz - \omega t)} [\hat{x} + \hat{y}] \quad (2.24)$$

−45^{deg} polarization ($\delta = -\frac{\pi}{4}$):Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = \frac{\sqrt{2}}{2} E_0 e^{i(kz - \omega t)} [\hat{x} - \hat{y}] \quad (2.25)$$

Circular polarization:

Right circular polarization (RCP) arises when $\hat{x}$ and $\hat{y}$ components of the $\vec{B}$ and $\vec{E}$ fields are out of phase by a factor of $-\frac{\pi}{2}$ such that Equation 2.20 equates to $-\frac{\pi}{2}$.

Hence the E and B fields using $\delta_y = 0$ and $\delta_x = \frac{\pi}{2}$ and Equation 2.14: Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = e^{i(kz - \omega t)} [iE_{0x}\hat{x} + E_{0y}\hat{y}] \quad (2.26)$$

Left circular polarization (LCP) arises when $\hat{x}$ and $\hat{y}$ components of the $\vec{B}$ and $\vec{E}$ fields are out of phase by a factor of $\frac{\pi}{2}$ such that Equation 2.20 equates to $\frac{\pi}{2}$.

Hence the E and B fields using $\delta_y = \frac{\pi}{2}$ and $\delta_x = 0$ and Equation 2.14: Fiber Optics For Sale [2024]

$$\tilde{\vec{E}}(z, t) = e^{i(kz - \omega t)} [E_{0x}\hat{x} + iE_{0y}\hat{y}] \quad (2.27)$$

2.3 Stokes Parameters and the Stokes Vector

The Stokes vector is a 4X1 column vector normalized with respect to total intensity (S_0) containing the polarization information (Stokes parameters) of an electromagnetic wave. In Equation 2.28 S_0 is the total intensity of the EM wave. S_1 is the vertical/horizontal polarization, S_2 is the +45° polarization and S_3 is the RCP/LCP polarization of the EM wave. Since we are working with a right circularly polarized wave (*RCP*) as seen in sections 1.1, 5.1.1, we define our initial Stokes vector as:

$$S = \begin{bmatrix} 1 \\ \frac{S_1}{S_0} \\ \frac{S_2}{S_0} \\ \frac{S_3}{S_0} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \\ -1.0 \end{bmatrix} \quad (2.28)$$

An important property of Stokes vectors is that they do not depend on the B-field (Equation 2.16) component of the EM wave, but only on the E-field (Equation 2.15) due to orthogonality where $\vec{E}(z, t) \cdot \vec{B}(z, t) = 0$ Fiber Optics For Sale [2024]

Taking the complex conjugate (*) of Equation 2.21:

$$\vec{E}^*(z, t) = e^{-i(kz - \omega t)} [\vec{E}_{0x}^* \hat{x} + \vec{E}_{0y}^* \hat{y}] \quad (2.29)$$

$$S_0 = E_{0x}^2 + E_{0y}^2 = I \quad (2.30)$$

$$S_1 = E_{0x}^2 - E_{0y}^2 = Q \quad (2.31)$$

$$S_2 = 2E_{0x}E_{0y}\cos(\delta) = U \quad (2.32)$$

Because $\cos(-\delta) = \cos(\delta)$ (even function) Fiber Optics For Sale [2024].

$$S_3 = 2E_{0x}E_{0y}\sin(\delta) = V \quad (2.33)$$

Because $\sin(-\delta) = -\sin(\delta)$ (odd function) Fiber Optics For Sale [2024].

A Stokes vector is related to the components of the electric field, known as the Jones vector:

$$E_i = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{pmatrix} E_{0x} \\ E_{0y}e^{i\delta} \end{pmatrix} \quad (2.34)$$

Such that:

$$E_{0x} = \sqrt{\frac{S_0 + S_1}{2}}, E_{0y} = \sqrt{\frac{S_0 - S_1}{2}} \quad (2.35)$$

Combining Equation 2.34 and 2.35 to our initial Stokes vector (Equation 2.28) such that $\delta = -\frac{\pi}{2}$ the associated *RCP* Jones vector is:

$$E_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \quad (2.36)$$

Same sense polarization refers to the polarization in the same sense as that of the incident signal. Using Equations 2.30 and 2.31, we define same sense linear polarization as:

$$SL = S_1 - S_0 \quad (2.37)$$

Opposite sense polarization refers to the polarization in the opposite sense of that of the incident signal. Using Equations 2.30 and 2.31, we define opposite sense linear polarization as:

$$OL = S_1 + S_0 \quad (2.38)$$

Using Equations 2.37-2.38, the linear polarization ratio is therefore:

$$LPR = \frac{SL}{OL} = \frac{S_1 - S_0}{S_1 + S_0} \quad (2.39)$$

Similarly, using Equations 2.30 and 2.33, we define same-sense circular polarization as:

$$SC = S_3 - S_0 \quad (2.40)$$

And opposite-sense circular polarization as:

$$OC = S_3 + S_0 \quad (2.41)$$

Using Equations 2.40-2.41, the circular polarization ratio is therefore:

$$CPR = \frac{SC}{OC} = \frac{S_3 - S_0}{S_3 + S_0} \quad (2.42)$$

The Degree of linear polarization:

$$DLP = \frac{\sqrt{(S_1)^2 + (S_2)^2}}{S_0} \quad (2.43)$$

Equation 2.43 refers to the degree of linear polarization. When a circularly polarized wave interacts with scatterers (impurities, cracks, or void spaces), it changes the phase shift (δ) from circular to elliptical, which introduces a linear component that was not present before, thus increasing the degree of linear polarization [Neish and Carter, 2014].¹

2.4 Circular Polarization Radar Backscatter

2.4.1 SC circular polarization radar backscatter

2.1: When an incident beam reflects from a surface, it changes its sense of polarization to the opposite (left circular) of that of the incident signal, and then it is known as *OC*. Similarly, when it reverses polarization twice after a double-bounce scattering event, it results in the same sense polarization of which it was transmitted, then it is known as *SC* [Calla et al., 2015]. Understanding *SC* is important because it helps us to isolate what physical processes affect the circular polarization ratio. High *SC* arises from the coherent-backscattering-opposition-effect (CBOE in Figure 2.2), where radar waves traveling in opposite directions due to multiple bounces from scatterers in Figure 2.2 (impurities, cracks or void spaces) in a weakly absorbing medium (such as water ice) add coherently, resulting in high *CPR* [Hapke, 1990]. There are two types of scattering mediums that generate CBOE; the first one is a scattering medium with high roughness (sharp scatterers). Rough and blocky surfaces that have high *CPR* [Fa and Cai, 2013] since it causes incident radar beams to bounce more than once (double-bounce) or dihedral scattering, leading to more *SC* signal [Calla et al., 2015]. High roughness results in the creation of corner-reflector-like

¹In some papers, *LPR* (2.39) and *CPR* (2.42) are written as μ_l and μ_c respectively.

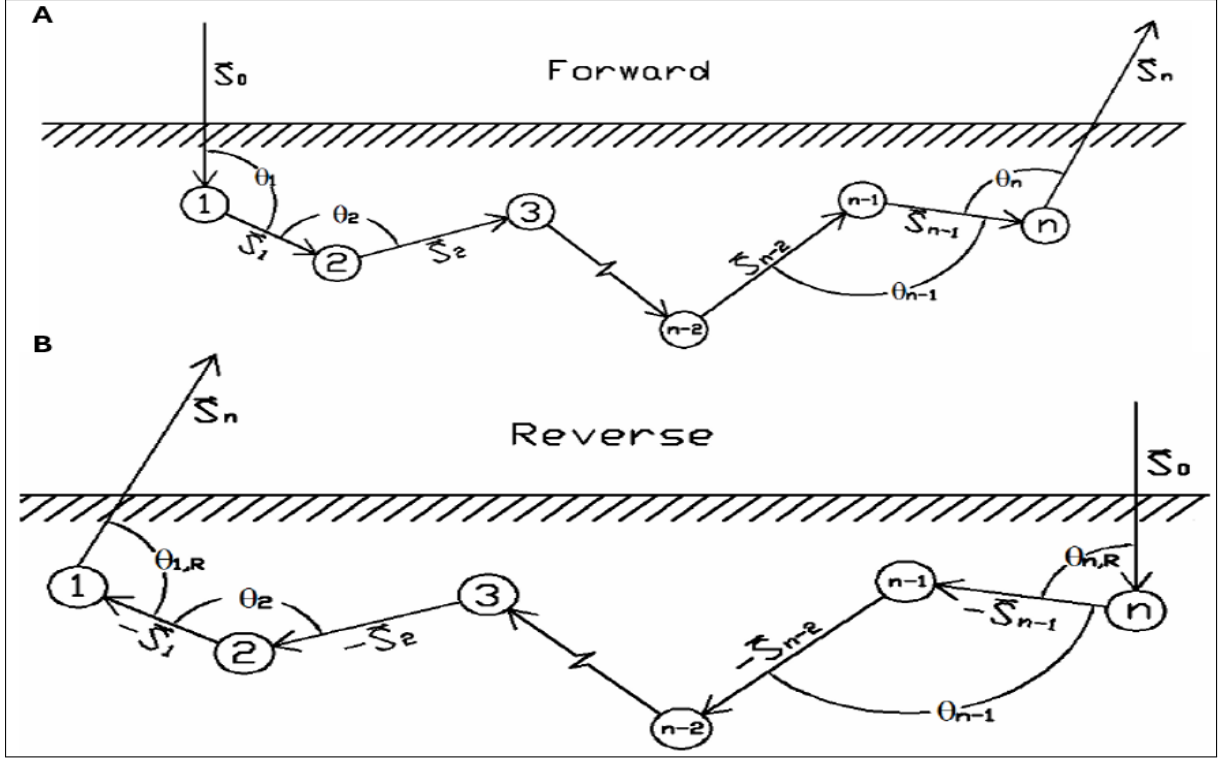


Figure 2.2: Coherent backscattering opposition effect (CBOE) schematic showing the forward path (A) and the reverse path (B) from Figure 1 in Sawicki et al. [2008]

structures that increase the amount of *SC* (double-bounce) reflections and thus, result in high *CPR* [Campbell, 2012]. The second process that gives rise to strong CBOE is diffuse (volume) scattering in a weakly absorbing medium such as water-ice. In weakly absorbing media, the radar signal can undergo a series of forward scattering events (Figure 2.2 A) off small imperfections (cracks or void spaces) in the material, each of which preserves the polarization properties of the signal. Those portions of the wave front that are scattered along the same path but in opposite directions (Figure 2.2 B) combine coherently to produce an increase in the *SC* radar backscatter. This coherent backscatter effect leads to large returns in the *SC* polarization and values for *CPR* that can exceed unity [Neish and Carter, 2014]. Hence, there are two physical processes that give rise to *SC*; the first is the roughness of the scattering medium that, due to corner-reflections, leads to dihedral (double-bounce) scattering, causing a surge in *SC*. Second is the diffuse (volume) scattering in low absorbing scattering media such as water-ice, that contributes to *SC* echoes from imperfections within the medium at all incidence angles (angle between the medium's normal and antenna) [Rivera-Valentin et al., 2022]. A similar conclusion was

found by Virkki and Bhiravarasu [2019], that at all incidence angles, the SC echoes arise primarily from diffuse scattering from wavelength-sized scatterers (cracks or void spaces) in the subsurface. It is worth noting that at high incident angles, high surface diffuse scattering prevents subsurface diffuse scattering that results in the randomization of SC , and OC such that $SC \sim OC$ with $CPR \sim 1$ with low-mod DLP as seen in 6.2.1, 6.3.2, and 6.3.3.

2.4.2 OC Circular Polarization Radar Backscatter

When an incident beam reflects from a surface, it reverses polarization, changing its sense of polarization to the opposite of that transmitted, and then it is known as opposite sense (OC) [Calla et al., 2015]. Because flat, mirror-like surfaces are dominated by single-bounce reflections, they tend to have high OC returns and low CPR values ($CPR \leq 1$) [Neish and Carter, 2014]. Specular scattering contributes only to OC echoes, while diffuse (volume) scattering contributes to both OC and SC echoes. Hence, for most surfaces, this leads to a return with more of an “opposite sense” (OC) polarization than a “same sense” (SC) polarization, so the CPR remains less than 1 [Neish and Carter, 2014]. At low incidence angles, OC echoes result from specular, single-scattering events from the surface while at high incidence angles OC echoes primarily result from diffuse scattering, such as rocks Ostro [1993], Black [2002]. Furthermore, the second process that gives rise to OC is diffuse (volume scattering), unlike SC that is affected by diffuse scattering at all incident angles, OC is affected by diffuse scattering at high incident angles only [Rivera-Valentin et al., 2022]. At high incident angles, diffuse scattering is strong enough to randomize SC and OC such that $SC \sim OC$ with $CPR \sim 1$. Thus, as SC there are two physical processes that give rise to opposite sense circular polarization. The first is specular scattering that, due to single-bounce reflections, tends to have high OC returns and low CPR at low incident angles. The second is the diffuse (volume) scattering in low-absorbing scattering media such as water-ice, that contributes to OC echoes from impurities (cracks or void spaces) at high incidence angles [Rivera-Valentin et al., 2022]. At low incidence angles, the OC -polarized returns are dominated by quasi-specular scattering, and at higher incidence angles, the OC echoes are produced by scattering from radar-facing facets as

well as by diffuse scattering from wavelength-scale roughness [Thompson et al., 2011]. Metallic surfaces, due to the free electrons surrounding the metal ions in metallic bonds, which, when interacting with incident polarized radiation, excite and emit photons in OC polarization, resulting in single bounce (specular scattering) [Bourget et al., 2024b] with $OC \gg SC$ and $CPR \ll 1$.

2.5 Polarization Ratios

2.5.1 Circular Polarization Ratio and Degree of Linear Polarization

Methods of studying potential ice deposits on planetary bodies involve the analysis of radar returns, specifically through measures like circular polarization ratio (defined as CPR) and degree of linear polarization (defined as DLP). These parameters provide information about the scattering properties of the ice and the impurities embedded within. In essence, studying the CPR and DLP of ice-bearing deposits can offer crucial insights into the composition, concentration, and distribution of impurities, as well as the scattering properties of the ice. These parameters, derived from the analysis of Stokes parameters (see Sections 2.3, 4.1, and 5.1.1), have dependence on radar configurations and geophysical parameters of the regolith within the radar wave’s penetration depth [Fa and Cai, 2013]. CPR is the medium’s tendency to conserve circular polarization in the SC . Conversely CPR is an index that shows how the received radar energy is allocated between different polarization states during the interaction process of radar waves with the target [Fa and Cai, 2013]. Ice deposits are known to have high CPR [Ostro, 1987, Campbell, 2002] caused by both volumetric backscatter and the coherent backscatter opposition effect (CBOE in Figure 2.2) [Mishchenko and Dlugach, 1992, Peters, 1992, Black et al., 2001]. Models for high CPR are based on radar waves traveling in opposite directions in a weakly absorbing medium (such as water ice) that add coherently, resulting in high CPR [Hapke, 1990]. To better understand subsurface scattering phenomena, the study examines the DLP , which can reveal more about the subsurface interface (a boundary) between two different scattering media [Neish and Carter, 2014]. The DLP

can be used to infer the presence of subsurface scattering and is defined in 2.3. *CPR* and *DLP* are defined such that high *DLP* and *CPR*: $DLP \geq 0.1$ and $CPR > 1$, moderate *DLP* and *CPR*: $\sim 0.05 \leq DLP < \sim 0.1$ and $\sim 0.5 \leq CPR \leq \sim 1$, low *DLP* and *CPR*: $\sim 0 \leq DLP < \sim 0.05$ and $\sim 0 \leq CPR < \sim 0.5$. A circularly polarized wave can be thought of as a combination of vertically and horizontally polarized waves. These two components have different complex power transmission coefficients, so if the wave penetrates the surface, the reflected wave will change from circular ($\delta = \pm \frac{\pi}{2}$) to elliptical ($\delta \neq \pm \frac{\pi}{2}$) [Neish and Carter, 2014]. This introduces a linear component to the signal that was not present before, increasing the *DLP*. High *CPR* values, approaching 2, may signify a rough surface with abundant impurities; while low *CPR* values suggest a smoother surface [Neish and Carter, 2014]. Similarly, high *DLP* values might indicate larger or non-spherical impurities, whereas low values could signify smaller, more spherical impurities or no subsurface scattering. Thus, echoes from subsurface interfaces (cracks or void spaces) will have a larger *DLP* than those from most types of surface interfaces. Combined with the *CPR*, these products can be used to infer differences in the surface and subsurface structure of the planetary body (Figure 53.13 in Neish and Carter [2014]).

Chapter 3

Modeling Radar Scattering in Icy Regolith Analogs

In this Chapter 3 we describe the modeling of the propagation of a polarized electric field through a scattering medium consisting of a colloidal solution of latex beads with dielectric properties analogous to that of water-ice with impurities (scatterers, cracks, or void spaces) 1.1. We use the definition of the electric field that was derived from Maxwell's equations in 2.1, we extend our derivation to polarized electromagnetic waves in 2.2, and the Stokes vector and polarization ratios in 2.3. These were used in our modeling in 3.2. We keep track of the polarized electric field using a steady state Monte-Carlo simulation 3.2.2, that uses the output of the expected values 3.1.5 of the Mie efficiencies 3.1.2-3.1.3, and of the scattering phase function 3.1.4 that are based on Mie scattering theory 3.1.1 and Gamma distribution (Equation 1.1). The Mie scattering program is developed in Python, and is based on Mätzler [2002].

3.1 Scattering Theory

3.1.1 Mie Scattering Theory

We define R_0 as the radius of a sphere (scatterer), μ and ϵ as the relative magnetic and electric permittivity. The Helmholtz field $H = \frac{B}{\mu}$ is the ability of a material to retain a magnetic field. The wave-number is $k = \frac{2\pi}{\lambda} [m^{-1}]$ where $\lambda [m]$ is the wavelength in the

ambient medium. The relative index of refraction (m) depends on the index of refraction of the sphere (m_2), and on the index of refraction of the medium m_1 , is given by:

$$m = \frac{m_2}{m_1} \quad (3.1)$$

We define the size parameter as:

$$X = kR_0 \quad (3.2)$$

Such that the Mie coefficients are dimensionless.

We modify the Maxwell equations from 2.1 for the H fields:

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \quad (3.3)$$

(the scatterer has no charges)

$$\nabla \cdot H = 0 \quad (3.4)$$

H field does not diverge

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (3.5)$$

$$\nabla \times H = -\sigma E + \epsilon \frac{\partial E}{\partial t} \quad (3.6)$$

such that $j = \sigma E$ The electric field solution from Equation 2.21:

$$\tilde{\vec{E}}(z, t) = e^{i(kz - \omega t)} [\tilde{E}_{0x} \hat{x} + \tilde{E}_{0y} \hat{y}] \quad (3.7)$$

The Helmholtz field equation using Equation 2.22:

$$\tilde{\vec{H}}(z, t) = -\frac{e^{i(kz - \omega t)}}{\mu c} [\tilde{E}_{0y} \hat{x} - \tilde{E}_{0x} \hat{y}] \quad (3.8)$$

From separation of variables:

$$E(z, t) = E(z)E(t) \quad (3.9)$$

Deriving the index of refraction m in terms of dielectric permittivity and magnetic

permeability:

$$\nabla \times H = \sigma e^{-i\omega t} - \epsilon i\omega e^{-i\omega t} \quad (3.10)$$

$$\sigma - \epsilon i\omega = -\frac{i\omega m^2}{c^2 \mu} \quad (3.11)$$

$$\frac{\mu c^2 (i\sigma + \omega \epsilon)}{\omega} = m^2 \quad (3.12)$$

Hence:

$$m = \sqrt{\frac{\mu(\epsilon + \frac{i\sigma}{\omega})}{\mu_0 \epsilon_0}} \quad (3.13)$$

Equation 3.13 is very important because it connects the index of refraction of a sphere of radius R_0 to its dielectric permittivity and magnetic permeability properties, which do affect the ability of the sphere to hold on to charge (electric field). Since polarization is related to the electric field (Equations 2.21 and 2.22), therefore a material with different dielectric properties will change or make irrelevant the polarization of the incident beam. Using boundary condition $z_l(k_1 r) = j_l(k_1 r)$ and the continuity conditions on the surface of the sphere [Bohren and Huffman, 1983]:

$$e_r \times [E^i + E^o] = e_r \times E^t, e_r \times [H^i + H^o] = e_r \times H^t \quad (3.14)$$

$$j_l(x) = a_l^{(r)} h_l^1(x) = a_l^{(t)} j_l(y), (e_\theta, E) \quad (3.15)$$

$$\mu_1 [x j_l(x)]' + \mu_1 a_l^{(1)} [x h_l^1(x)]' = \mu_2 a_l^{(t)} [y j_l(y)]', (e_\theta, H) \quad (3.16)$$

$$m [x j_l(x)]' + m b_l^{(r)} [x h_l^{(1)}(x)]' = b_l^{(t)} [y j_l(y)]', (e_\phi, E) \quad (3.17)$$

$$\mu_1 [j_l(x)] + \mu_1 b_l^{(r)} h_l^1(x) = \mu_2 b_l^{(t)} m j_l(y), (e_\phi, H) \quad (3.18)$$

Where $x = k_2 R_0$ and $y = k_1 R_0 = n k_2 R_0$. We can write the above system of equations (Equations 3.15-3.18):

With a little less generality we will now assume $\mu_1 = \mu_2$. Using the Riccati-Bessel functions we can solve Equation 3.15-3.18:

$$a_1^r = \frac{-\psi_1(y)\psi_1'(y) - m\psi_1'(y)\psi_1(y)}{\psi_1(y)\zeta_1'(x) - m\psi_1'(y)\zeta_1(x)} \quad (3.19)$$

$$b_1^r = \frac{\psi_1(y)\psi_1'(y) - m\psi_1'(x)\psi_1(y)}{\psi_1'(y)\zeta_1(x) - m\psi_1(y)\zeta_1'(x)} \quad (3.20)$$

3.1.2 Mie Coefficient

To compute the scattering properties of a sphere, we need the Mie coefficients c_n d_n , the scattering function $F_{11}(\theta)$ $F_{22}(\theta)$ and related amplitude A_1 A_2 , which require the Mie coefficients a_n b_n . The computation of these parameters is the most computationally intensive part in Mie computations due to the involvement of spherical Bessel functions up to high order. Modifying the algorithm from [Mätzler, 2002] by using Python SciPy's built-in double-precision Bessel functions, we are able to compute the Mie coefficients for size parameters up to 10^2 . The Mie coefficients can be derived from Equations 3.19-3.20, and are given by Bohren and Huffman [2008]:

$$a_n = \frac{m^2 j_n(mx)[x j_n(x)]' - \mu_1 j_n(x)[m x j_n(mx)]'}{m^2 j_n(mx)[x h_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[m x j_n(mx)]'} \quad (3.21)$$

$$b_n = \frac{\mu_1 j_n(mx)[x j_n(x)]' - j_n(x)[m x j_n(mx)]'}{\mu_1 j_n(mx)[x h_n^{(1)}(x)]' - h_n^{(1)}(x)[m x j_n(mx)]'} \quad (3.22)$$

$$c_n = \frac{\mu_1 j_n(mx)[x h_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[x j_n(x)]'}{\mu_1 j_n(mx)[x h_n^{(1)}(x)]' - h_n^{(1)}(x)[m x j_n(mx)]'} \quad (3.23)$$

$$d_n = \frac{\mu_1 m j_n(x)[x h_n^{(1)}(x)]' - \mu_1 m h_n^{(1)}(x)[x j_n(x)]'}{m^2 j_n(mx)[x h_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[m x j_n(mx)]'} \quad (3.24)$$

Considering $\mu_1 = \mu_2 = 1$, then the Mie coefficient simplifies to:

$$a_n = \frac{m^2 j_n(mx) [x j_n(x)]' - j_n(x) [mx j_n(mx)]'}{m^2 j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [mx j_n(mx)]'} \quad (3.25)$$

$$b_n = \frac{\mu_1 j_n(mx) [x j_n(x)]' - j_n(x) [mx j_n(mx)]'}{\mu_1 j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [mx j_n(mx)]'} \quad (3.26)$$

$$c_n = \frac{\mu_1 j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [x j_n(x)]'}{\mu_1 j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [mx j_n(mx)]'} \quad (3.27)$$

$$d_n = \frac{\mu_1 m j_n(x) [x h_n^{(1)}(x)]' - m h_n^{(1)}(x) [x j_n(x)]'}{m^2 j_n(mx) [x h_n^{(1)}(x)]' - h_n^{(1)}(x) [mx j_n(mx)]'} \quad (3.28)$$

Equations 3.25-3.26 are crucial for deriving the efficiencies Q_i (3.34-3.36) and the scattering phase function (3.42) with the associated scattering amplitude (3.44). The efficiencies (Q_i) (3.34-3.36) and the scattering phase function (3.42) are combined with the Gamma distribution (1.1) to calculate the expected values in 3.1.5 (weighted average), being the input parameter for the polarized Monte-Carlo simulation in 3.2.

3.1.3 Scattering Efficiencies

The coefficients $\beta_i [m^{-1}]$ describe the interaction of radiation with a sphere of radius R_0 , and depend on the Mie efficiencies Q_i , the scattering cross section of the sphere (πR_0^2), and on the number of scatterers (spheres) per unit volume $C [m^{-3}]$. We define i as the extinction ($i = ext$), absorption ($i = abs$), scattering ($i = sca$), and backscattering ($i = bs$) respectively.

$$\beta_i = C \pi R_0^2 Q_i [m^{-1}] \quad (3.29)$$

Using the definition of the coefficients from Equation 3.29, we define $\sigma(r) [m^2]$ to be the cross-section of the scatterers.

$$\beta_i [m^{-1}] = C \sigma(r) Q_i \quad (3.30)$$

The cross section as a function of r [m] is:

$$\sigma(r) = \pi r^2 [m^2] \quad (3.31)$$

The extinction efficiency is defined as:

$$Q_{ext} = Q_{sca} + Q_{abs}, \beta_{ext} = \beta_{sca} + \beta_{abs} \quad (3.32)$$

We can rearrange Equation 3.32 such that:

$$Q_{abs} = Q_{ext} - Q_{sca}, \beta_{abs} = \beta_{ext} - \beta_{sca} \quad (3.33)$$

The scattering efficiency Q_{sca} from Equation 3.32 follows from the integration of the scattered power over all directions, and the extinction efficiency Q_{ext} follows from the Extinction Theorem in Ishimaru [1978], Hulst and van de Hulst [1981] pages 14 and 31 respectively, also called the Forward-Scattering Theorem, leading to:

$$Q_{sca} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2) \quad (3.34)$$

$$Q_{ext} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)Re(a_n + b_n) \quad (3.35)$$

Thus, since we have the formulas for both Equations 3.34 and 3.35 we can find Q_{abs} using equation 3.32:

$$Q_{abs} = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1)[Re(a_n + b_n) - (|a_n|^2 + |b_n|^2)] \quad (3.36)$$

The backscattering coefficient can be obtained from Equation 3.37:

$$Q_{bs} = \frac{1}{x^2} \left| \sum_{n=1}^{\infty} (2n+1)(-1)^n [a_n - b_n] \right|^2 \quad (3.37)$$

All infinite series can be truncated after n_{max} terms. For this number Bohren and Huffman [2008] proposed the value:

$$n_{max} = x + 4x^{1/3} + 2 \quad (3.38)$$

and this value is used here as well. Furthermore, the asymmetry parameter $g = \langle \cos(\theta) \rangle$ indicates the average cosine of the scattering angle θ with respect to power.

In section 3.1.4, we defined the scattering phase function (equation 3.42), that has to be normalized with respect to the scattering cross section (Equation 3.31)

$$Q_{rr} = k^2 r^2 Q_{sca} = \frac{k^2}{\pi} \sigma(r) Q_{sca} \quad (3.39)$$

$$Q_{xx} = x^2 Q_{sca}(x), \quad r = k^{-1}x \quad (3.40)$$

Equation 3.40 is used in the normalization of the scattering phase function (Equation 3.42). Using 3.34, and 3.36 we define the albedo (α), which is a dimensionless quantity that represents the ratio of the electromagnetic radiation that is scattered back with respect to the total amount of incident radiation.

$$\alpha = \frac{Q_{sca}}{Q_{ext}} = \frac{Q_{sca}}{Q_{sca} + Q_{abs}} \quad (3.41)$$

Equations 3.32-3.41 are combined with the Gamma distribution $n(r)$ (Equation 1.1) and with Equation 3.29 to obtain the associated expected values of the scattering coefficients $\langle \beta_i \rangle$ in section 3.1.5 that are the starting parameters for the polarized Monte-Carlo simulation 3.2.

3.1.4 The Scattering Phase Function

In this section, we derive the scattering phase function F_{11} and F_{22} that represents the shape of the scattered field and is related to its amplitude (Equation 3.44) that results from the interaction between an electromagnetic wave and a scatterer. The scattering phase function is a crucial step in the initialization of the polarized Monte-Carlo 3.2 as it is used for computing the scattering matrix F (Equation 3.50), and the Henyey-Greenstein scattering phase function $P(\cos(\theta))$ (Equation 3.49). The F matrix in Sawicki et al. [2008]

uses the amplitude of the scattering phase function (Equation 3.44) and is used in the scattering of the electric field when it interacts with scatterers 3.2. The Henyey-Greenstein function was originally proposed by Henyey and Greenstein [1941] for galactic scattering and is used in 3.2.4 to initiate the light forward path (B3 in Figure 3.2) and for computing the probability of the photon exiting the scattering medium (B9 in Figure 3.2). The Henyey-Greenstein function (Equation 3.49) requires the anisotropy (g) (Equation 3.48) that is the average cosine of the scattering phase function, and has a $-1 \leq g \leq 1$ range representing the "preferred" direction of scattering. A value of $g \sim 0$ indicates isotropic (uniform) scattering, a value $g < 0$ means backscattering, and a value of $g > 0$ means forward-directed scattering [Jacques et al., 1987] relative to the direction of the incident beam. The scattering phase function F_{11} and F_{22} from Mätzler [2002]

$$F_{11}(\cos(\theta)) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n), \quad F_{22}(\cos(\theta)) = \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n) \quad (3.42)$$

The scattering phase functions are bounded such that: $F_{11}(x, \theta) 0 \leq \theta \leq \pi$ and $F_{22}(x, \theta) \pi \leq \theta \leq 2\pi$ where θ is the scattering angle.

Combining Equations 3.40, and 3.42 to obtain the normalized scattering phase function with respect to the scattering cross-section:

$$\hat{F}_{11}(\cos(\theta)) = \frac{F_{11}(\cos(\theta))}{\sqrt{\pi Q_{xx}}}, \quad \hat{F}_{22}(\cos(\theta)) = \frac{F_{22}(\cos(\theta))}{\sqrt{\pi Q_{xx}}} \quad (3.43)$$

We define the normalized amplitude of the scattering phase function such that:

$$\hat{A}_1 = |\hat{F}_{11}|^2 = \hat{F}_{11} \hat{F}_{11}^*, \quad \hat{A}_2 = |\hat{F}_{22}|^2 = \hat{F}_{22} \hat{F}_{22}^* \quad (3.44)$$

Where $*$ denotes the complex conjugate of a complex number.

$E_{s\theta}$ is the scattered far-field component in the scattering plane, defined by the incident and scattered directions, and $E_{s\phi}$ is the orthogonal component. The angle ϕ is the angle between the incident electric field and the scattering plane. The functions $\pi_n(\cos(\theta))$ and $\tau_n(\cos(\theta))$ describe the angular scattering patterns of the spherical harmonics used to

describe F_{11} and F_{22} and follow from the recurrence relations [Deirmendjian, 1969]:

$$\pi_n = \frac{2n-1}{n-1} \cos(\theta) \pi_{n-1} - \frac{n}{n-1} \pi_{n-2}, \tau_n = n \cos(\theta) \pi_n - (n+1) \pi_{n-1} \quad (3.45)$$

From Deirmendjian [1969] on Page 15:

$$\pi_0 = 0, \pi_1 = 1, \pi_2 = 3 \cos(\theta) \quad (3.46)$$

$$\tau_0 = 0, \tau_1 = \cos(\theta), \tau_2 = 3 \cos(2\theta) \quad (3.47)$$

We can use Equation 3.42 to determine the anisotropy (g), using the formula from Steven L. Jacques, Scott A. Prahl [Jacques et al., 1987]:

$$g = \frac{\int_0^\pi |\hat{F}_{11}(\theta)| \cos(\theta) \sin(\theta) d\theta}{\int_0^\pi |\hat{F}_{11}(\theta)| \sin(\theta) d\theta} [\text{dimensionless}] \quad (3.48)$$

The normalization factor (Equation 3.40) used in the normalization of the scattering phase function (Equation 3.43) cancels out in Equation 3.48 and thus has no impact on the value of the anisotropy g . Using the anisotropy g , we can construct the Henyey-Greenstein function $P(\cos(\theta))$ [Henyey and Greenstein, 1941]:

$$p(\cos(\theta)) = \frac{1 - g^2}{2(1 + g^2 - 2g \cos(\theta))^{\frac{3}{2}}} \quad (3.49)$$

Finally, using the normalized scattering amplitudes (Equations 3.44), we can make the scattering (F) matrix from Sawicki et al. [2008]:

$$F = \begin{bmatrix} \hat{A}_2(\theta) & 0 \\ 0 & \hat{A}_1(\theta) \end{bmatrix} = \begin{bmatrix} \hat{F}_{22} \hat{F}_{22}^* & 0 \\ = 0 & \hat{F}_{11} \hat{F}_{11}^* \end{bmatrix} \quad (3.50)$$

Equation 3.50 is used in the construction of the scattering matrix (F) in the EMC simulation (B7 in Figure 3.2). Equations 3.42-3.44, and 3.48 are combined with the Gamma distribution $n(r)$ (Equation 1.1) to obtain the expected values of the normalized scattering phase function $\langle \hat{F}_{11} \rangle$, $\langle \hat{F}_{22} \rangle$ with the associated scattering amplitude

$< \hat{A}_1 >$, $< \hat{A}_2 >$ in section 3.1.5 that are the starting parameters for the polarized Monte-Carlo simulation 3.2.

3.1.5 Expected Values

In this section we derive the expected value of the scattering coefficients (Equations 3.32-3.41), and of the normalized scattering phase function and amplitude (Equations 3.43-3.44) as well as the expected mean free path $l_s[m]$, being the initial parameters for the EMC simulation (A5 in Figure 3.2). Using the definition of size parameter in Equation 3.2, we rewrite Equation 1.1 such that:

$$n(x) = K(k^{-1}x)^{\frac{1-3b}{b}} e^{-\frac{x}{bX_0}}, r = k^{-1}x \quad (3.51)$$

$n(x)$ is a norm probability size distribution of scatterers (beads) of radius $r [m]$ centered at $R_0 [m]$ illuminated by a monochromatic beam of wavelength $\lambda [m]$ with size parameter x , and centered at X_0 , with wave-number $k [m^{-1}]$, and normalization constant $K [m^{-1}]$. The expected value of a continuous function $f(x)$ over $[x_1, x_2]$ is:

$$< f > = \int_{x_1}^{x_2} f(x)n(x)dx \quad (3.52)$$

The expected size parameter is:

$$< x > = \int_{x_1}^{x_2} n(x)x dx, x_1 = \frac{X_0}{2}, \text{ and } x_2 = 2 * X_0 \quad (3.53)$$

Equation 3.53 is useful for calculating the expected scattering regime for a distribution of sizes $n(x)$ with a central size parameter X_0 . Using Equation 3.31, we can rewrite the scattering cross section as a function of x :

$$\sigma(x) = \frac{\pi}{k^2} x^2 [m^2] \quad (3.54)$$

Hence Equation 3.30 can be rewritten as a function of size parameter x :

$$\beta_i(x) = \frac{\pi C}{k^2} [x^2 Q_i(x)] [m^{-1}] \quad (3.55)$$

Hence, combining Equations 3.52 and 3.55, the expected values of the coefficients are:

$$\langle \beta_i \rangle = \frac{\pi C}{k^2} \int_{x_1}^{x_2} [x^2 Q_i(x)] n(x) dx [m^{-1}] \quad (3.56)$$

The expected value of the anisotropy factor (g) using Equations 3.48 from Jacques et al. [1987], 3.51, and 3.52 is:

$$\langle g \rangle = \frac{\int_0^\pi | \langle \hat{F}_{11}(\theta) \rangle | \cos(\theta) \sin(\theta) d\theta}{\int_0^\pi | \langle \hat{F}_{11}(\theta) \rangle | \sin(\theta) d\theta} \quad (3.57)$$

The expected value of the normalized scattering phase function $\hat{F}_{11}$, $\hat{F}_{22}$:

$$\langle \hat{F}_{11}(\theta) \rangle = \int_{x_1}^{x_2} \frac{F_{11}(x, \theta)}{\sqrt{\pi Q_{xx}}} n(x) dx, \quad 0 \leq \theta \leq \pi \quad (3.58)$$

$$\langle \hat{F}_{22}(\theta) \rangle = \int_{x_1}^{x_2} \frac{F_{22}(x, \theta)}{\sqrt{\pi Q_{xx}}} n(x) dx, \quad \pi \leq \theta \leq 2\pi \quad (3.59)$$

Where Q_{xx} is given by Equation 3.40, Equations 3.58-3.59 are used as entry in the scattering matrix F (Equation 3.50) in the EMC simulation (B7 in 3.2). Finally, the expected value of the Henyey-Greenstein scattering function $\langle p(\cos(\theta)) \rangle$ by combining Equations 3.49, and 3.52:

$$\langle p(\cos(\theta)) \rangle = \frac{1 - \langle g^2 \rangle}{2(1 + \langle g^2 \rangle - 2 \langle g \rangle \cos(\theta))^{\frac{3}{2}}} \quad (3.60)$$

The volume of a sphere of radius in terms of the size parameter x is:

$$V(x) = \frac{4\pi}{3k^3} x^3 [m^3] \quad (3.61)$$

Using Equations 3.51, and 3.61, the volume number of solid ϕ within a colloidal solution of latex beads is:

$$\phi = C \langle V(x) \rangle = \frac{4C\pi}{3k^3} \int_{x_1}^{x_2} [x^3 n(x)] dx \quad (3.62)$$

Therefore, the % volume fraction of solid is:

$$\phi(\%) = \frac{400C\pi}{3k^3} \int_{x_1}^{x_2} [x^3 n(x)] dx \quad (3.63)$$

Let $l_s [m]$ be the mean free path in terms of size parameter x such that:

$$l_s(x) = \frac{4k^{-1}x}{3\phi_{ext}(x)} [m] \quad (3.64)$$

Combining Equations 3.51 and 3.64, the expected value of the mean free path:

$$\langle l_s \rangle = \frac{4k^{-1}}{3\phi} \int_{x_1}^{x_2} \frac{xn(x)}{q_{ext}(x)} dx [m] \quad (3.65)$$

Let $\tau [m]$ be the thickness of a scattering medium and z_{max} be the number of times that the mean free path fits into the thickness of the sample.

$$\therefore z_{max} = \frac{\tau}{\langle l_s \rangle} \quad (3.66)$$

3.2 Electric Field Monte-Carlo Simulation for Coherent Backscattering of Polarized Light in a Scattering Media

3.2.1 What is and Why a Monte-Carlo Simulation?

Monte-Carlo simulations in the simplest definition are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle [Modest and Mazumder, 2021, Modest, 2013]. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. Monte Carlo methods are mainly used in three problem classes: optimization, numerical integration, and generating draws from a probability distribution [Kroese et al., 2014]. Monte Carlo simulation is, in essence, the generation of random objects or processes by means of a computer. These objects could arise ‘naturally’ as part

of the modeling of a real-life system, such as a complex road network, the transport of neutrons, or the evolution of the stock market [Kroese et al., 2014]. More formally, in all applications of the Monte Carlo method, a stochastic model is constructed in which the expected value of a certain random variable (or of a combination of several variables) is equivalent to the value of a physical quantity to be determined. This expected value is then estimated by the average of multiple independent samples representing the random variables introduced above [Lux and Koblinger, 2018]. For the construction of the series of independent samples, random numbers following the distribution of the variable to be estimated are used [Lux and Koblinger, 2018]. In our case, a Monte-Carlo simulation will be useful to compute the propagation of a large number $10^5 - 10^7$ of photons through a turbid medium containing scatterers in 3.2.7, and 6.5. Single scattering, albeit being simple and having a nice analytical solution in Deirmendjian [1969] that is reliable and computationally cheap, falls short when addressing the question of multiple scattering events from which CBOE arises ((Figure 2.2) in 2.4). Equation for a polydisperse system can be generalized for a multi-layered scattering medium, becoming a solution for the radiative transfer equation. The radiative transfer equation (RTE) is a five-dimensional integrodifferential equation, which is extremely difficult and expensive to solve. In fact, while the spherical harmonics and discrete ordinates methods and, to a lesser extent, the zonal method each enjoy a certain popularity and can be applied to fairly general problems, to this day no truly satisfactory RTE solution method has emerged. The problem is exacerbated by strong spectral variations of radiative properties (gases as well as particulates), so that up to one million RTE evaluations are needed to achieve acceptable accuracy [Modest, 2013]. These challenges, combined with the fact that Monte Carlo methods, unlike conventional RTE solvers, are ideal candidates for parallel computing, have led to their rapidly increasing popularity during the past few years [Modest, 2013].

3.2.2 Type of Monte-Carlo Simulation

There are two types of Monte-Carlo simulations for photon transport into a scattering medium, the first one being a steady-state Monte-Carlo simulation (spatially resolved), the second one being a time-resolved Monte-Carlo simulation. A steady state means that

a stable distribution of light has been achieved, and there are no time dynamics involved [Jacques, 2010], such that only the spatial location of the photon is being updated using a step size based on the generation of a random number (ξ). A time-resolved Monte-Carlo simulation is almost identical to the steady-state simulation discussed above, except that the step size is based on the time duration of the photon propagation defined by $t = \frac{L}{c}$, where L is the photon path [Jacques, 2010]. In our case, we will be using a steady state Monte-Carlo simulation in 3.2, since the polarization 2.3 is time-independent. The step size of a steady state Monte-Carlo simulation is provided by a pseudo-random number provided by the *random.rand()* Python built-in function. The *random.rand()* function has properties such that:

$$\xi = \int_a^x p(x)dx, [0, 1] \quad (3.67)$$

Equation 3.67 generates a random variable ξ , which is uniformly distributed over the interval $[0, 1]$. The NIR $1.064\mu m$ *RCP* beam propagates in the $-\hat{z}$ direction (Figure 5.1), the photon position and directions are given by:

$$(x, y, z) = (0, 0, 0), (u, v, w) = (0, 0, -\xi\hat{z}) \quad (3.68)$$

Equation 3.68 moves the photon (B7 in Figure 3.2) using a pseudo-random number generator from Equation 3.67. The cosine of the critical (reflection) angle is defined by:

$$\sqrt{|1 - \frac{1}{m^2}|} \quad (3.69)$$

The specular reflection R_s from the medium is defined by:

$$R_s = |\frac{(m-1)^2}{(m+1)^2}| \quad (3.70)$$

The sampling of the scattering angle is based on the same pseudo number generator from Equation 3.67. The probability distribution for the cosine of the scattering angle $< p(\cos(\theta)) >$ is described by the expected value of the Henyey and Greenstein scattering function (Equation 3.60) that initiates the light forward path (B3 in Figure 3.2) and the

probability of the photon exiting the scattering medium (B9 in Figure 3.2).

3.2.3 Boundary Condition

Monte-Carlo simulations for the propagation of a photon in a scattering medium have 3 types of boundary conditions. First, the infinite medium that simulates light propagation from a point source in an infinite medium (Figure 3.1 A) akin to Jacques [2010], second, the semi-infinite medium that simulates light propagation from normal irradiation of a semi-infinite medium where the photon interacts with the top surface of the scattering medium (Figure 3.1 B), similar to Sawicki et al. [2008] resulting in specular reflection R_s (Equation 3.67). Finally, the third one is a Monte-Carlo simulation for layered boundary conditions (Figure 3.1 C) where the scattering medium is sandwiched between two boundary conditions, similar to Wang et al. [1995] and Prahl [1989] for layered tissues. The layered boundary condition in Figure 3.1 C is still considered a semi-infinite medium due to the absence of boundary in the x-y plane. The method developed by Sawicki et al. [2008] works for semi-infinite medium with photon-to-surface medium interaction which is sufficient for repeating the results from Sawicki et al. [2008], Mishchenko [1992b], and Hapke and Blewett [1991], but insufficient for our experiment in 5 due to photon-to-sample platform interaction from Bourget et al. [2024b]. To address this dilemma, we combine the methods from Sawicki et al. [2008] to keep track of the polarized photon into the scattering medium while accounting for photon-to-surface interaction, and from Prahl [1989] to account for photon-to-sample platform interaction.

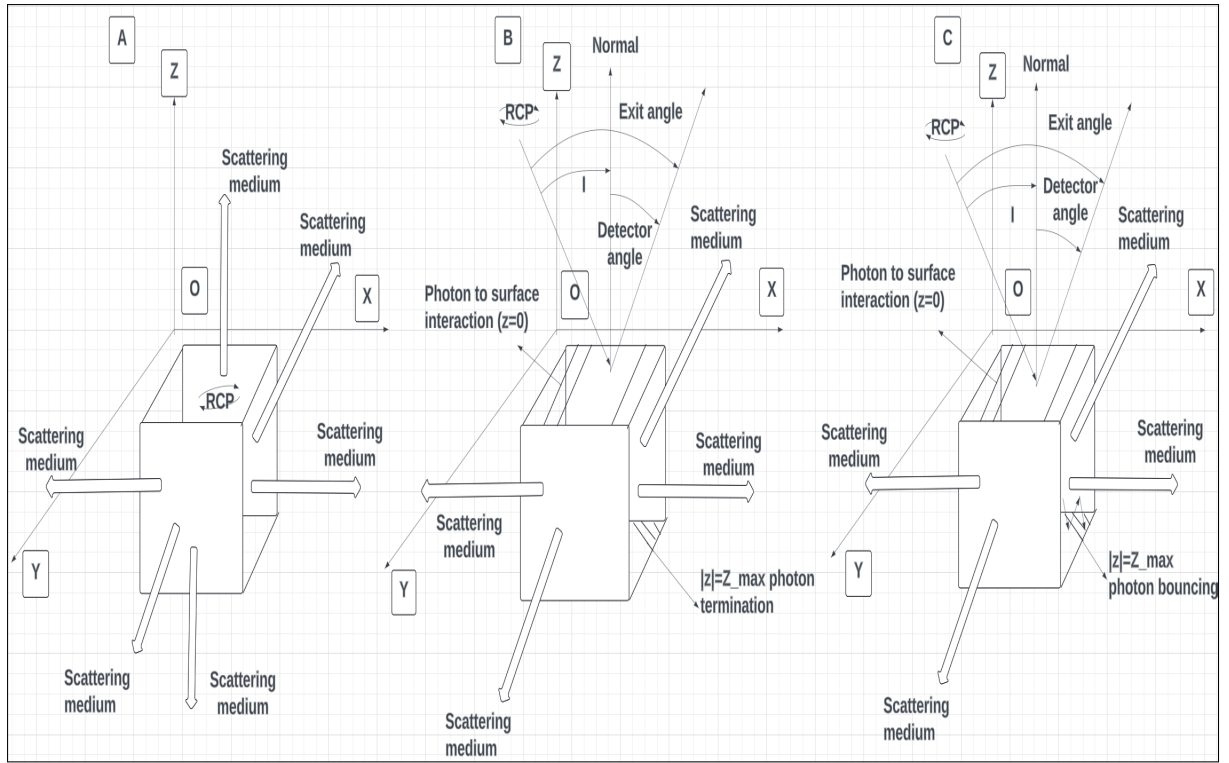


Figure 3.1: Flow chart for the photon propagation through a scattering medium. (A) Infinite medium for a *RCP* point source without boundary condition. (B) Semi-infinite medium with surface boundary condition. (C) Semi-infinite medium with layered boundary condition.

In the case of a lossless medium, a Russian roulette technique is used to determine if the photon is still alive after each scattering event; for a medium with absorption, a portion of the photon (weight) is absorbed before the Russian roulette technique is applied at each scattering event. In Figure 3.1 A, the scattering medium is unconstrained in all directions with the photon propagating from a point source. Figure 3.1 B shows a semi-infinite medium with a surface boundary, the angle between the *RCP* source and the normal of the surface is the incident angle I , the angle between the normal of the surface and the exiting photon is the detector angle, and the angle between the exiting photon and the *RCP* beam is the exiting (phase) angle. Photons with position $z \geq z_{max}$ are terminated at $Z = Z_{max}$ boundary in Figure 3.1 B. Figure 3.1 C is the same as Figure 3.1 B, but with the photon bouncing off the surface for $z \geq z_{max}$ at the $z = z_{max}$ boundary instead of photon termination. Using Equations (6) and (7) from Prahl [1989] we update the position (Equation 3.71) and the direction (Equation 3.72) of the photon for $z \geq z_{max}$:

$$(x'', y'', z'') = \begin{cases} (x, y, -z), & \text{if } z \geq z_{max}. \\ (x, y, z), & z < z_{max}. \end{cases} \quad (3.71)$$

$$(\mu'_x, \mu'_y, \mu'_z) = \begin{cases} (\mu_x, \mu_y, -\mu_z), & \text{if } z \geq z_{max}. \\ (\mu_x, \mu_y, \mu_z), & z < z_{max}. \end{cases} \quad (3.72)$$

In the case of a polarized beam, the rotation matrix $R(\phi)$ (B7 in Figure 3.2) is also updated at the boundary condition $z \geq z_{max}$ such that:

$$R'(\phi) = \begin{cases} R(-\phi), & \text{if } z \geq z_{max}. \\ R(\phi), & z < z_{max}. \end{cases} \quad (3.73)$$

The rotation matrix has the following properties:

$$R^T(\phi) = R(-\phi), \quad R^T(-\phi) = R(\phi) \quad (3.74)$$

Such that T denotes the transpose, in the reverse path the rotation matrix reads:

$$R'^T(\phi) = \begin{cases} R(\phi), & \text{if } z \geq z_{max}. \\ R(-\phi), & z < z_{max}. \end{cases} \quad (3.75)$$

Where ϕ is the angle between the perpendicular component of the electric field (E_y) and the scattering plane.

3.2.4 Outline of the Program (Polarized)

We simulate the propagation of a *RCP* beam with initial Stokes vector $S_i = [1.0, 0.0, 0.0, -1.0]$. Details about the experimental set-up and the scattering regime of our samples can be found in Bourget et al. [2024b]. In Figure 3.2, we outline the program where the polarized Monte-Carlo has been integrated into a Python workflow. The initial parameters are: N_{ph} as the number of photons, the number of scattering angles N_{scat} , the incident angle range R_{inc} , the exit (phase) angle range R_{exit} , the concentration $C[\frac{beads}{m^3}]$ of beads, the central radius of the beads R_0 , and S_i as the initial Stokes vector. The thickness (τ) of the scattering medium to the scattering mean free path l_s is defined as $z_{max} = \frac{\tau}{l_s}$. In 1941, Henyey and Greenstein proposed a scattering phase function $p(\cos(\theta)) = \frac{1-g^2}{2(1+g^2-2g\cos(\theta))^{\frac{3}{2}}}$ to explain galactic scattering in Henyey and Greenstein [1941]. The Henyey-Greenstein function gives the probability distribution of the scattering angle cosine ($\cos(\theta)$) and uses the anisotropy ($-1 \leq g \leq 1$) as an argument. A value of 0 indicates isotropic scattering, and a value near 1 indicates forward-directed scattering [Jacques et al., 1987]. The Henyey and Greenstein function is used in Section 3.2.4 to initiate the light forward path (B3 in Figure 3.2). The position of the photon within the scattering medium is given by (x, y, z) . If the z position of the photon is greater than or equal to the z_{max} (boundary condition $z \geq z_{max}$), the photons bounce off in the opposite direction, where the z -position (Equation 3.71), direction (Equation 3.72) and rotation matrix (Equations 3.73, 3.75) are updated accordingly. The interaction between *RCP* photons and the boundary $z = z_{max}$ changes the polarization from circular ($\delta = 90^\circ$) to elliptical ($\delta \neq 90^\circ$) that introduces a linear component that was not present before, thus increasing the *DLP* [Neish and Carter, 2014].

Hence, for high transparency Rayleigh samples (Figures 6.3, 6.4, 6.8, 6.9), more photons will meet boundary condition $z \geq z_{max}$, causing significant $OC \gg SC$ with $CPR \ll 1$ and high DLP relative to classical Mie and large Mie beads (Figures 6.3, 6.5, 6.6, 6.8, 6.10, 6.11). Details about the initial parameters can be found in Tables 2-5 in .3-6. To illustrate the photon propagation in the simulation, we present a flow-chart in Figure 3.2.

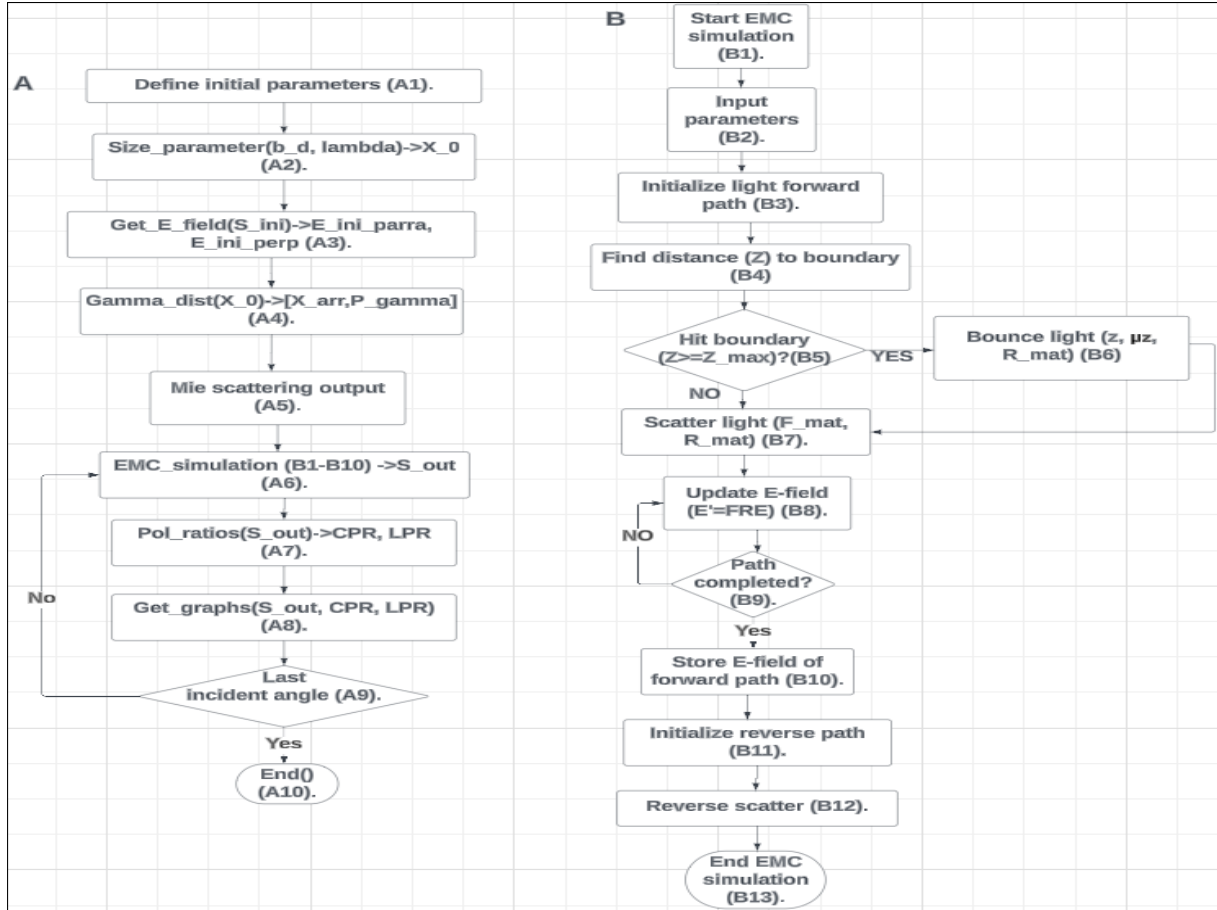


Figure 3.2: Flow chart for the photon propagation through a scattering medium. (A) The integration of the EMC simulation into a python workflow. (B) EMC simulation itself.

3.2.5 Monte-Carlo Electric-Field Integration Into A Python Workflow:

This subsection describes the Python workflow (A in Figure 3.2), here is a detailed description of the workflow:

Step 1 (A1 in Figure 3.2 A): Initialization, the initial parameters are defined.

Step 2 (A2 in Figure 3.2 A): The size parameter calculation, a function of the particle radius and wavelength.

Step 3 (A3 in Figure 3.2 A): Converts the initial stokes vector into the initial parallel (E_x), and perpendicular (E_y) E-field components.

Step 4 (A4 in Figure 3.2 A): Calculation of the Gamma distribution that generates an array of 100 gamma-distributed size parameters of bound $[0.5X_0, 2X_0]$ to mirror the distribution in Mishchenko and Dlugach [1992], Nelson et al. [2000].

Step 5 (A5 in Figure 3.2 A): Calculates the expected values of the scattering coefficient and amplitudes (see 3.1.5) using Mie scattering, and the Gamma distribution (A1 and A4 respectively in Figure 3.2 A).

Step 6 (A6 in Figure 3.2 A): The EMC is initialized where the expected values of A5 are used as input parameter to initiate the simulation.

Step 7 (A7 in Figure 3.2 A): Converts the output of A6 (electric field components) into polarization ratios.

Step 8 (A8 in Figure 3.2 A): Generate the graphs of the polarization ratios from A7 as a function R_{exit} and R_{inc} .

We calculate the average and associated error bar of the modeled polarization ratios over 5 trials using the same method as in Bourget et al. [2024b]. Finally, steps 6-9 (A6-A9 in Figure 3.2 A) repeat until the last incident angle in the R_{inc} array.

3.2.6 EMC Simulation:

This subsection describes the Electric-field Monte-Carlo (EMC) simulation itself (B in Figure 3.2). Step 1 (B1 in Figure 3.2 B) the EMC simulation is initiated using the Mie scattering output (A5 in Figure 3.2 A) as an input parameter. The distance z to boundary (z_{max}) is calculated in B4 of Figure 3.2. If the position of the photon satisfies $z \geq z_{max}$ (B5 in Figure 3.2), the photon bounces off the boundary where the position (Equation 3.74), direction (Equation 3.72), and rotation matrix (Equations 3.73-3.75) are updated accordingly (B6 in Figure 3.2). The azimuthal rotations in the time-reversed path (B11 and B12 in Figure 3.2 B) are a crucial aspect of the Electric-field Monte Carlo (EMC) simulation of coherent backscattering of light [Sawicki et al., 2008]. These rotations (B7, B8 in Figure 3.2 B) are essential for accurately modeling the behavior of light as it retraces its path in reverse order (Figure 2.2 B, also known as "time reverse path" (B12 in Figure 3.2 B) through a turbid medium. Polarization from which the Stokes vector arises (see 2.3) is time-independent, and the term "time reverse path" means that the path of the photon is retraced in the reverse order, such that the order of scattering events is reversed as seen in Figure 2.2 B. Here is a more detailed description of these special rotations:

Concept of time reverse path (B11, B12 in Figure 3.2 B): In a time-reversed path, light travels the same path through the medium but in the opposite direction to the forward path. This is key in simulating coherent backscattering, where light traveling in forward and reverse paths interferes constructively.

Need for special azimuthal rotations (B7, B8 in Figure 3.2 B): When light is scattered in a medium, its electric field vector is rotated to align with the local scattering plane at each scattering event. In the time-reversed path (B11 in Figure 3.2 B), the sequence of scatterings is retraced in reverse order. However, simply reversing the order of scatterings is not sufficient; the orientation of the electric field at each scattering event must also be correctly accounted for. This requires specific azimuthal rotations (B7, B8 in Figure 3.2 B) at certain points (first and last scatterer).

Rotations at the first scatterer in reverse path (B11 in Figure 3.2 B): The first scatterer in the reverse path is actually the last scatterer that light encounters in

the forward path. Two special rotations (B12 in Figure 3.2 B) are needed at this scatterer: The first rotation aligns the incoming (E_y) component with the normal of the scattering plane (defined by the directions of light before and after scattering). The second rotation adjusts the reference frame for the upcoming scattering event at the second scatterer in the reverse path.

Rotation at the last Scatterer in reverse path (B12 in Figure 3.2 B): A special rotation is also required at the last scatterer in the reverse path (which is the first scatterer in the forward path). This rotation aligns the (E_y) component with the normal of the upcoming scattering plane, ensuring that the electric field vector is correctly oriented for the final scattering event in the reverse path.

From the output E_x E_y , the Stokes parameters, and polarization ratio and their error bars (A6, A7 respectively in Figure 3.2 A) are calculated. To confirm the reliability of our model, CPR , and LPR of Hapke and Blewett [1991] were modeled and compared using identical initial conditions.

3.2.7 Examples and Tests

Here we perform several examples to test the Monte-Carlo simulation; the first test is identical to the test case from Sawicki et al. [2008], the second test is the study of CPR and LPR over incident angle for different indices of refraction m from Mishchenko [1992b], and the third test attempts to recreate the results from Figure 1 in Hapke and Blewett [1991].

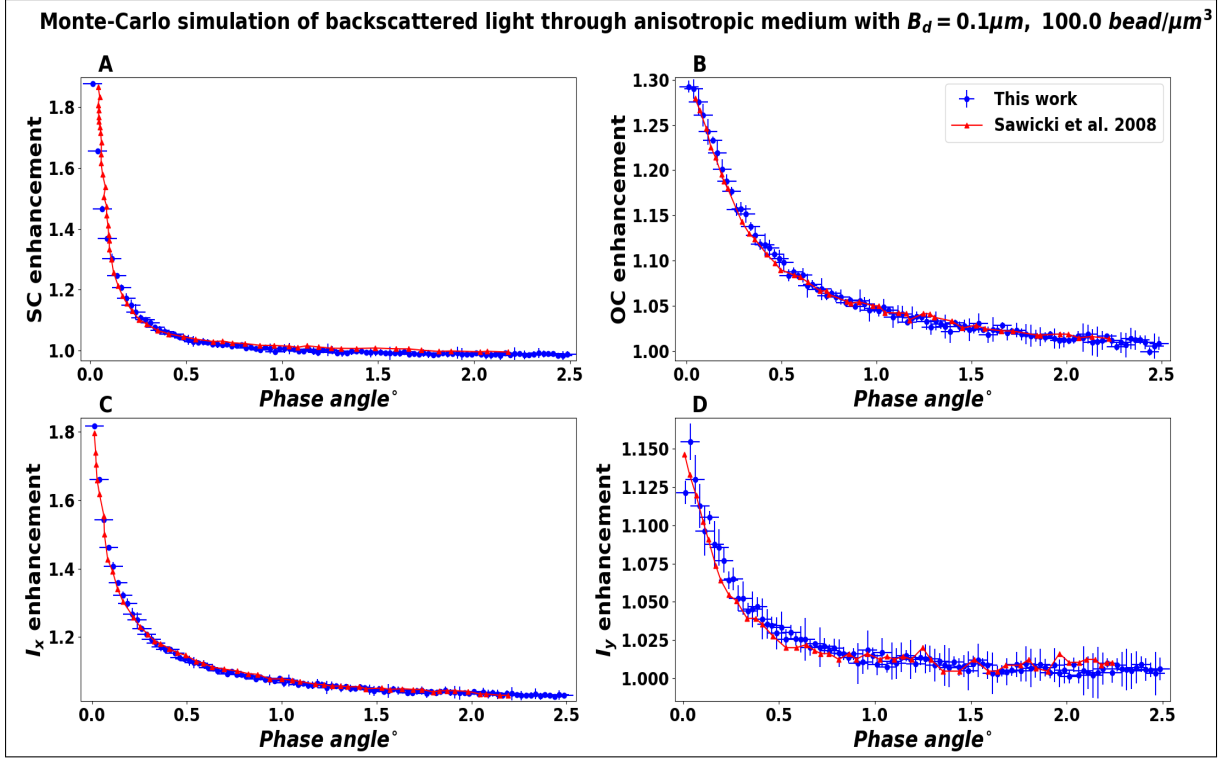


Figure 3.3: A: SC circular polarization enhancement as a function of phase angle. B: OC circular polarization enhancement as a function of phase angle. C: linear polarization enhancement alongside the x-axis. D: linear polarization enhancement alongside the y-axis. The scattering input parameters: $\langle X_0 \rangle = 0.611$, $\langle \beta_{sca} \rangle = 3.4266 \times 10^4$, $\langle \beta_{abs} \rangle = 1.6622 \times 10^{-11}$, $\langle g \rangle = 0.076$.

Enhancement of polarized light around the backscattering axis: In this section, we revisit the papers from Sawicki et al. and use the Monte Carlo simulation to reproduce the results from Figure 4 and 6 in Sawicki et al. [2008]. We compare the enhancement of the backscattered intensity in the *SC* and *OC* circular polarization (A and B in Figure 3.3) to Figure 4 in Sawicki et al. [2008] for *RCP* light with initial Stokes vector $S_{ini} = [1, 0, 0, -1]$. We also compare the enhancement of the backscattered intensity along the x-axis (C in Figure 3.3), and y-axis (D in Figure 3.3) to Figure 6 in Sawicki et al. [2008] for linearly polarized light with initial Stokes vector $S_{ini} = [1, 1, 0, 0]$. For all curves in Figure 3.3 the enhancement decreases as the phase angle increases, which is typical in backscattering studies where the intensity of scattered light usually decreases with increasing angle due to the geometry and physics of light scattering. The blue data points (this work in Figure 3.3) show a nearly identical trend to those from Sawicki et al. Figure 4 and 6 in Sawicki et al. [2008] indicate a reliable model. Study of sensitivity to initial Stokes vector shows stable behavior in the enhancement along the x-axis and y-axis, with more variation for

the enhancement along the y-axis. However, the enhancement along the x-axis and y-axis has a stable negative Lyapunov exponent from $0^\circ - 5^\circ$ indicating a stable non-chaotic behavior.

Relationship to Incident Angle: In this section, we plot the circular polarization and linear polarization ratio (CPR and LPR) as a function of incident angle at a 0° phase angle, and compare the results to Mishchenko [1992b]. In Figure 3.4 we illustrate

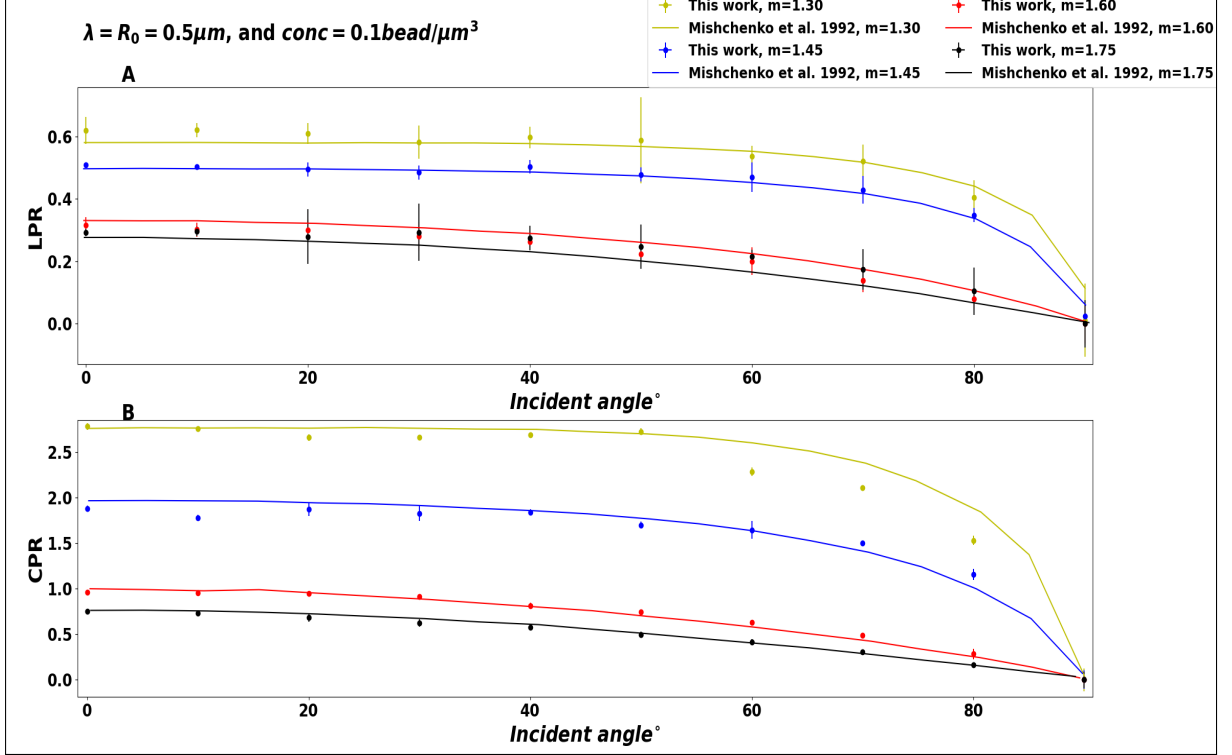


Figure 3.4: A: LPR as a function of incident angle at 0° phase angle for different index of refraction (yellow-black curves). B: CPR as a function of incident angle at 0° phase angle for different index of refraction (yellow-black curves). The scattering input parameters: $\langle X_0 \rangle = 6.087$, $\langle \beta_{sca} \rangle = 3.36855 \times 10^5$, $\langle \beta_{abs} \rangle = 8.5415 \times 10^{-11}$, $\langle \beta_{bs} \rangle = 1.43520 \times 10^5$, $\langle g \rangle = 0.7717$.

the (LPR) and the (CPR) as functions of the incident angle, with both measured at a 0° phase angle. The plots compare the LPR and CPR simulation results with those reported by Mishchenko (Fig. 6, and Fig. 9 in Mishchenko [1992b]) for different indices of refraction (m) of the scatterers. In the case of LPR in Figure 3.4 A, a similar trend to CPR is evident. However, the LPR values are generally lower than those of CPR , suggesting that the backscattering effect is less pronounced in linear polarization compared to circular polarization for the range of incident angles considered, being consistent with Mishchenko in Mishchenko [1992b]. For all refractive indices, LPR decreases with the increasing incident angle, although the rate of decrease and the absolute values of LPR are influenced by the refractive index. For CPR in Figure 3.4 B, it's observed that as the index of refraction increases, the peak of the CPR decreases. This trend is consistent

across all refractive indices, with the CPR starting near unity at a 0° incident angle for $m = 1.6$ (red curve in Figure 3.4 A) and $m = 1.75$ (black curve in Figure 3.4 B) and gradually declining as the incident angle increases. The decline is the most pronounced for lower refractive index values, indicating a strong dependence of CPR on both the incident angle and the refractive index of the scatterers. In both models (Figure 3.4 A, B), and Fig. 6, Fig. 9 in Mishchenko [1992b] CPR , and LPR are inversely proportional to the index of refraction. The simulation results derived from the implementation of the EMC method show a qualitative agreement with Mishchenko's findings, displaying similar trends in the behavior of polarization ratios with changing incident angles and refractive indices.

Hapke et al. 1991 results revisited: Here we reproduce the *CPR*, and *LPR* results in Figure 1 in Hapke and Blewett [1991]. We choose our concentration such that the modeled amount of solid (10%) within the colloidal latex beads solution matches that of Hapke and Blewett [1991]. The plots of Figure 3.5 represent different aspects of

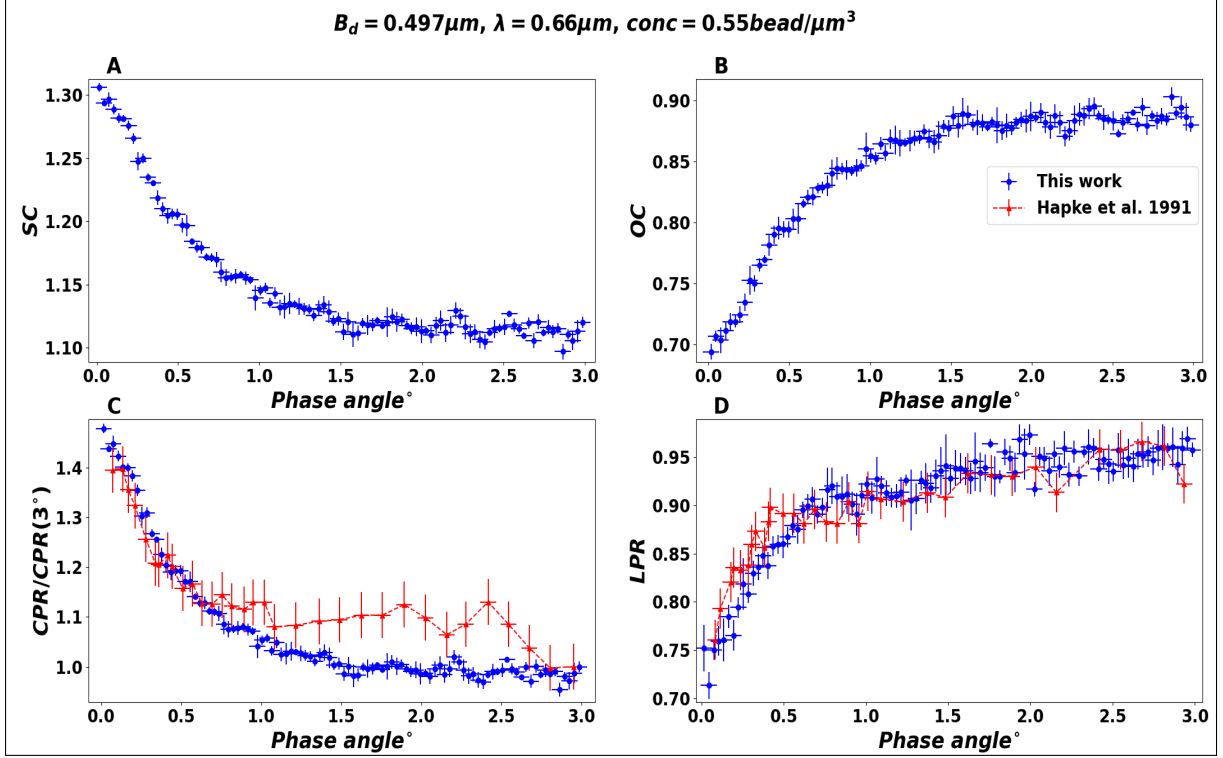


Figure 3.5: (A) is the modeled *SC* (blue dots). (B) is the modeled *OC* (blue dots). (C) is the modeled *CPR* (blue dots), and comparison to fig 1 (red dashed line) in [Hapke and Blewett, 1991]. (D) is the modeled *LPR* (blue dots), and comparison to fig 1 (red dashed line) in [Hapke and Blewett, 1991].

polarized light scattering and their trends with respect to the phase angle. The blue dots represent our simulation results, and the red dots are the experimental results from Hapke and Blewett [1991, Figure 1]. For the *SC*, the simulation results (blue dots in Figure 3.5 A) show a steep decrease in *SC* with increasing phase angle. This indicates that, as the phase angle increases, the intensity of light that is circularly polarized in the *SC* decreases sharply. The *OC* curve increases with phase angle, but the rate of increase is less steep compared to the rate of decrease of the *SC* curve. The graph shows that *CPR* decreases with increasing phase angle, indicating that the proportion of same sense polarization compared to opposite sense polarization diminishes with phase angle. The *CPR* simulation (blue dots in Figure 3.5 C) had to be normalized by the value at 3° in order for agreement

with the [Hapke and Blewett, 1991] results. Our *CPR* curve shows a similar behavior to the *CPR* from Hapke and Blewett [1991] with some discrepancies between $1.5^\circ - 2.5^\circ$; however, the peak area and HWHM between simulation and experimental *CPR* data are within the margin of error (Table 5 in .6). The agreement in the trend suggests that the simulation captures the relative changes in *SC* and *OC* correctly. Finally, the simulation results (blue dots in 3.5 D) for the *LPR* show an upward trend with increasing phase angle, which could indicate that the amount of linear polarization relative to the amount of circular polarization increases with phase angle. Compared to the results of Figure 1 in Hapke and Blewett [1991], the simulation appears to capture the general behavior of *LPR* where the HWHM and peak area are within the margin of error (Table 5 in .6) for simulation and experimental data, with some discrepancies in the exact values due to experimental error in Hapke and Blewett [1991]. In summary, our simulations capture the general behavior of *CPR* and *LPR* being within the margin of error of Hapke and Blewett [1991] data indicating a reliable model.

Chapter 4

Experimental Methods

In this chapter, we detail the technique to measure the Stokes parameters in 5.1.1 to understand the backscattering condition at 0° phase angle of our Lambertian surfaces in 5.4.1, and 6.1, our alumina powders in 5.4.2, and 6.2.1, our colloidal solution of latex beads (non-dye and dye) in 5.4.3-5.4.4, and 6.2.2-6.4. However, the beam splitter in 5.1.3 that enables measurement of backscattering conditions induces an increase in linear (S_1) and 45° (S_2) that we address in 4.2-4.2.1, and 5.1.4.

4.1 Measuring Stokes Parameters

4.1.1 Rotating Quarter Wave Plate Technique

It is important to understand that lower case θ refers to the scattering angle of the phase function from 3.1.4, while upper case Θ refers to the rotation angle of the QWP (see Figure 5.1 in chapter 5). Static polarimeter set-ups utilize multiple detectors with no moving optical components in the optical path. Dynamic methods are based on moving optical components in the optical path and utilize a single detector. In this project, we used the dynamic method called the Rotating Quarter Wave Plate Technique (RQWP). Advantages of a dynamic polarimeter include the elimination of multiple detectors and the need to develop error reduction algorithms that arise in static multi-channel polarimeters Arnoldt [2011]. The Stokes parameter of a signal can be measured through the RQWP technique as shown in the schematic setup in Figure 4.1. The test signal is modulated by

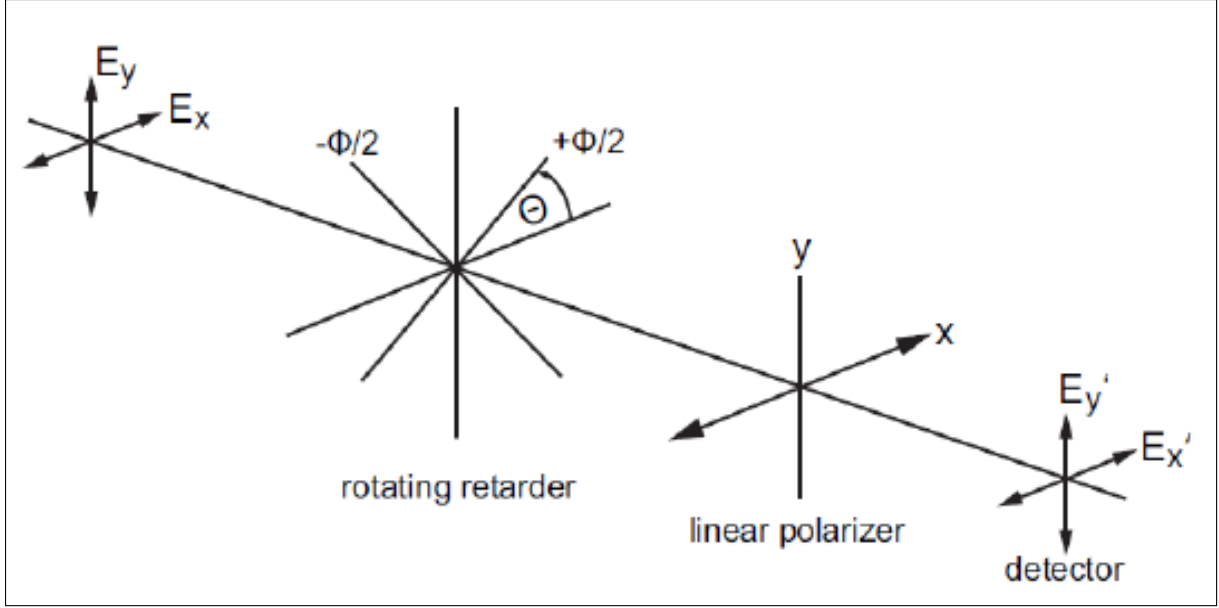


Figure 4.1: Schematic for the Rotating Quarter Wave Plate Technique Arnoldt [2011]

a rotating quarter wave plate, after which it passes through a linear polarizer and into the detector as seen in 5.1.1. The measured intensity $I(\Theta)$ at the detector is a function of the quarter wave plate rotation angle Θ from Arnoldt [2011].

$$I(\Theta) = \frac{1}{2}[A - B\sin(2\Theta) + C\cos(4\Theta) + D\sin(4\Theta)] \quad (4.1)$$

The intensity (Equation 4.1) is a truncated Fourier Series whose coefficients from Arnoldt [2011] are:

$$A = \frac{1}{\pi} \int_0^{2\pi} I(\Theta) d\Theta \quad (4.2)$$

$$B = \frac{2}{\pi} \int_0^{2\pi} I(\Theta) \sin(2\Theta) d\Theta \quad (4.3)$$

$$C = \frac{2}{\pi} \int_0^{2\pi} I(\Theta) \cos(4\Theta) d\Theta \quad (4.4)$$

$$D = \frac{2}{\pi} \int_0^{2\pi} I(\Theta) \sin(4\Theta) d\Theta \quad (4.5)$$

From the Fourier coefficients (Equations 4.2-4.5) we can describe the Stokes parameters of an electromagnetic wave:

$$S_0 = A - C, S_1 = 2C, S_2 = 2D, S_3 = B, \quad (4.6)$$

As the quarter wave plate angle is rotated from $0 - 360^\circ$, $(0 - 2\pi)$ in 5° increments, the intensity values at each interval are recorded. The coefficients of the truncated Fourier Series (Equations 4.2-4.5) are calculated by integrating the observed intensity values (Equation 4.1) over the rotation angle range (Θ) . The Stokes vector that describes the polarization state of a signal is derived from the coefficients in the above Equation 4.6 as seen in 5.1.1.

4.2 Measuring Mueller Matrix

The polarization state of an electromagnetic wave can be determined by measuring the Stokes vector of the signal. The Stokes vector has four components that represent its total intensity and polarization state. The signal propagating through an optical medium can undergo polarization change that alters the Stokes parameters of the signal propagating outward from the medium. A circularly polarized signal can alter its polarization state to elliptically polarized while propagating through an optical medium. We can correct for these polarization alterations by measuring the Mueller matrices of the optical medium and determining its impact on the Stokes parameters [Goldstein, 1992]. In our case, we measure the Mueller matrix of the beam splitter in 5.1.4 to correct for the increase in (S_1) and (S_2) induced by the beam splitter.

4.2.1 Dual Rotating Quarter Wave Plate Technique

The Mueller matrix of our beam splitter in 5.1.4 is measured using the dual rotating quarter waveplate technique that determines its polarization properties and its impact on the propagating signal. The errors in retardance of the quarter wave plate, imperfect retardation increment, and misalignment in the polarizing components can be corrected using error analysis [Arnoldt, 2011, Goldstein, 1992]. The dual rotating quarter wave plate

technique measures a chopped signal that is modulated by rotating the polarizing optical elements. The signal is Fourier analyzed after passing through linear polarizers and dual rotating retarders to determine the Mueller matrix elements (element 9 and 10 in Figure 5.1. As shown in Figure 4.2 the two fixed linear polarizers and rotating quarter wave plates are aligned with respect to their transmission axes and fast axes. The second retarder is rotated at five times the rate the first retarder is rotated, which generates twelve harmonic frequencies in the Fourier Spectrum of the modulated intensity [Goldstein, 1992]. The intensity values are measured from the output signal as both the QWP's are incremented in a 1 : 5 ratio such that the first retarder rotates through 180° . The minimum number of equations required to solve for the coefficients uniquely is 25, with the first retarder rotating at 7.2° through 180° . There are many optimal methods to formulate the Fourier coefficients from the intensity values [Smith, 2002] one of which was used by Goldstein in Goldstein [1992] and was applied to the calculation. The first retarder was incremented in 5° degree increments through 180° , resulting in 36 equations that were solved to obtain the Fourier coefficients.

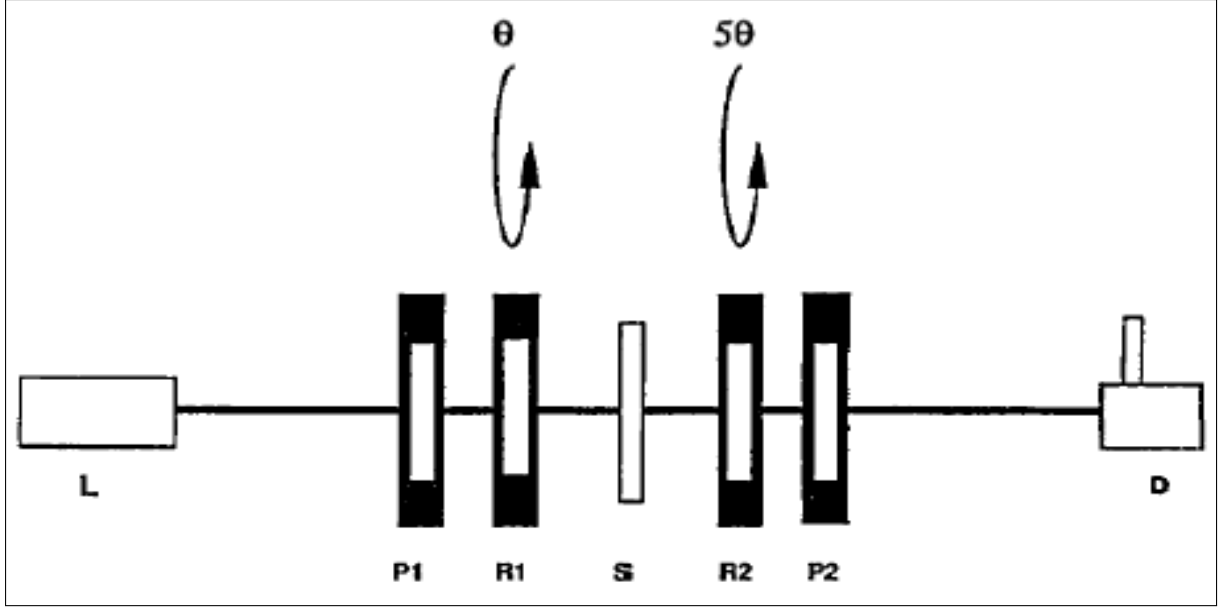


Figure 4.2: Dual Rotating Quarter Wave Plate Technique Schematic Goldstein [1992]

The resulting system of equations is written as:

$xa = I$, I : vector of 36×1 intensity values.

a :, vector of 25 Fourier coefficients.

x :, matrix of 36×25 harmonic frequencies of the form shown below where Θ values for each row are fast axis angle for the first retarder such that

$$(\cos(2\Theta)\cos(4\Theta)\dots\cos(24\Theta)\sin(2\Theta)\sin(4\Theta)\dots\sin(24\Theta)) \quad (4.7)$$

The solution for the formulated Equation 4.7 is represented as follows;

$$a = (x^T x)^{-1} x^T I \quad (4.8)$$

Solution acquired through this method (Equation 4.8) was similar to the least squares solution expressed as:

$$I(\Theta) = a_0 + \sum_{j=0}^{12} (a_j \cos(2_j \Theta) + b_j \sin(2_j \Theta)) \quad (4.9)$$

Solving the Equation 4.7 results in 12 sine and 12 cosine harmonic Fourier coefficients that are used to calculate the individual 4×4 Mueller matrix elements. Furthermore, the Mueller matrices are normalized using their first element (m_{11}) to acquire the normalized

Mueller matrix with values ranging from $-1 < x < +1$ [Arnoldt, 2011, Goldstein, 1992].

Chapter 5

Experiment

This chapter is based on Bourget et al. [2024b], the alignment procedure in 5.2, the data acquisition set-up in 5.3, and finally the sample section and preparation in 5.4. All acronyms are defined in Table 9 in .9.

5.1 Experimental Procedure

5.1.1 Measurement Procedure

We use a Multi-Axis Goniometer instrument (MAGI, Figure 5.1) composed of a linear polarizer (LP, 5 in Figure 5.1) and a quarter-wave plate (QWP, 6 in Figure 5.1). We generate an incident right circularly polarized (RCP) signal defined by $S_i = [S_0, S_1, S_2, S_3] = [1.0, 0, 0, -1.0] + / - [0.02, 0.02, 0.02, 0.02]$. The analyser (components 9 and 10 in Figure 5.1) is composed of a rotating QWP and a LP that enables the measurement of vertical and horizontal polarization to calculate the Stokes parameters in 4.1.1. To prevent unwanted scattered light from reaching the detector, the incident signal is split by the beam-splitter (BS, 7 in Figure 5.1), and half is sent through a beam trap consisting of neutral density filters (12 in Figure 5.1). The reflected Stokes vector is calculated using the QWP technique which consists of averaging the scattered light from the sample through a rotating QWP (9 in Figure 5.1) over the $[0 - 2\pi]$ domain, a LP (10 in Figure 5.1), and a photo-diode detector (11 in Figure 5.1) [Arnoldt, 2011], 4.1.1. The BS (7) in Figure 5.1 is mounted on a optical rail and starts at 42mm at 0° phase angle (Figure 5.3 in 5.1.3). For phase angle

$\sim 1.5^\circ$ the BS (7) in Figure 5.1 had to be moved -4mm (left) on the 45° plane, for phase angle $\sim 2^\circ$ the BS (7) had to be moved $+6\text{mm}$ (right) relative to the start position on the 45° plane to prevent the incident beam from interacting with the edges of the BS (7). The sample platform is made out of metal painted in black.

Stokes' parameters can be expressed as a vector such that $S = [S_0, S_1, S_2, S_3]$, where S_0 is the unpolarized intensity, S_1 is the horizontal/vertical polarization, S_2 is the $+/-45^\circ$ polarization, and S_3 is right/left circular polarization 2.3, [Arnoldt, 2011]. The Stokes parameters are expressed independently of intensity such that $[\frac{S_0}{S_0} = 1, \frac{S_1}{S_0}, \frac{S_2}{S_0}, \frac{S_3}{S_0}]$, the same is done in our experiment and model 3.2. Our experiment in the Figure 5.1 consists of measuring unpolarized S_0 and circular polarization S_3 at low phase angle ($0^\circ - 5^\circ$), at 0.5° interval. From Stokes parameters, same sense ($SC = S_0 - S_3$) 2.4.1 and opposite sense ($OC = S_0 + S_3$) 2.4.2 circular polarization and circular polarization ratio ($CPR = \frac{SC}{OC}$) and degree of linear polarization (DLP) are calculated in 2.5 and plotted over phase angle in Figures 6.1-6.7. One notable advantage of MAGI is its ability to facilitate polarimetric measurements at a 0° phase angle (angle between laser source and detector) using a 45° beam-splitter (BS, 7).

We measure reflection Stokes' parameters of latex beads in solution. Detailed characteristics of the latex beads are documented in Tables 2-4 in .3-5. It's worth noting that aqueous solutions of micro-beads were used due to their utility as optical analogs, given that the size, concentration, and shape of the beads can be varied to produce a variety of scattering medium analogues. This study considers the size parameter $X = \frac{\pi d}{\lambda}$, signifying the bead size with respect to the illumination wavelength. For the various bead sizes: $X \geq 1000$ is in the geometric regime where classical optics can be applied, $10 \leq X \leq 1000$ it is in the large Mie regime, $0.3 \leq X \leq 10$ is in the classical Mie regime, for $X \leq 0.3$ is in the Rayleigh scattering regime [Zhao et al., 2019]. Studied samples have 3 different scattering regimes, the Rayleigh regime ($0.08 \mu m$), the classical Mie regime ($0.81 \mu m, 2.1 \mu m, 4.0 \mu m$) and the Mie regime for large particles ($8.3 \mu m, 30 \mu m$). Our alumina powder samples and colloidal solutions of latex beads follow the same gamma distribution $n(r)$ of sizes as in Mishchenko and Dlugach [1992]. To confirm the scattering regime of the alumina samples and latex beads samples we computed the expected value

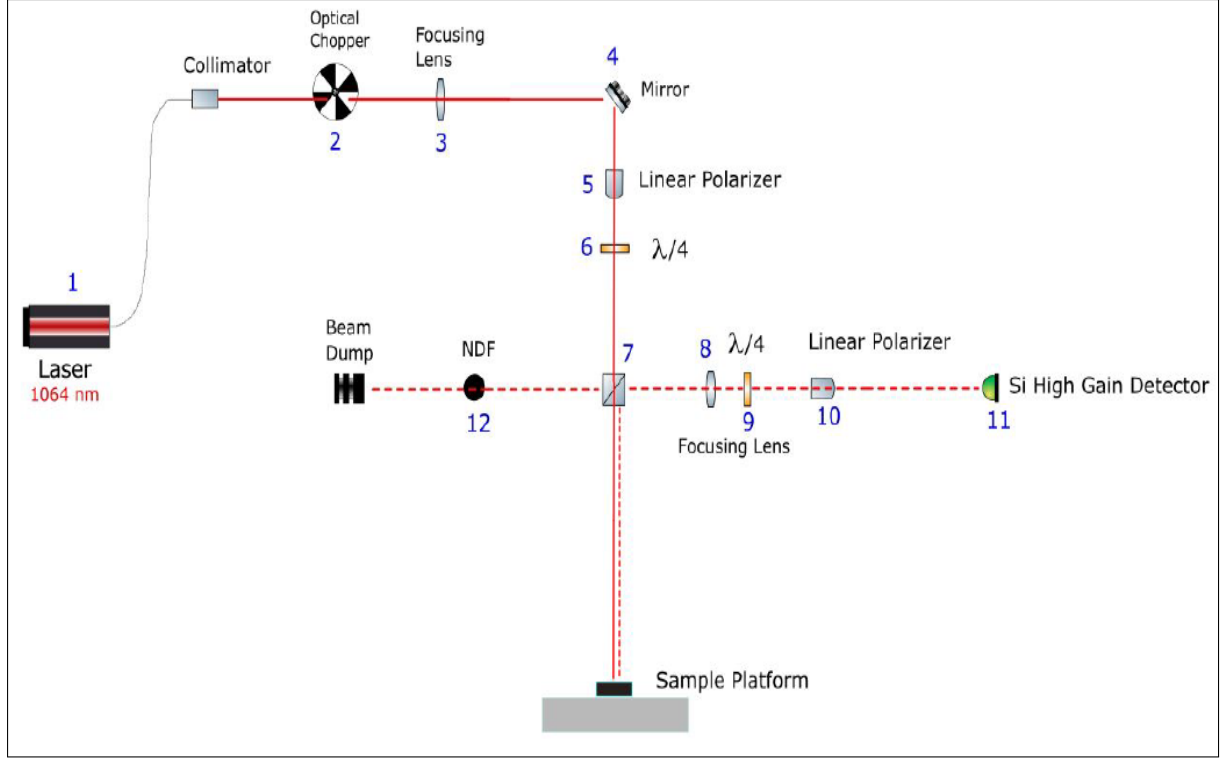


Figure 5.1: MAGI schematic Blewett et al. [2018], optical components 1-6 generate right circular polarization and are installed on a caddy. Components 8-11 are installed on a polarimetric arm and serve as an analyser for the scattered light from sample. Both caddy and arm can move independently.

of the size parameter $< x >$ in .2 using Equation 3.53 from 3.1.5 for the central size parameter X_0 . The expected values, $< \beta_{sca} >$, $< \beta_{abs} >$, $< \beta_{bs} >$, and $< g >$ are defined and derived in 3.1.5 for a gamma distribution $n(x)$, and are computed in Table 1 in .2 for alumina powders, Table 2 in .3 for Rayleigh ($0.08\mu m$), Table 3 in .4 for classical Mie ($0.81\mu m$), and Table 4 in .5 for large Mie ($8.3\mu m$) beads.

The bead solutions were stored in a translucent cylindrical glass cup with dimensions $R = 2.5Cm$, $H = 2.0Cm$ while the alumina powders were kept in a black metallic cup of identical dimensions; the black metallic cup is of the same material as our black metallic sample platform. To determine the influence of the reflection from the sample platform we record the scattered intensity from the sample platform and pour the samples in increment of 1mm thickness and record the intensity as a function of thickness in Figure 1 at a 0° phase angle for both alumina and latex beads, and at 0° incident angle (alumina powders) and 15° incident angle (latex beads). Unlike the Al_2O_3 powders, latex beads are in solution thus they will generate unwanted surface specular reflection characterized by a

surge in OC circular polarization (2.4.2) from the top of the solution. Hence, the Stokes' parameters of latex beads are taken at a 15° incidence angle (angle between the sample's normal and the laser source) such that we only measure subsurface (volume) scattering. To verify that the surface specular reflections are negligible we look at the SC , OC , and CPR curves for the sample platform (yellow curves in Figure 6.1), the water glass cup (blue curves in Figure 6.1), the glass water cup on top of the 60% Lambertian surface (magenta curves in Figure 6.1), and the 60% Lambertian surface alone (black curves in Figure 6.1) at $i = 0^\circ$ (squares in Figure 6.1), and $i = 15^\circ$ (triangles in Figure 6.1). If the surface specular reflections are negligible then we should expect the glass water cup on top of the 60% Lambertian surface (magenta curves in Figure 6.1) to have SC , OC and CPR curves matching that of the 60% lambertian surface alone at $i = 15^\circ$ incident angle. If the surface specular reflections are not negligible, we should expect the SC , OC and CPR curves of the glass water cup on top of the 60% Lambertian surface (magenta curves in Figure 6.1) at $i = 15^\circ$ incident to have SC , OC and CPR curves matching that of the water glass cup at $i = 0^\circ$ incident angle. The Lambertian surfaces are cylindrical, and are diffuse reflectance standards made by Labsphere and have reflectance from 99% – 2% of the incident light from 400nm – 1500nm. The standards have dimensions $R = 3.025Cm$, $H = 1.52Cm$, and are composed of a fluoropolymer [LabSphere, 2024]. To illustrate the results we plot the average (5-trials) same sense (SC) and opposite sense (OC) circular polarization, circular polarization ratio (CPR), and degree of linear polarization (DLP) over a phase angle range from $0 - 5^\circ$. The reflected Stokes parameters S_0 (unpolarized) and S_3 (circular polarized) are computed for each phase angle, with SC and OC defined as $S_0 - S_3$ and $S_0 + S_3$, respectively. The error bars in Figures 6.1-6.7 are calculated by computing the standard deviation (population) of the data set (5-trials). To confirm the reliability of the instrument, measurements from standardized AL_2O_3 powders by WCA Micro Abrasives Corp were compared with the results from Nelson et al. [2000]. Information regarding alumina powders can be found in Table 1 in .2. The relationship between void space and particle size is shown in Fig. 1 in Nelson et al. [2018]. The particle sizes $< \sim 1.5\mu m$ (shown as 'equant' in the Figure and designated 'GB') have much more void space than the particles of size $> \sim 2.1\mu m$ (Shown as 'Platelet' in Fig. 1 in Nelson et al. [2018] and

are designated WCA). This is consistent with what is reported by the manufacturer as reported in Nelson et al. [2000, 2002], and Nelson et al. 2016. Figure 1 in Nelson et al. [2018] shows that the void space of alumina samples depends on the particle size and is inversely proportional to the particle size.

5.1.2 Alignment Procedure

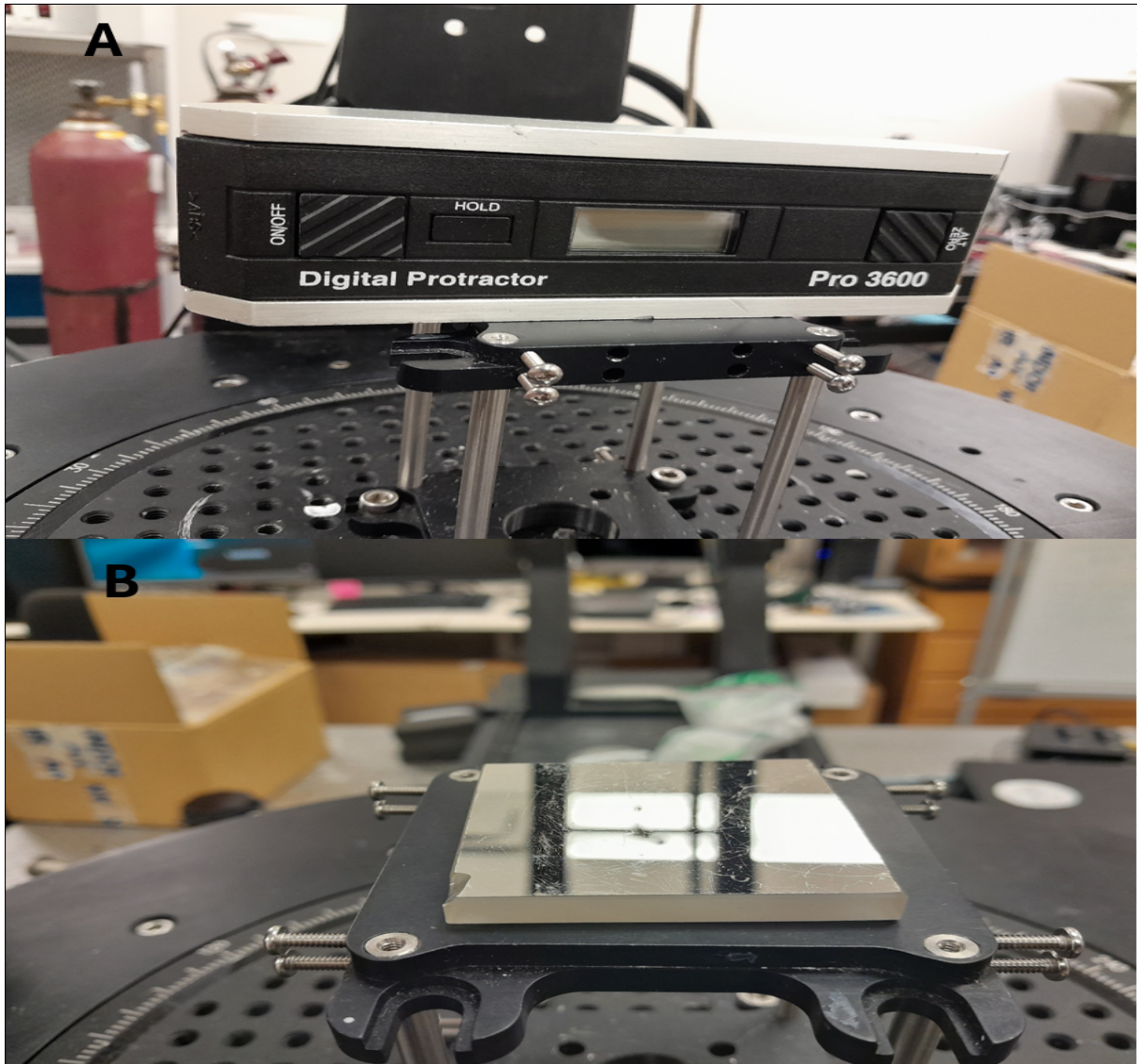


Figure 5.2: A: The digital level meter to align the Caddy and arm platform with respect to the sample platform. B: Square Alumina coated mirror for alignment.

To align MAGI we use two methods, first we use a level meter in Figure 5.2 (A) to level out the Caddy platform in Figure 5.7 and the arm platform in Figure 5.8 with respect to the sample platform. Second we use a square alumina coated mirror in Figure 5.2 (B) to align the laser beam with the arm platform in Figure 5.8.



Figure 5.3: Beam splitter (BS) mounted on a 45° rail to enable $0 - 5^\circ$ phase angle measurements without any setup restrictions

5.1.3 Moving the Beam Splitter

For phase angle (angle between the source and detector), $1.5^\circ \leq \theta \leq 2.5^\circ$ the beam splitter (BS, 7 in Figure 5.1) will interact with the incident laser beam, thus generating unwanted changes in polarization. The data points between $1.5 - 2.5^\circ$ range cannot be acquired due to mounting restrictions on the beam splitter. To make measurement from $0 - 5^\circ$ possible, we mount the beam splitter on the arm platform with a 45° rail in Figure 5.3 to enable acquisition of measurements over $0 - 5^\circ$ without any setup restrictions. The mounted beam splitter is able to move along the rail platform but the platform would have to be very precisely mounted on the 45° plane ($\pm 0.1^\circ$) as any small deviation $\geq 0.2^\circ$ in the angle would cause the back-scattered signal to miss the analyzer or interact with the edges of the beam splitter, causing unwanted changes in polarization. To check the precision of the beam splitter set-up in Figure 5.3, we measure the back-scattered Stokes vector $S = [S_0, S_1, S_2, S_3]$ for each displacement of the beam splitter along the 45° rail in Figure 5.3 relative to the start position 42mm of the BS(7). If the displacement of the beam splitter is such that the back-scattered signal misses the analyzer or is interacting with

the edges of the beam-splitter, we should observe a change in the backscatter signal in Figure 5.4 greater than the margin of error in the absence of a change in phase-angle or incident-angle.

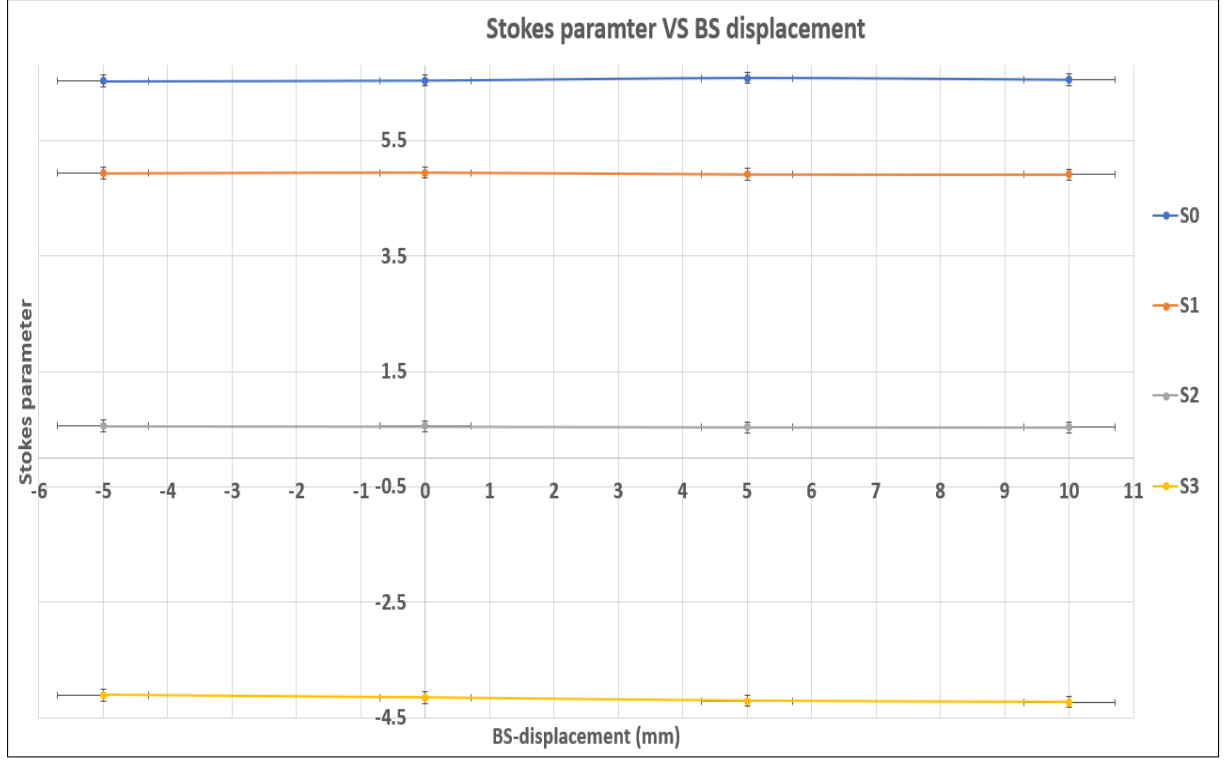


Figure 5.4: bacs-scattered Stokes parameters [mV] $S_0 - S_3$ versus beam-splitter position, left (-) and right (+)

In Figure 5.4 we measure the backscattered Stokes vector $S = [S_0, S_1, S_2, S_3]$ as a function of the beam splitter displacement in mm relative to the start position 42mm of the BS (7) that represents the range in which the beam splitter can be displaced without missing the analyser or causing changes to the polarization. Hence using Figure 5.4, for phase angle ~ 1.5 the BS (7) can be moved -4mm (left) on the 45° plane, for phase angle $\sim 2^\circ$ the BS (7) can be moved $+6\text{mm}$ (right) relative to the start position on the 45° plane in Figure 5.4 without changing the geometry of the system nor interacting with the edges of the BS (7), leaving the scattered Stokes vector relatively (within the margin of error) intact.

5.1.4 Mueller Matrix Correction

In the experimental setup in Figure 5.1, the incoming signal was propagated and reflected through the beam splitter mounted on the arm platform. Several inconsistencies were measured from the received backscatter due to uneven perpendicular and parallel polarization reflections from the beam splitter. These inconsistencies would change the Stokes parameter of the signal when interacting with the beam splitter. The polarization change caused by the beam splitter on the signal was analyzed using the Mueller matrix of the beam splitter. Figure 5.5 shows the signal propagating through the beam splitter (BS, 7)

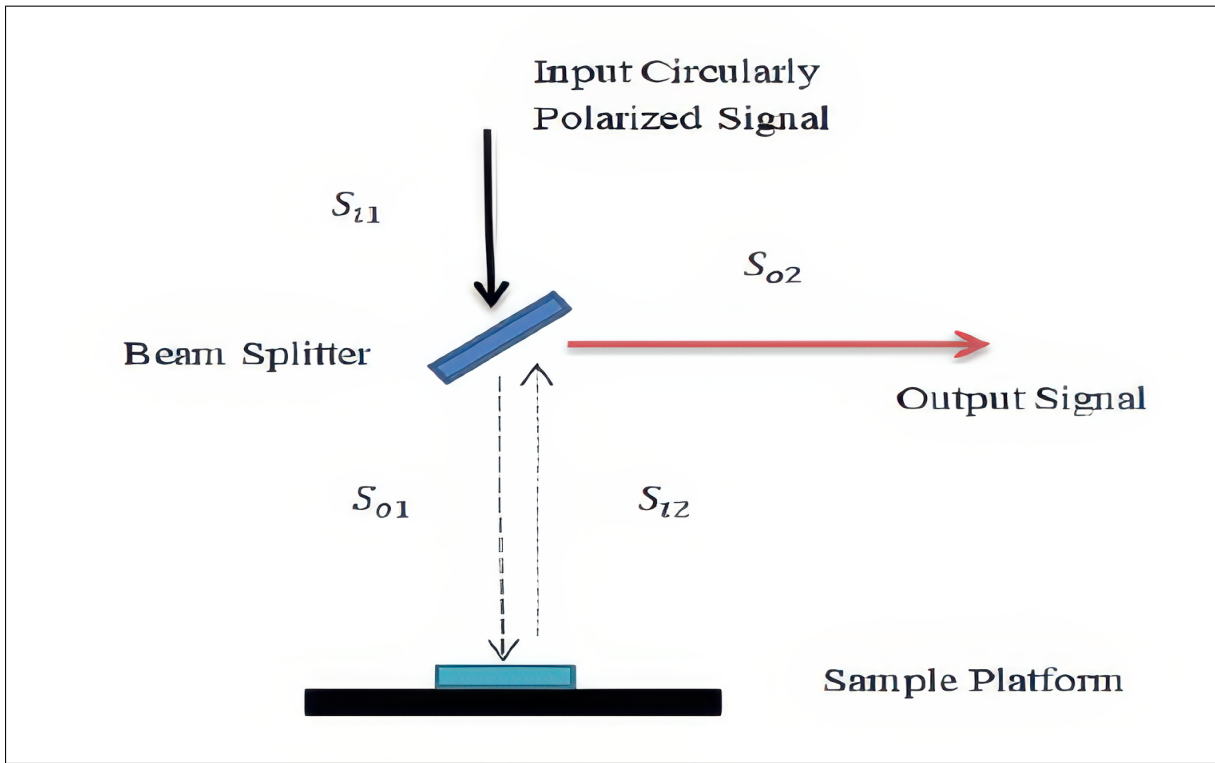


Figure 5.5: Variations in the polarized signal propagating and reflecting from the beam splitter

before and after interacting with the sample platform with individual polarization states represented through different Stokes vectors. The incoming signal S_{i1} was right/left handed circularly polarized, which changes its polarization state after propagating through the beam splitter to S_{o1} . After diffused scattering from the sample platform, the backscattered signal can be represented as an incoming signal towards the beam splitter as S_{i2} . The signal changes its polarization state to S_{o2} after reflecting from the 45° angled beam splitter. The beam splitter changes the polarization state of the signal twice; this change

can be measured by observing the Mueller matrix of the beam splitter. The relationship between the different polarization states of the signal is as follows:

$$S_{o1} = M_1 * S_{i1}, S_{i1} = (M_1)^{-1} * S_{o1} \quad (5.1)$$

$$S_{o2} = M_2 * S_{i2}, S_{i2} = (M_2)^{-1} * S_{o2} \quad (5.2)$$

Where M_1 and M_2 are Mueller matrices of the beam splitter. The Mueller matrix of the beam splitter was calculated for the first case (Equation 5.1) where the incoming signal propagated through the beam splitter and was incident on the sample platform. Using the dual rotating quarter wave plate technique, the intensity values were measured and Fourier analyzed to calculate the Mueller matrix as shown in section 4.2.1 in chapter 4. The Mueller matrix associated with Equation 5.1 is:

$$M_1 = \begin{bmatrix} 1 & -0.0401 & 0.2138 & 0.0032 \\ -0.0453 & 0.7694 & -0.0033 & -0.0003 \\ 0.0017 & -0.0033 & 0.7601 & 0.0008 \\ 0.0069 & 0.0112 & -0.0165 & 0.8763 \end{bmatrix} \quad (5.3)$$

Equation 5.3 is the Mueller matrix M_1 that resembles the no sample identity matrix from the calibration procedure. A signal propagating through the beam splitter would retain much of its linear and circular polarization state as the diagonal elements are positive and close to unity while the off-diagonal elements are near zero. This implies that the polarization state of the circularly polarized signal S_{o1} incident on the sample platform would be similar to the polarization state of the signal S_{i1} generated from the polarizer and quarter wave plate. For a completely circularly polarized signal propagating through the beam splitter, 87% of the polarization state will be retained. Secondly, using the observed backscatter signal S_{o2} and the second Mueller matrix M_2 (Equations 5.2, 5.4), the original diffused signal S_{i2} from the sample surface was calculated. By calculating the polarization state of the signal before it reflected from the beam splitter, we eliminated the effects of polarization change from the beam splitter. The method of eliminating the

polarization change from an optical element by calculating the Stokes vector before the interaction is referred to as the Mueller matrix correction.

$$M_2 = \begin{bmatrix} 1 & -0.0233 & 0.0257 & 0.0122 \\ 0.2350 & 0.2517 & -0.7709 & -0.0832 \\ 0.0810 & -0.7794 & -0.2454 & -0.0965 \\ -0.0302 & 0.0497 & 0.1011 & -0.8539 \end{bmatrix} \quad (5.4)$$

The degree of change in the polarization state of the backscattered signal from the sample platform (Equation 5.2) towards the analyzer can be represented through the second Mueller matrix (Equation 5.4). The m_{44} element in the matrix represents the degree to which the circular polarization state would change. As the element is negative, it would flip the polarization state of the incoming signal from *RCP* to *LCP* and vice versa. The value of m_{44} is near unity, which implies that it would allow only 86% of the original polarization state to propagate. The low values for m_{22} and m_{33} elements and high negative values for m_{23} and m_{32} indicate that the linear and 45° polarization state of the output signal would be skewed. A signal propagating through the beam splitter would not undergo much change in the polarization state; however, a signal reflecting from the beam splitter would have a significantly different polarization state. This change in the state of the output signal would not allow for correct interpretation of the backscatter results. To avoid this error, the Mueller matrix correction technique was applied and the effects of the beam splitter were eliminated from which the original backscattered data was calculated. The S_{i2} signal (backscattered signal from the sample) was calculated for all samples from the observed S_{o2} signal in Equation 5.2 and M_2 Mueller matrix (Equation 5.4) as shown in Chapter 6.

5.2 Multi-Axis-Goniometer-Instrument

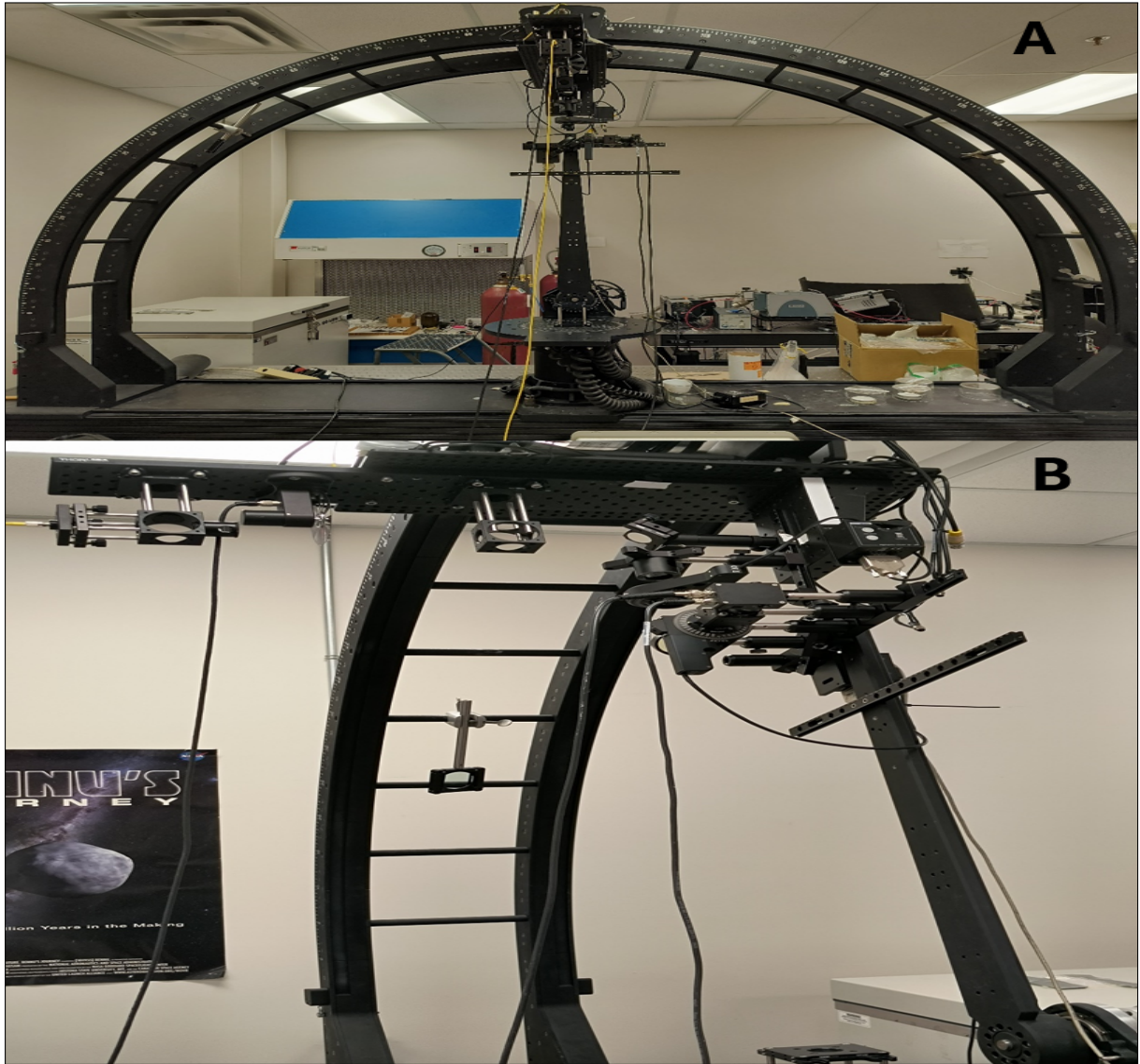


Figure 5.6: A: Multi-Axis Goniometric Instrument used for near zero phase angle measurements at 0° incident angle. B: 15° incident angle (Bottom). The ND plate at the back of the Goniometer corresponds to 12 in Figure 5.8 (A) and Figure 5.9 (B).

The near zero phase angle setup is built upon the Multi Axis Goniometric Instrument (MAGI) in Figure 5.6. MAGI is a long arm goniometric photopolarimeter in Figure 5.8 capable of measuring backscatter over 180° region with precise 0.1° increments. The MAGI has two moving components called the caddy platform (Figure 5.7) and long arm (Figure 5.8) that allow for large phase angle measurements. The caddy resembles a satellite transmitter and the arm acts as the receiver enabling various phase angle (angle between the source and receiver) and incident angle orientations (angle between the

normal of the sample and the source). The circularly polarized incident light source that illuminates the sample platform (Figure 5.2) is mounted on the caddy platform (Figure 5.7). The backscatter signal from the sample inserted on the sample platform is received and analyzed using the components mounted on the long arm (Figure 5.8). Near zero phase angle measurements were recorded from $0 - 5^\circ$ phase angle in 0.5° increments. The long arm is kept in 0° (alumina samples) or 15° (beads samples) incident angle position as the caddy platform moves from $0 - 5^\circ$ incidence angle. Similar results are expected when keeping the caddy platform stationary and moving the long arm platform. The heavy weight of the analyzer component mounted on the long arm significantly limits the arm's movement.

5.2.1 Caddy Platform

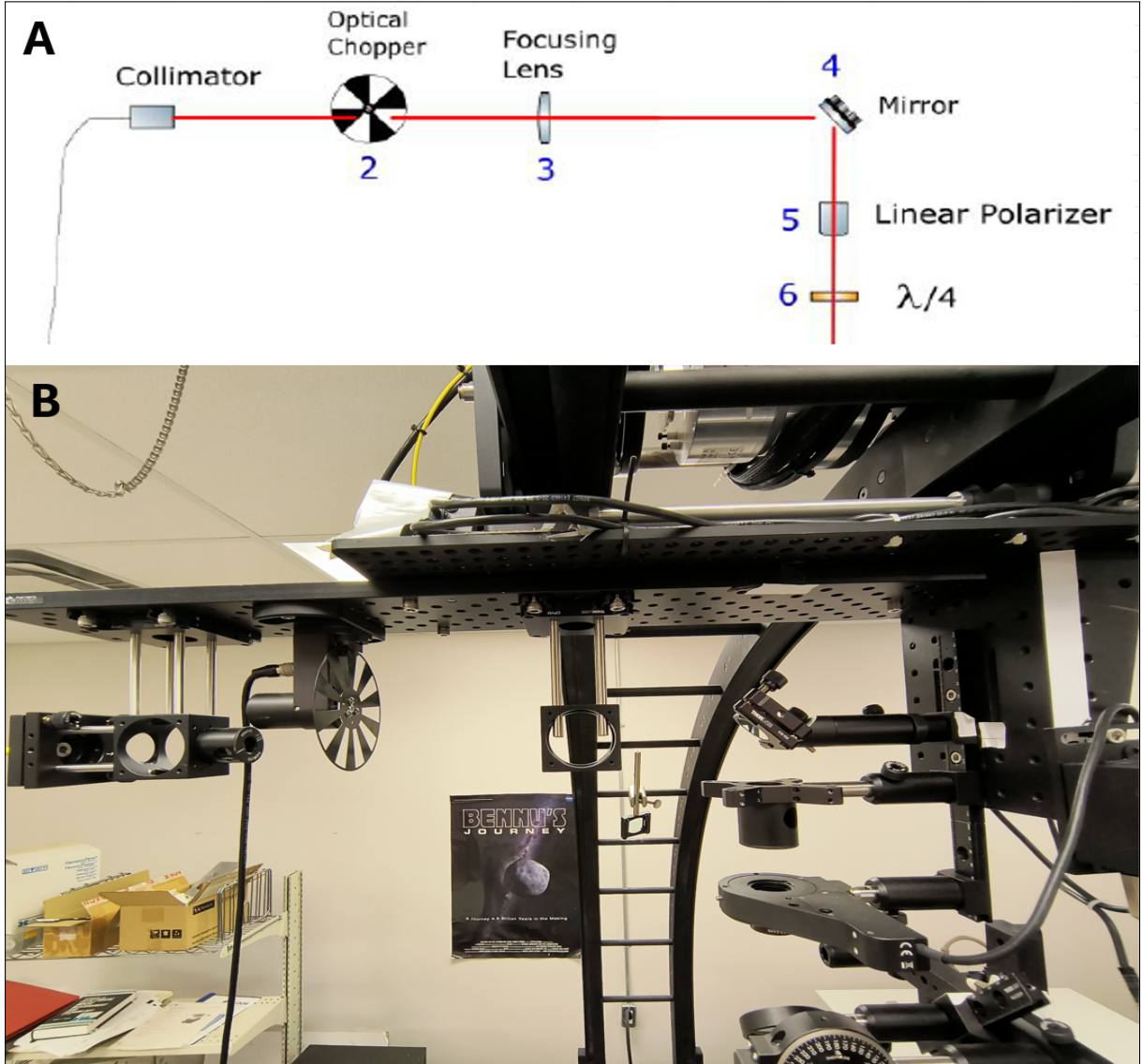


Figure 5.7: A: Schematic of the optical components mounted on the caddy platform. B: Photo of the caddy platform.

Components (2)-(6) are installed onto the caddy platform in Figure 5.7, and act as a transmitting instrument that generates the desired polarization state for the incident radiation. The incident laser beam was directed to the caddy platform (Figure 5.7) from the laser source through an optical fiber cable followed by a collimator for low beam divergence. The laser signal was chopped by an optical chopper (2 in Figures 5.1, 5.7) at a 250Hz frequency setting which offers low noise input. Certain components are far apart, which causes beam divergence and a blurred beam spot, so a focusing lens (3 in Figures 5.1, 5.7) was used to ensure the beam was centered upon propagation. An angled 45°

mirror (4 in Figures 5.1, 5.7) was used to direct the incoming radiation towards the sample platform. The polarization state of the laser beam was random after the mirror reflection. A linear polarizer (5 in Figures 5.1, 5.7) with its transmission axis parallel to the sample platform axis and a quarter wave plate (6 in Figures 5.1, 5.7) with its fast axis offset by 45° were mounted on the platform. The orientation of the LP and QWP ensured that the incident radiation would be circularly polarized. The components mounted on the caddy platform are listed in Table 6 in Appendix .7 with their settings and functionality.

5.2.2 Arm Platform

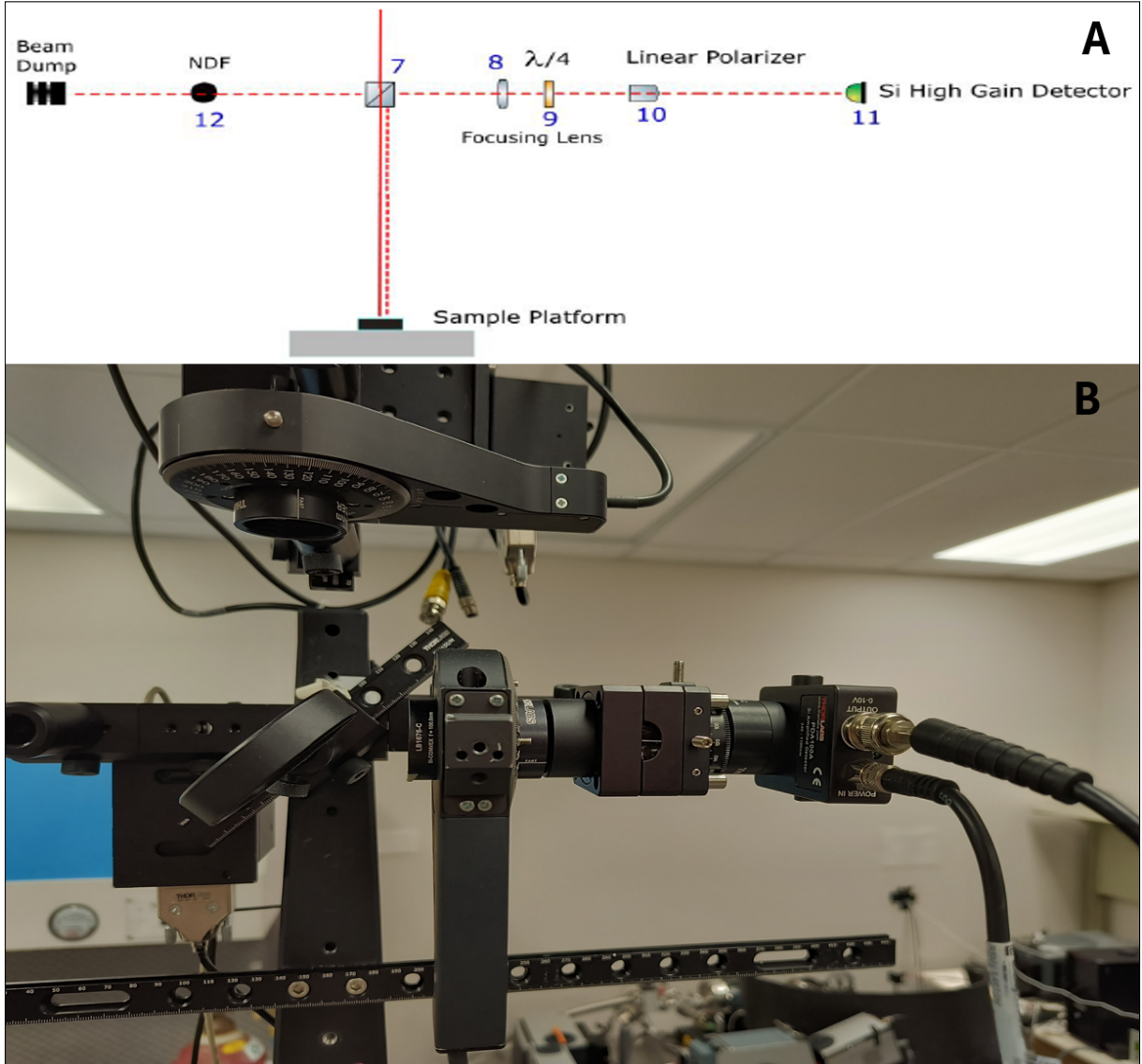


Figure 5.8: A: Schematic of the optical assembly on the arm platform. B: Photo of the arm platform.

The arm platform in Figure 5.8 of the MAGI acts as a receiving instrument that measures the backscattered signal from the sample platform. The circularly polarized signal incident from the caddy platform in Figure 5.7 propagates through the beam splitter mounted on the arm platform in Figure 5.8. The 50 – 50 beam splitter splits the beam into primary and secondary components. The primary incident beam was directed towards the sample platform in Figure 5.2 while the secondary beam was contained using a neutral density filter (12 in Figures 5.1, 5.8) and beam dump. The backscattered radiation from the sample platform in Figure 5.2 was focused towards the analyzer section through the

45° angled beam splitter (7 in Figure 5.8). The analyzer section in Figure 5.8 consisted of a quarter wave plate (9 in Figures 5.1, 5.8) and a linear polarizer (10 in Figures 5.1, 5.8) followed by the detector (11 in Figures 5.1, 5.8). The LP and QWP decomposed the backscattered radiation into Fourier components by using the rotating quarter wave plate technique in 4.1.1. The components mounted on the arm platform are listed in Table 6 in Appendix .7 with their functionality and settings.

5.3 Data Acquisition Set-Up

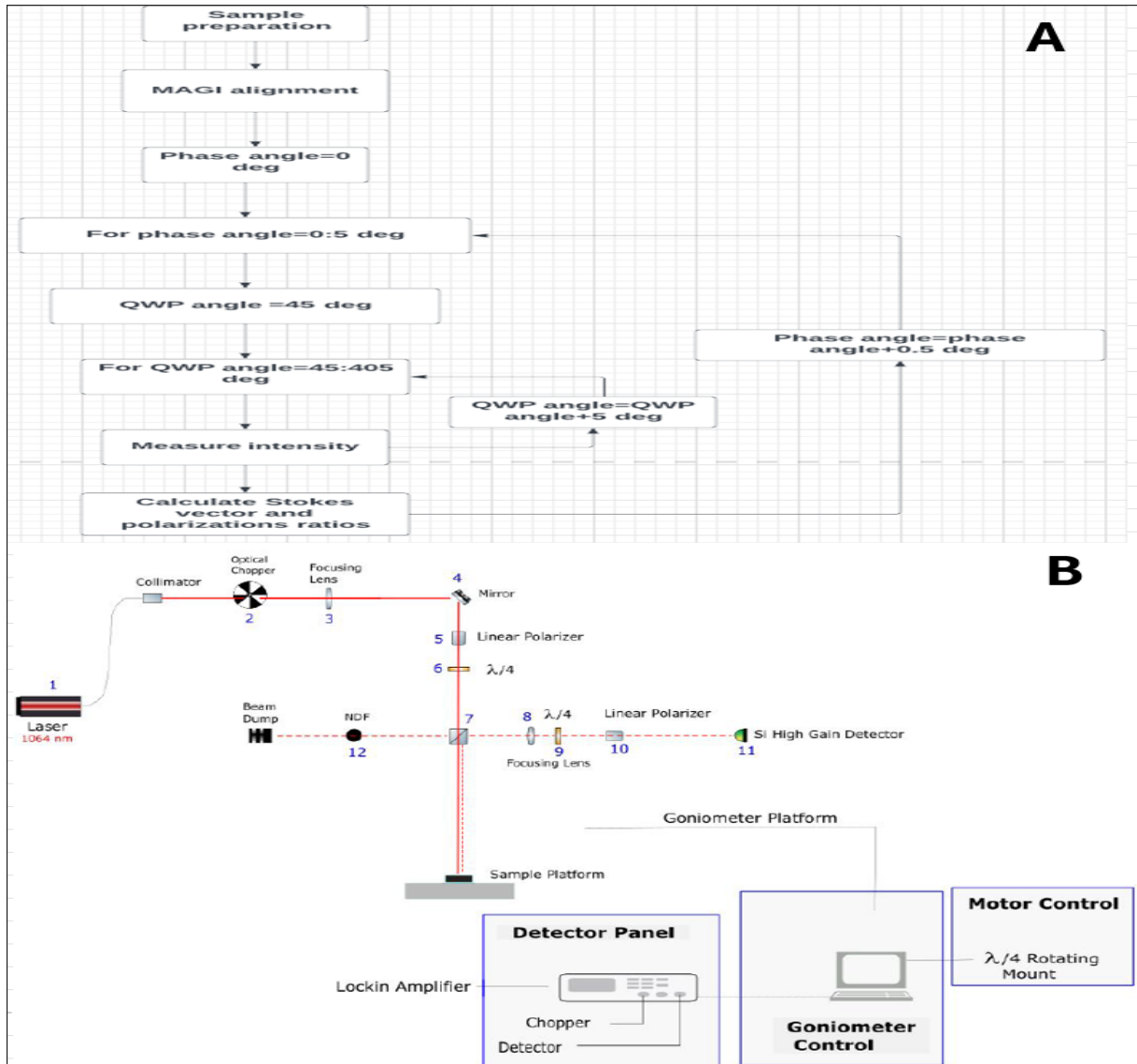


Figure 5.9: A: flow chart data acquisition procedure. B: Schematic of the Data Acquisition Programs for the Optical Setup.

The data for near zero phase angle measurements were collected using three programmed software elements that controlled the MAGI setup parameters. The three software elements were Motor Control in Figure 5.10, Detector Control in Figure 5.11 and Goniometer Control in Figure 5.12. Using the programs simultaneously, we were able to perform the RQWP technique, through which we acquired the Stokes parameters for a desired phase angle orientation on a sample. From the RQWP technique described in section 4.1.1 we controlled the QWP using the motor control software in Figure 5.10. Detector control software in

Figure 5.11 was used to display the measurements from the Lock-in amplifier in Figure 5.13 that were measured using a high-gain Si detector (11 in Figures 5.1, 5.8). Labview programmed script called the Goniometer control in Figure 5.12 was used for the MAGI setup that controlled the caddy platform in Figure 5.7 and long arm movements in Figure 5.8. Figure 5.9 A is a flow chart outlining the data acquisition procedure; Figure 5.9 B is the schematic of the data acquisition programs for the optical setup.

5.3.1 Quarter waveplate control

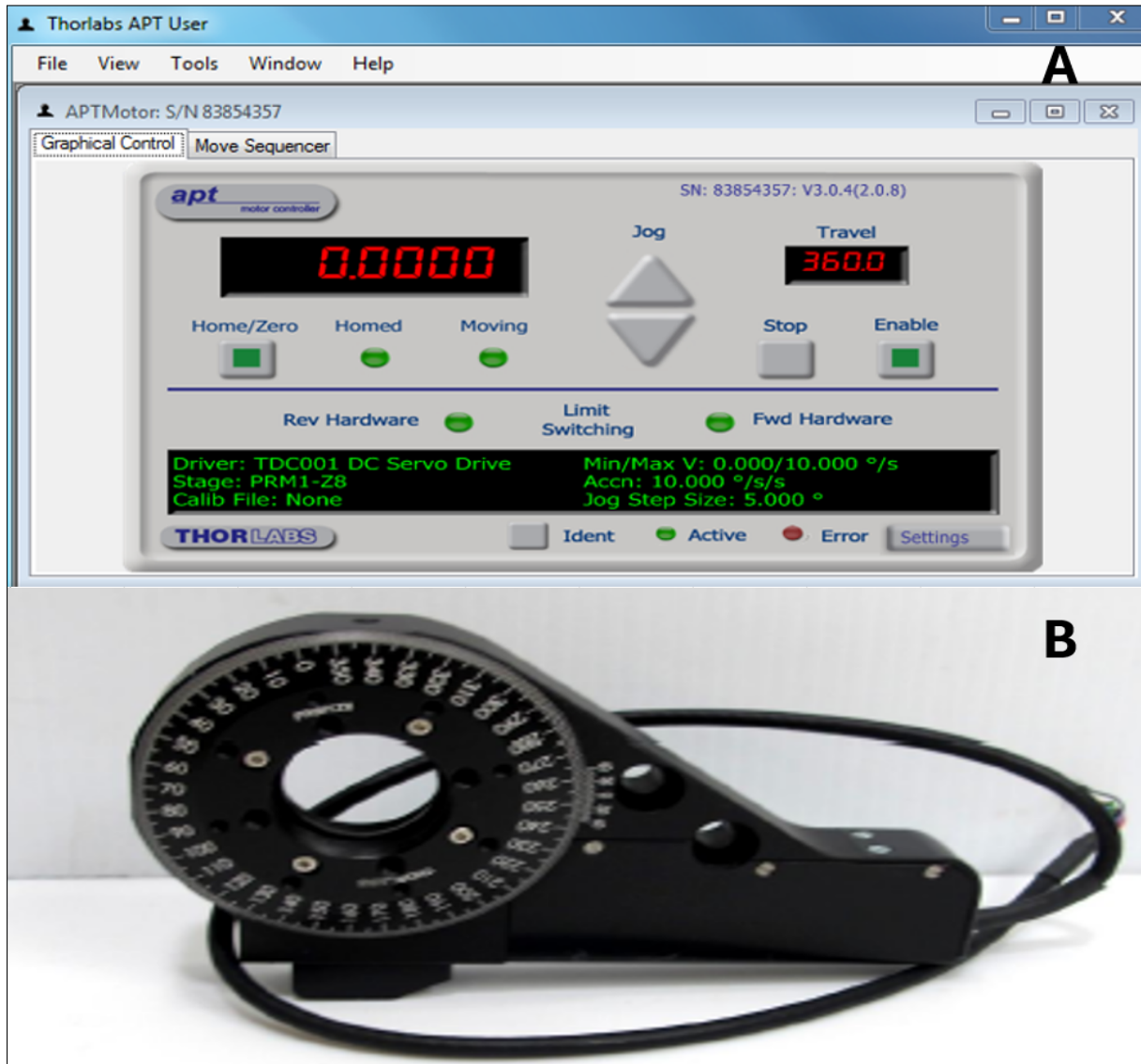


Figure 5.10: A: The motor control software used to run the QWP. B: Rotation stage where the QWP was mounted.

The motor control software in Figure 5.10 (A) was used to rotate the quarter wave plate in Figure 5.10 (B) according to the rotating quarter wave plate technique. In the RQWP technique in section 4.1.1 intensity values were recorded while the quarter wave plate was rotated 360 degrees in 5° increments. The QWP rotation was controlled using the motor control program. The program enabled us to precisely rotate the QWP with an accuracy of 0.01°.

5.3.2 Detector Control

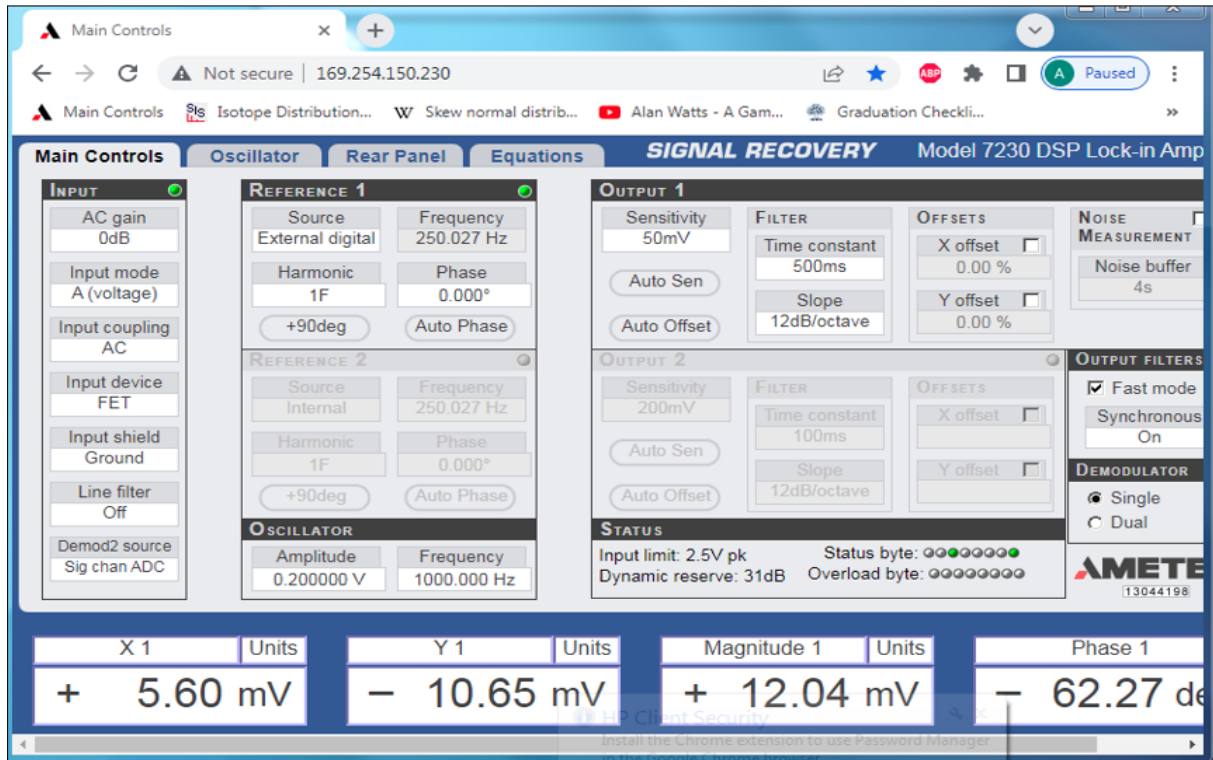


Figure 5.11: Front panel display for the detector control software run through the Lock-in Amplifier

The Detector control program running through the Lock-in Amplifier was used to measure the voltage values of the Si-High gain detector. The intensity measurements were acquired from the Si-High gain detector (Thorlabs PDA100a) using the detector control graphical user interface in Figure 5.11. The detector control program monitored and outputted the data read from the detector via the lock-in amplifier in Figure 5.13. The detector control in Figure 5.11 allowed for stable measurement display with custom AC gain settings and monitored the background noise levels.

5.3.3 Goniometer Control

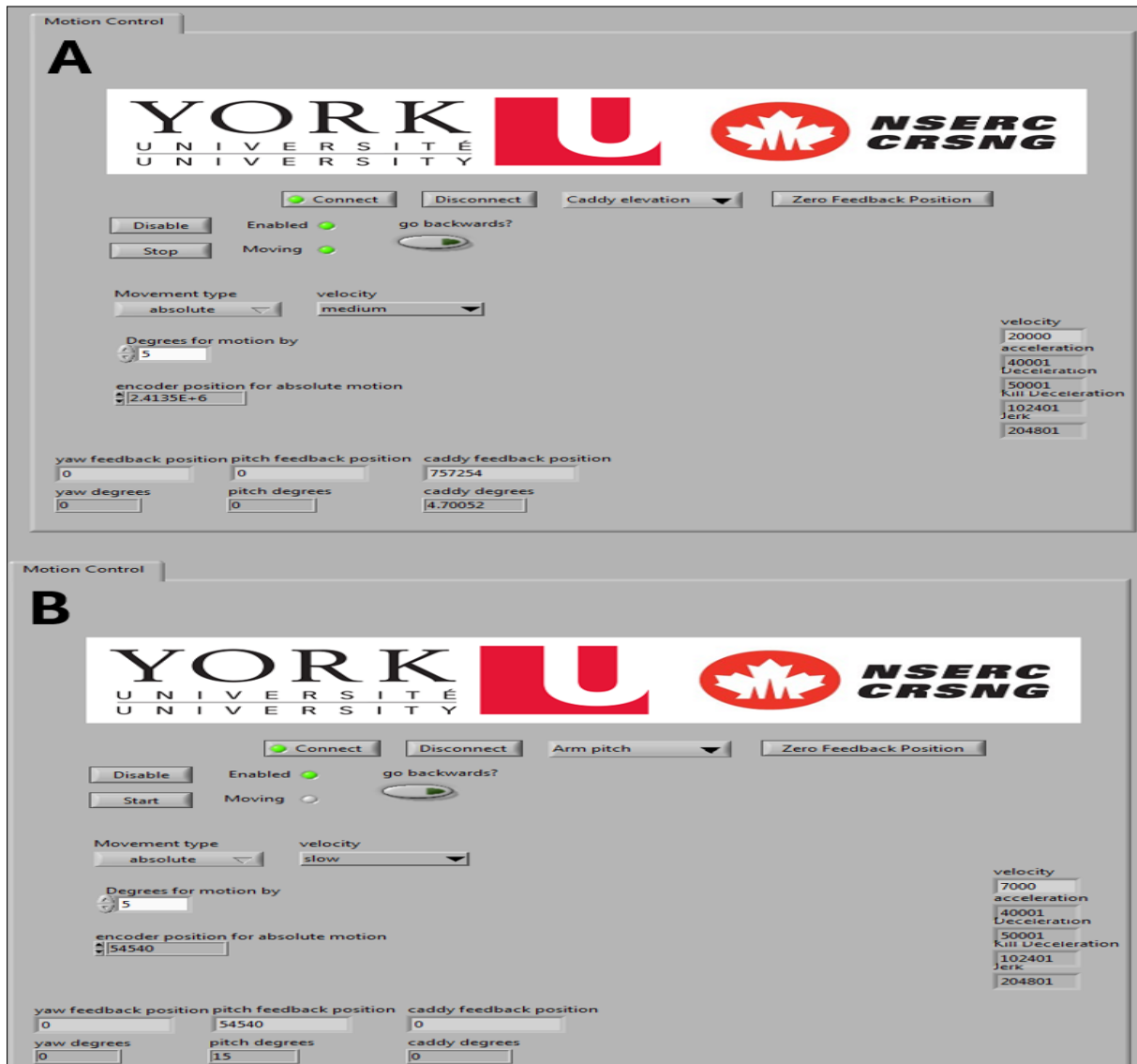


Figure 5.12: A: Front panel display for the MAGI caddy control software written in Labview environment. B: Front panel display for the MAGI arm control software written in Labview environment.

Multi-axis goniometer instrument was controlled using the goniometer caddy control in A of Figure 5.12 and arm control in B of Figure 5.12 program script as shown in the Figure 5.12. Goniometer control was programmed in LabVIEW programming environment and controlled using the graphical user interface display. The goniometer control program controlled the movements of the caddy platform and long arm platform to 0.05° precision. The MAGI control was used to move the caddy and arm platforms, creating a difference in incidence and emergence angles starting from 0° phase angle to 5° phase angle. In

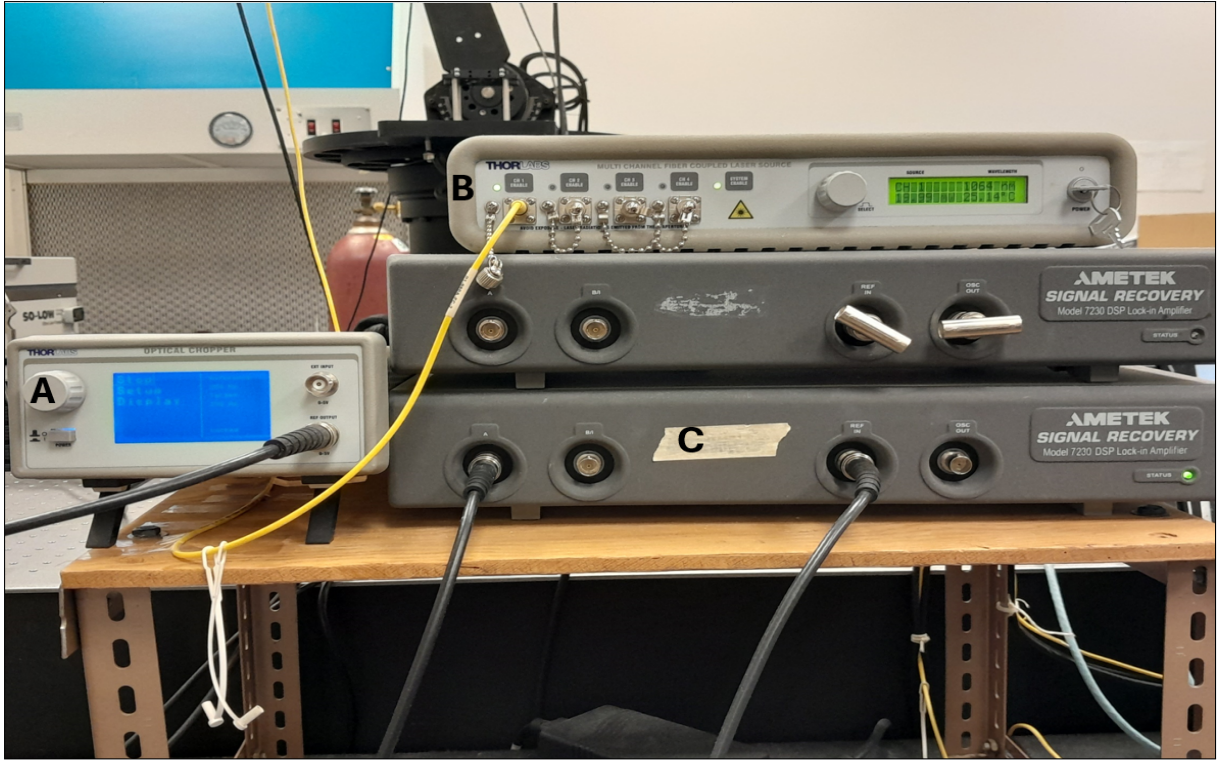


Figure 5.13: A: Optical (2 in Figures 5.1, 5.7) chopper control panel. B: laser control panel. C: Lockin-amplifier

Figure 5.13 we show the hardware for the data acquisition; A in Figure 5.13 is the control panel for the optical chopper (2 in Figures 5.1 and 5.7). C in Figure 5.13 is the lock-in amplifier to detect the chopped back-scattered light from the sample. Finally, B in Figure 5.13 represents the laser that generates the incident $1.064\mu\text{m}$ laser at a power of 20.0mW .

5.4 Samples Selection and Preparation

5.4.1 Lambertian Surfaces

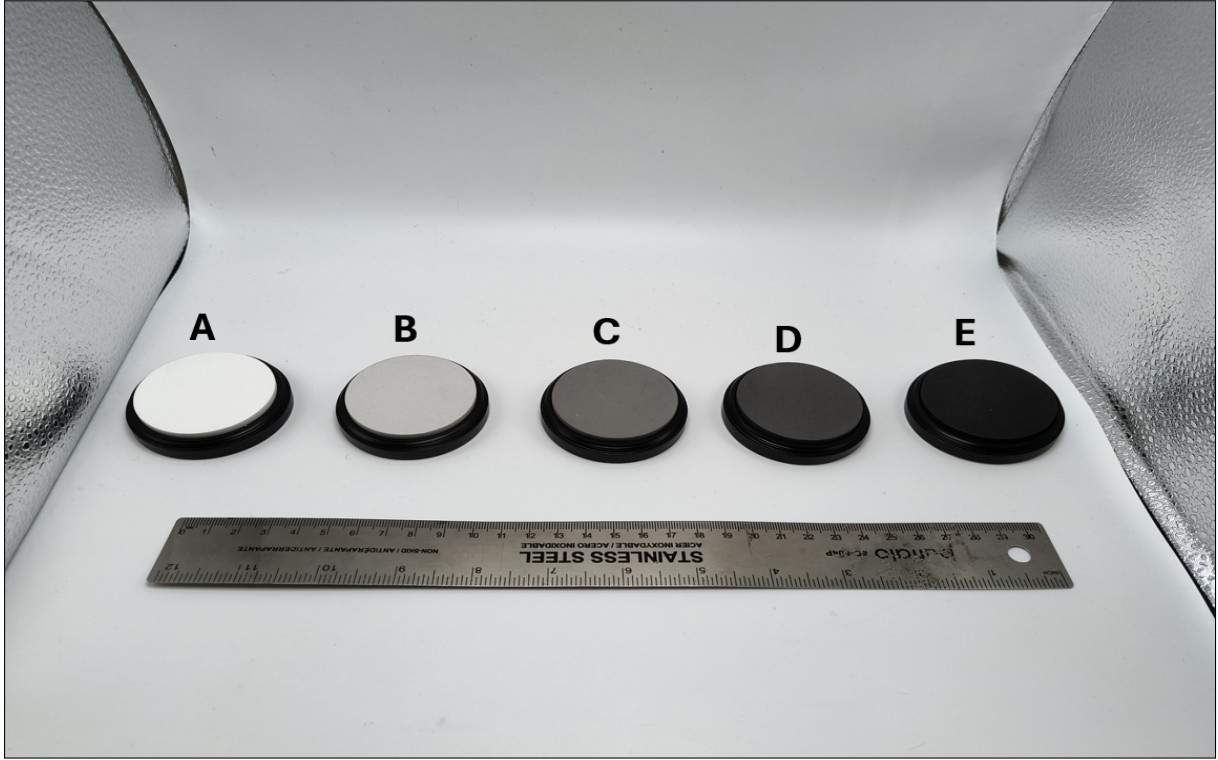


Figure 5.14: From A to E: 99%, 60%, 20%, 10%, 5% Lambertian surfaces (Spectralon samples)

Lambertian surfaces are diffuse reflectance laboratory standards that, due to their wavelength-scale roughness, generate diffuse (volume) scattering, explained more in depth in section 6.1 in chapter 6. Hence, Lambertian surfaces, as shown in Figure 5.14 provide an excellent frame of reference to study diffuse (volume) scattering present in alumina and latex beads samples under NIR wavelength. As shown in Figure 5.14 the standard Spectralon of varying reflectance values from highest to lowest was used in the phase angle study. The diffuse samples from Labsphere were studied as baseline measurements for powdered alumina ($i = 0^\circ$) in 6.2.1 and latex beads $i = 15^\circ$ samples in 6.2.2. The Spectralon samples had a varying range of reflectance values ranging from 5% – 99% with true Lambertian surface features. The samples were made of pressed transparent thermoplastic particles that produce close to true Lambertian surface reflections [Svensen et al., 2012].

5.4.2 Alumina Samples

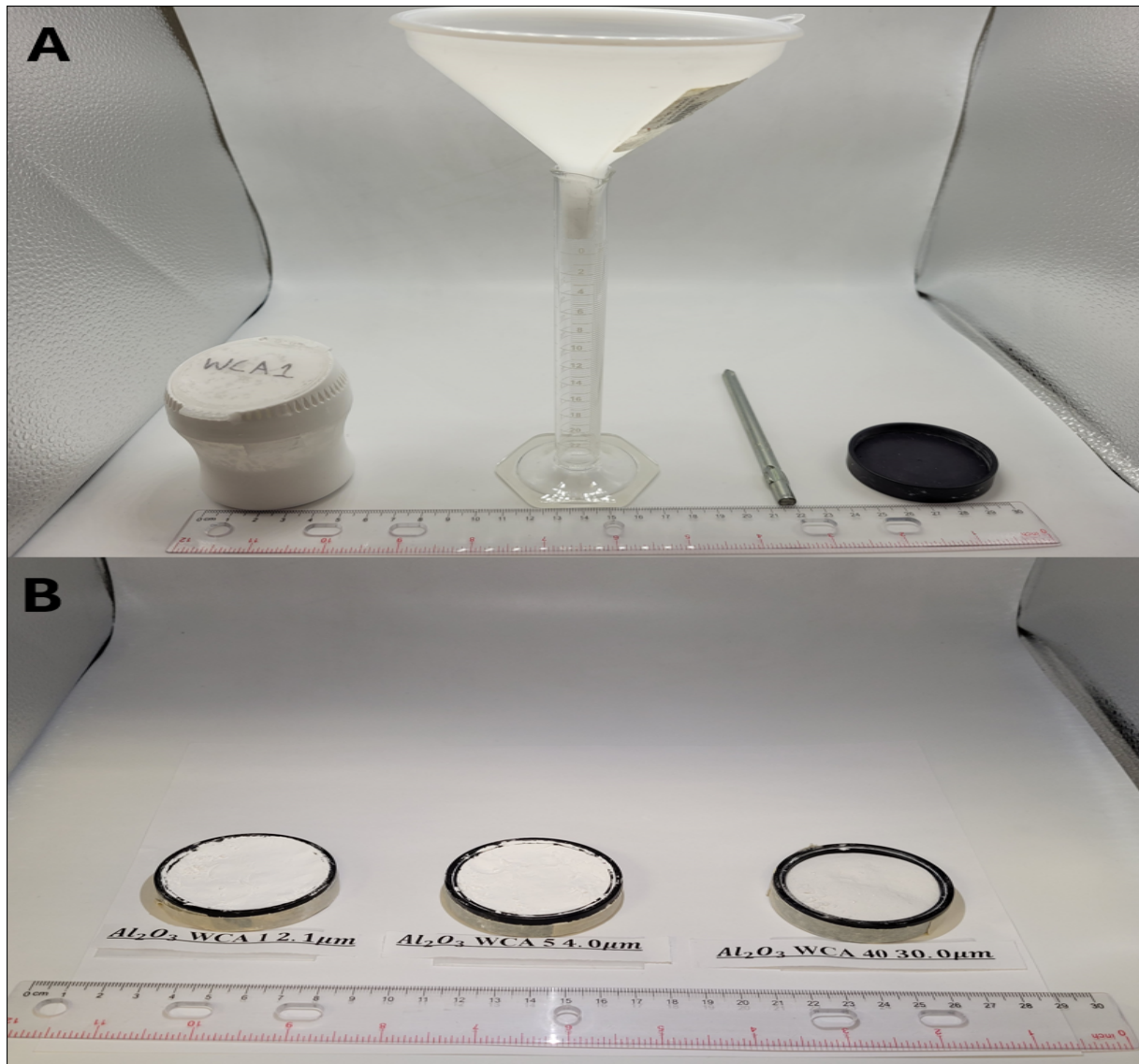


Figure 5.15: A: Alumina samples preparation. B: Alumina samples.

Industrial grade pure white alumina was selected as the main powdered sample for study. Alumina powder is commonly used as polishing grit due to its abrasive nature/hardness and is the second most abundant element found on the moon. Alumina powder was studied as it is bright in color and comes in various grain sizes which enables us to measure backscatter for wavelength-scale particles [Piatek, 2003]. Moreover, alumina samples have a purity over 99.20% Al_2O_3 in MICROAbrasives [2022]. Purity is measured using the mass ratio of alumina over the total mass of the sample. Al_2O_3 is an excellent analogue due to its hexagonal crystal structure, which matches that of water ice. Crystals of H_2O

tend to form platelets that are highly reflective and tend to align themselves upon settling; this is also true for our powders of Al_2O_3 [Nelson et al., 2018]. The powdered samples in Figure 5.15 were gently poured into a dark sample cup and the cups were lightly shaken to allow for natural settling. After allowing the sample to settle, the top surface was leveled by cutting through the excess sample with a glass slide. The sample was not packed in any manner; instead, the cups were shaken lightly to simulate a powdered surface of a planetary regolith. Similar preparation procedures were followed for all four alumina oxide samples under study such that surface deviations would not affect the measurements. Information regarding the physical properties of the alumina samples can be found in Table 1 in the Appendix .2. The size distribution of alumina powder samples is approximated by a narrow gamma distribution from Mishchenko [1992a]. The size distribution $n(X)$ is normalized such that the sum of all probabilities equals 1. The samples used for this study were intended to be used as optical abrasives and they could vary from the theoretical particle size distribution estimates [Nelson et al., 2000, Piatek, 2003].

5.4.3 Non-dye Latex Beads

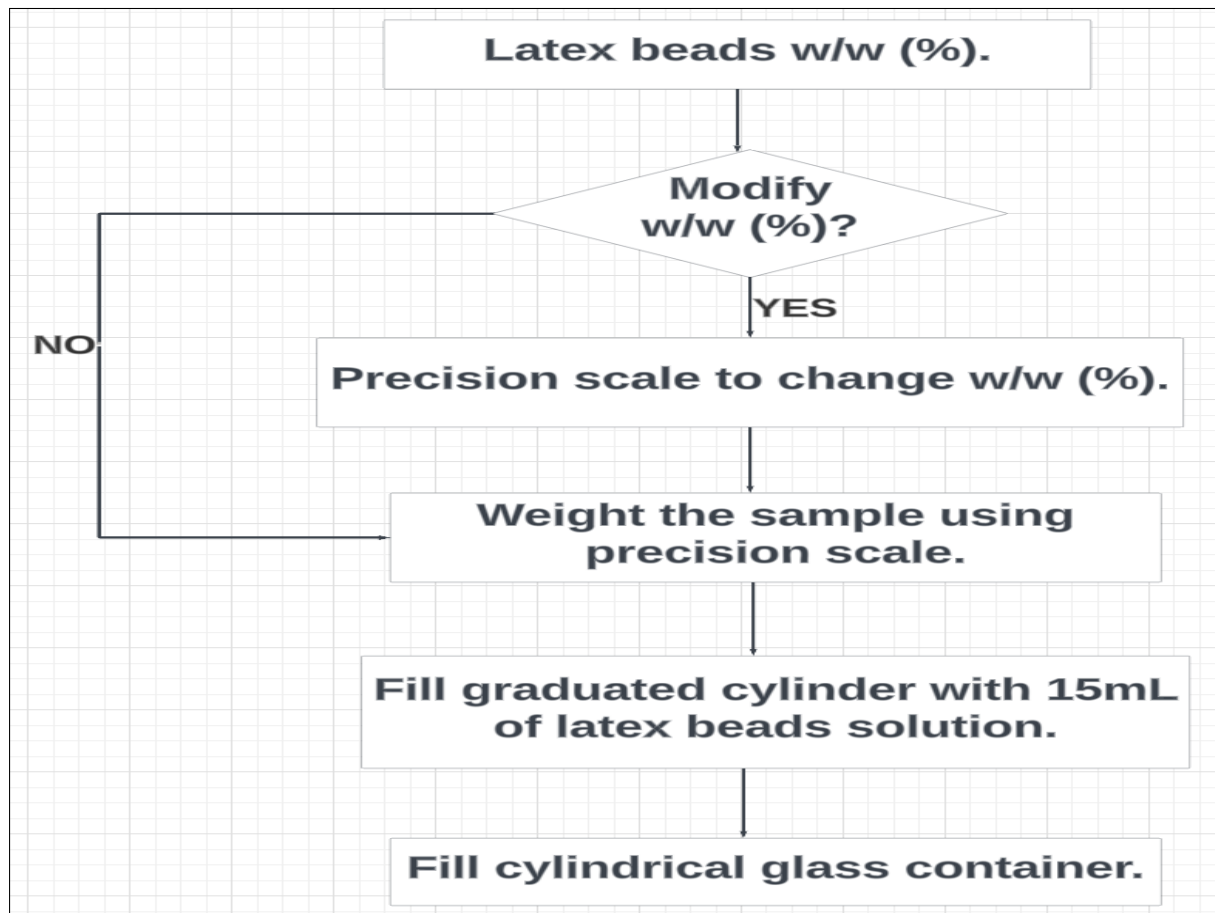


Figure 5.16: Flow chart for the sample preparation

The characteristic of the non-dye and blue-dye latex beads can be found in [Thermo Fisher Scientific, 2019]. In Figure 5.16 we show the flow chart for prepping the dye and non-dye latex beads solution; this is the first step in the flow chart of Figure 5.9 to perform measurements with MAGI. From the flow chart in Figure 5.16 we decide if we want to change the $\frac{w}{w}(\%)$ of the sample. If no, we weigh the sample using the precision scale in Figure 5.17 and we fill the graduated cylinder in Figure 5.17 with 15mL of solution and pour the solution into the glass cylindrical container in Figure 5.17. The precision scale is able to measure the mass of a sample up to a μg precision, enabling very precise measurements of the mass of our samples. If we wish to change the $\frac{w}{w}(\%)$ of the sample, we use the precision scale to either add or remove water to reach the desired concentration of the latex beads solution. Then we fill the graduated cylinder in Figure 5.17 with 15mL of solution and pour the solution into the glass cylindrical container in Figure 5.17.

The manufacturer [scientific, 2015] has given the concentration of the latex beads in weight concentration $\frac{w}{w}(\%)$. Information regarding the physical properties of non-dye bead samples and water can be found in the Appendix .3, .4, .5 section. Using the density of latex ρ_{latex} and the density of water ρ_{water} , we can convert the weight concentration to the volume concentration of the solution using Equation 5.5:

$$\frac{v}{v}(\%) = \frac{\frac{w}{w}(\%) \rho_{latex}^{-1}}{(100 - \frac{w}{w}(\%)) \rho_{water}^{-1} + \frac{w}{w}(\%) \rho_{latex}^{-1}} \quad (5.5)$$

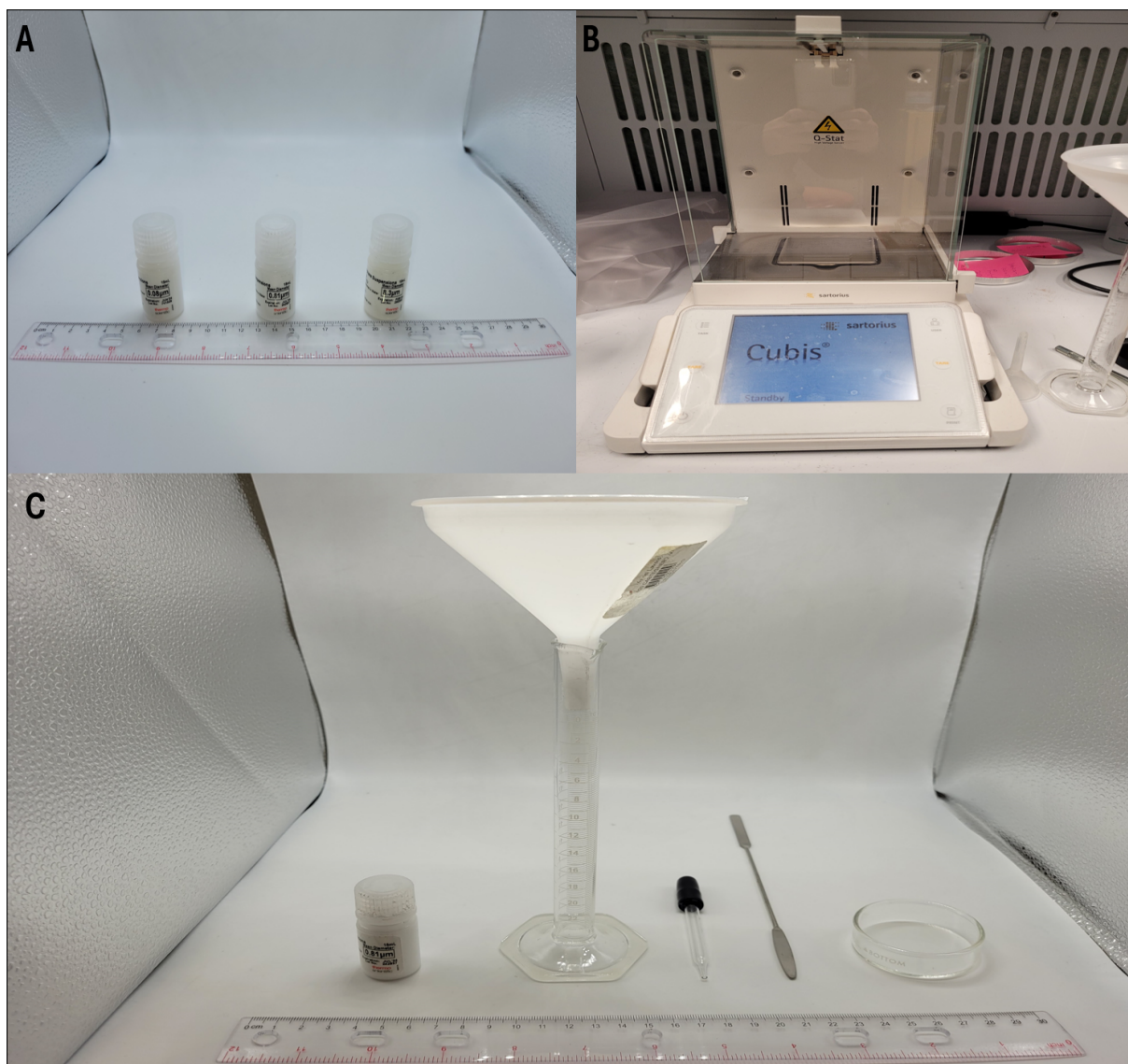


Figure 5.17: A: Non-dye latex beads samples. B: Precision Scale to weight the samples. C: Prepping the latex beads samples.

The non-dye latex beads solution (A in Figure 5.17) with different scatterer (beads) sizes to represent the different scattering regimes from .3-.5. A in Figure 5.17 shows from left to right the scatterers in the Rayleigh ($0.08\mu m$), Mie ($0.81\mu m$) and large Mie ($8.3\mu m$) scattering regimes respectively. For further explanation on the physical meaning of those scattering regimes, please refer to the 5.1.1 section. B in Figure 5.17 represents the precision scale we use to measure the mass of our samples up to a μg accuracy. C in Figure 5.17 represents the last two steps of the flow chart in Figure 5.16 where we pour $15mL$ of latex beads solution into the glass graduated cylinder and into the glass cylindrical container for both non-dye (Figure 5.17 bottom), and dye (Figure 5.18).

5.4.4 Dye Latex Beads

In section 6.4 we test the reliability of our instrument by using Mie blue dye colored beads (Figure 5.18) and comparing the SC , OC , CPR , and DLP return to the Mie non-dye beads under identical volume concentration (2.5% – 0.1%) from 0° – 5° phase angle at $i = 15^\circ$ incident angle. Our incident wavelength is in the NIR ($1.064\mu m$), therefore the absorption should not change between blue-dye and non-dye beads, because there is no blue component to be reflected from the incident NIR laser. Hence, under identical concentration the polarimetric curves in Figure 6.7 should be within the margin error (error bars) between the Mie blue dye and the Mie non-dye beads.

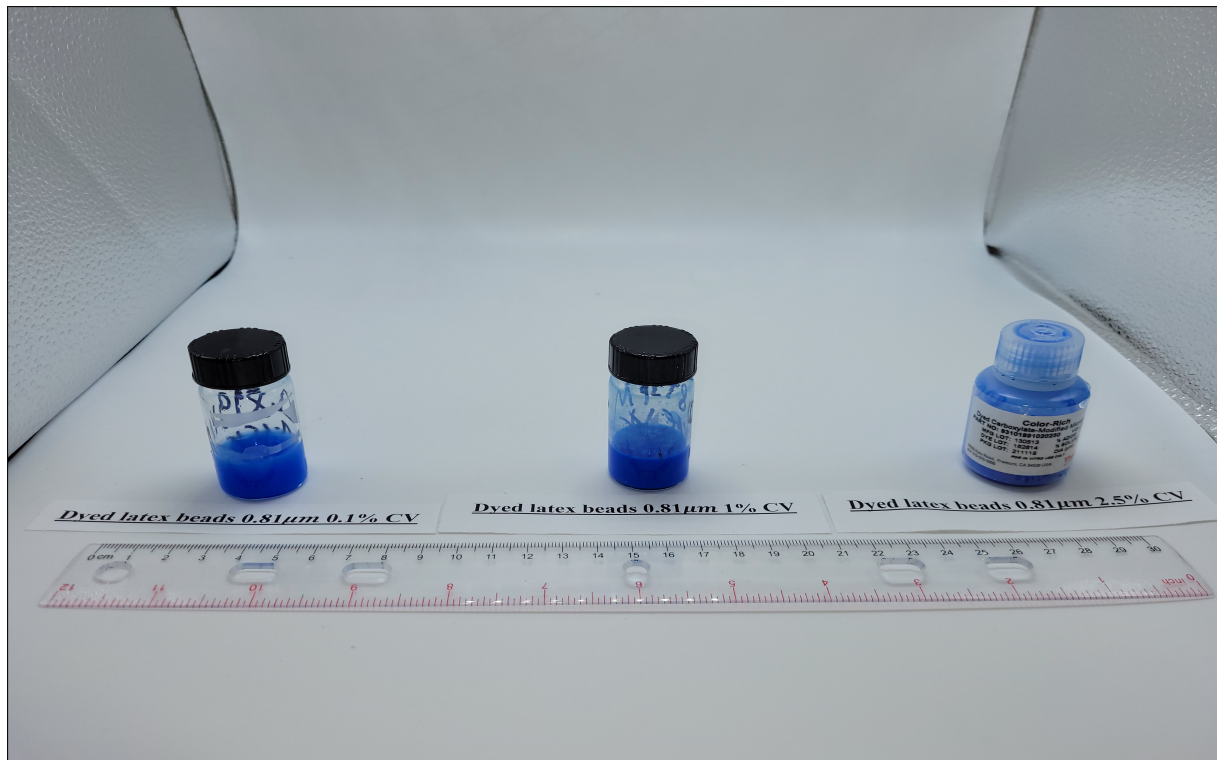


Figure 5.18: Flow chart for the sample preparation

In Figure 5.18, we show the dye latex beads with a 2.5%, 1%, and 0.1% volume concentration and use the same preparation technique as in 5.4.3.

Chapter 6

Data Analysis

In this chapter we characterize the effect of incident angle i° on surface and subsurface scattering in 6.1 by measuring the SC , OC , CPR , and the DLP of Lambertian surfaces with and without water glass cup (5.1.1). Lambertian surfaces are useful as laboratory standards for characterization of diffuse scattering due to their ability to uniformly diffuse incident light regardless of the detector angle due to their microfiber structure LabSphere [2024]. Standardised WCA Micro abrasive corp Alumina powders prove useful due to their crystal structure identical to that of water-ice, and to confirm the reliability of our instrument by comparing our measurements in 6.2.1 to the results from Nelson et al. [2000, 2002]. In this experiment, SC , OC , CPR , and DLP prove useful to characterize the imperfections like impurities in a weakly absorbing medium in 6.2.2, and 'strongly' absorbing medium in 6.4.1. As the scattered signal undergoes multiple forward scattering events, it retains its polarization properties. This scattering process can result in a coherent backscatter effect, amplifying the SC radar backscatter and leading to CPR values exceeding unity [Neish and Carter, 2014]. However, these polarization measurements alone cannot definitively determine if the observed CPR is due to surface (Figure 53.13 d in Neish and Carter [2014]) or subsurface impurities (Figure 53.13 c in Neish and Carter [2014]) or both (Figure 53.13 f in Neish and Carter [2014]). When specular reflections occur, we expect single bounce scattering that is characterized by the opposite sense (OC) to be much greater than same sense (SC) circular polarization such that $OC \gg SC$ with $CPR \ll 1$ [Neish and Carter, 2014]. As the thickness of the sample increases, reflection

from the sample platform decreases, reaching a minimum at $(1mm - 2mm)$ thickness for alumina powders and up to $\tau = 7mm + / - 1mm$ for the $0.08\mu m, 1\%v/v$, then the reflectance (scattered intensity) from the samples (latex beads) increases linearly as the thickness of the samples increases (Figure 1). We use this thickness $\tau = 7mm + / - 1mm$ for all samples (latex beads and alumina powders). To validate our experimental results in 6.2.2, we compare our results to the Monte-Carlo simulation in 6.5 from 3.2.

6.1 Effect of Incidence Angle

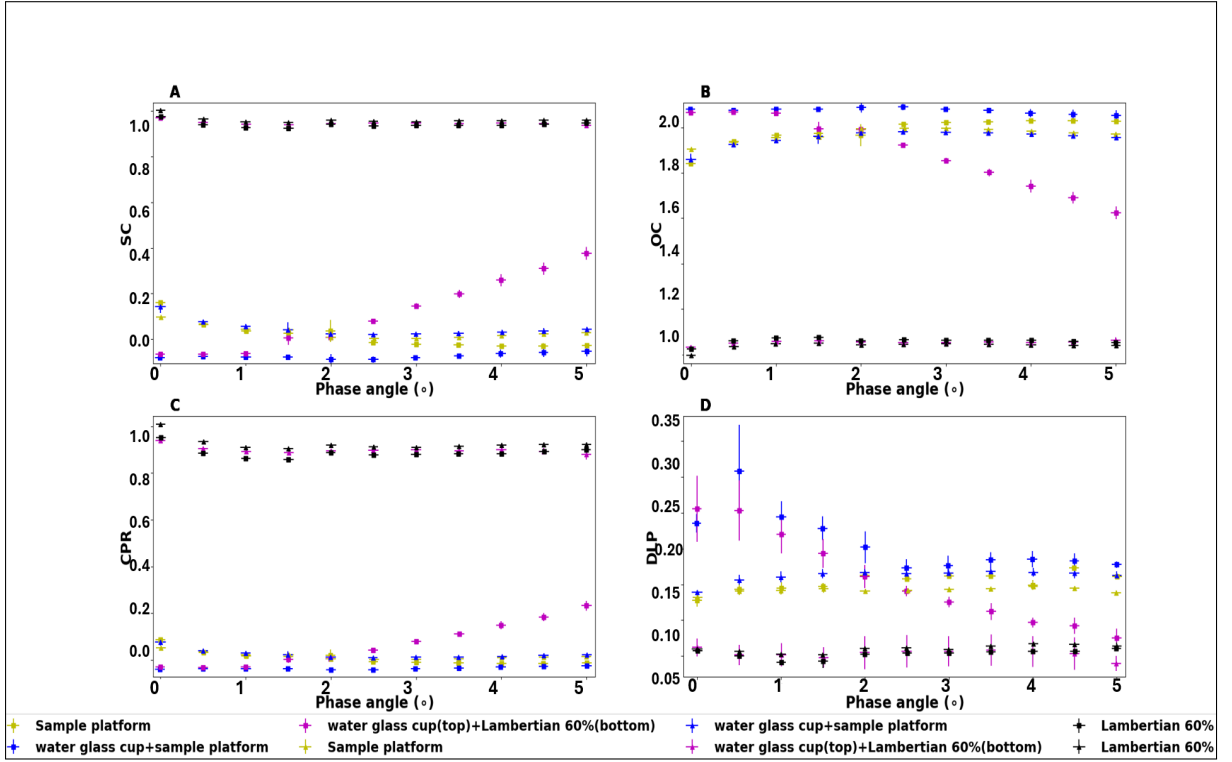


Figure 6.1: SC radar backscatter (A). Top right: OC radar backscatter (B). Bottom left: CPR (C) Bottom right: DLP (D), $i = 0^\circ$ (squares), $i = 15^\circ$ (triangles).

This section explores the behavior of SC , OC circular polarization, CPR , and the DLP in different samples at $i = 0^\circ$ (Figure 6.1 square) and $i = 15^\circ$ (Figure 6.1 triangles) incidence angles. The purpose is to look for an optical configuration such that surface specular reflection is minimized by studying the polarization return at $i = 0^\circ$ (squares in Figure 6.1) and $i = 15^\circ$ (triangles in Figure 6.1). The measurements include the sample platform (yellow curves in Figure 6.1 squares and triangles), water glass cup (blue curves in Figure 6.1 squares and triangles), a 60% Lambertian surface (bottom) with water glass

cup (top) (magenta curves in Figure 6.1 squares and triangles), and a 60% Lambertian surface (black curves in Figure 6.1 squares and triangles). The 60% Lambertian surface serves as a reference point for the polarimetric measurements of our alumina and latex beads samples. At both incidence angles, the metallic background exhibits low CPR and high DLP , attributed to the flipping of SC to OC due to its metallic nature such that $OC \gg SC$ with $CPR \ll 1$. Similarly, pure water and the Lambertian surface with water on top exhibit dominant specular surface reflections at 0° incidence angle, evidenced by a flip of SC to OC , low CPR such that $OC \gg SC$ with $CPR \ll 1$, and relatively high DLP . The Lambertian surface's microfiber structure contributes to an $SC \sim OC$ with $CPR \sim 1$, and low DLP , indicating surface diffuse scattering at both $i = 0^\circ$ and $i = 15^\circ$ incident angles. At 15° incidence angle, diffuse scattering becomes dominant over specular scattering. This pattern can be observed when comparing the 60% Lambertian surface with and without the water glass cup on top at $i = 15^\circ$ that have SC , OC , and CPR curves that match the SC , OC , and CPR curves of the Lambertian 60% at 0° incidence angle confirming the measurement of diffuse scattering at high incident ($i = 15^\circ$) for samples that have a specular surface with diffuse subsurface properties. Consequently, samples with a specular surface are dominated by surface specular scattering at $i = 0^\circ$, whereas at $i = 15^\circ$ and $1 - 5^\circ$ phase angles, the specular ray misses the detector, allowing for the observation of other scattering processes, such as diffuse and dihedral scattering (Figure 53.11 in Neish and Carter [2014]).

6.2 Scatterers of Varying Sizes with Constant Total Volume of Scatterers

6.2.1 Alumina Powders

Here we measure the SC , OC , CPR and DLP curves of alumina classical Mie ($2.1 \mu m$, $4.0 \mu m$) particles (yellow and blue curves in Figure 6.2) and alumina large Mie ($30 \mu m$) particles (magenta curves in Figure 6.2) compared to a Lambertian 60% surface (black curves in Figure 6.2) at 0° incident angle. Alumina samples have a purity over 99.20% Al_2O_3

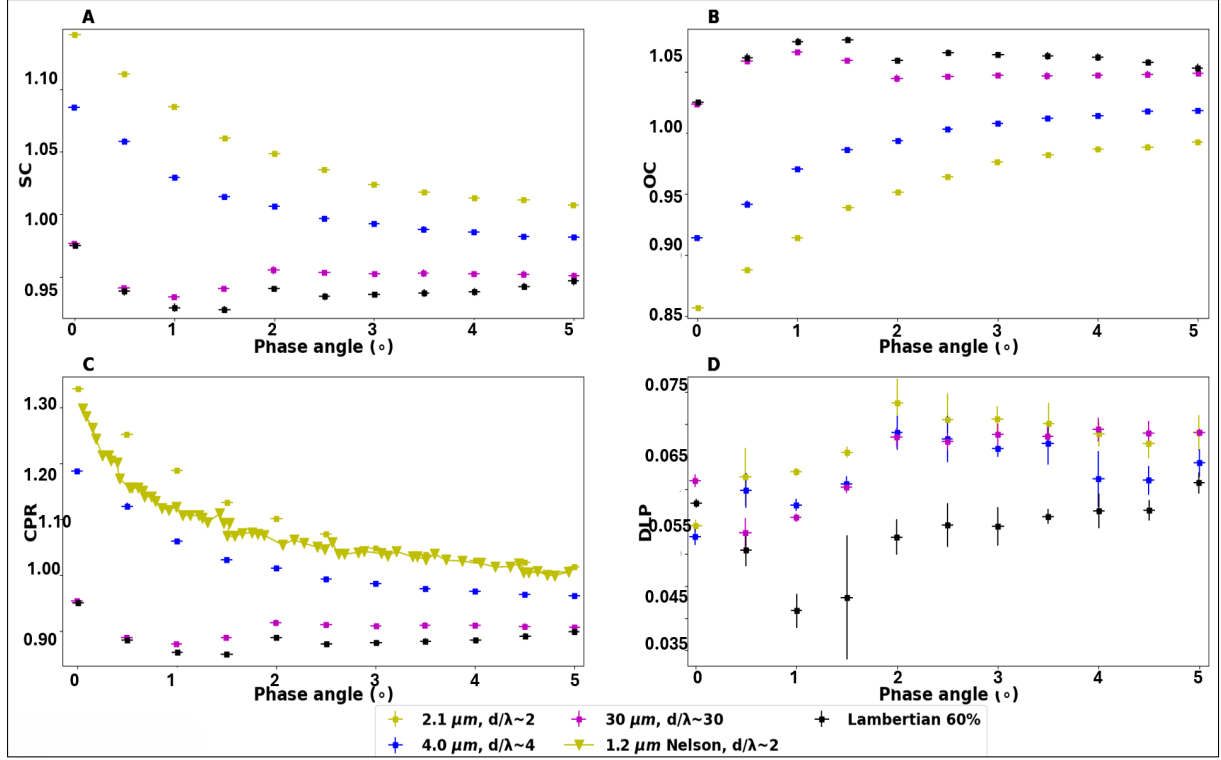


Figure 6.2: Backscatter of Alumina powders at low phase angles. (A) Alumina oxide powder *SC* radar backscatter. (B) Alumina oxide powder *OC* radar backscatter. (C) Alumina oxide powder *CPR* MAGI and Nelson et al. [2000], Nelson et al. [2002]. (D) Alumina oxide powder *DLP*.

[MICROAbrasives, 2022]; purity is measured using the mass ratio of alumina over the total mass of the sample. Our Al_2O_3 ($2.1\ \mu\text{m}$, $4.0\ \mu\text{m}$, $30\ \mu\text{m}$) are excellent analogues of water ice due to their hexagonal crystal structure. Crystals of H_2O tend to form platelets that are highly reflective; this is also true for our powders of Al_2O_3 [Nelson et al., 2018]. Our Alumina Mie ($2.1\ \mu\text{m}$, $4.0\ \mu\text{m}$) scatterers are of WCA class and according to Nelson et al. [2018], should have a platelet rather than isometric shape. At 0° phase angle our alumina displayed high (> 1) *CPR* ($2.1\ \mu\text{m}$, $4.0\ \mu\text{m}$) and low-mod *DLP* ($\sim 0.05 - 0.075$) ($2.1\ \mu\text{m}$, $4.0\ \mu\text{m}$). The large Mie $30\ \mu\text{m}$ alumina sample has mod *CPR* (1-0.95) and low-mod *DLP* (0.04-0.055) and is within the margin of error (1-2 sigma) of the Lambertian 99% surface from $0 - 5^\circ$ phase angle. MAGI, when compared to Nelson et al. [2000, 2002], is within the margin of error for *CPR* from $1.5 - 5^\circ$ phase angle for a similar size parameter (Figure 6.2. c). All alumina *DLP* curves are within the margin of error (1 sigma); however, the high void space of Mie ($2.1\ \mu\text{m}$, $4.0\ \mu\text{m}$) generates a higher *DLP* (subsurface scattering) relative to large Mie ($30\ \mu\text{m}$) samples.

6.2.2 Non-Dye Latex Beads

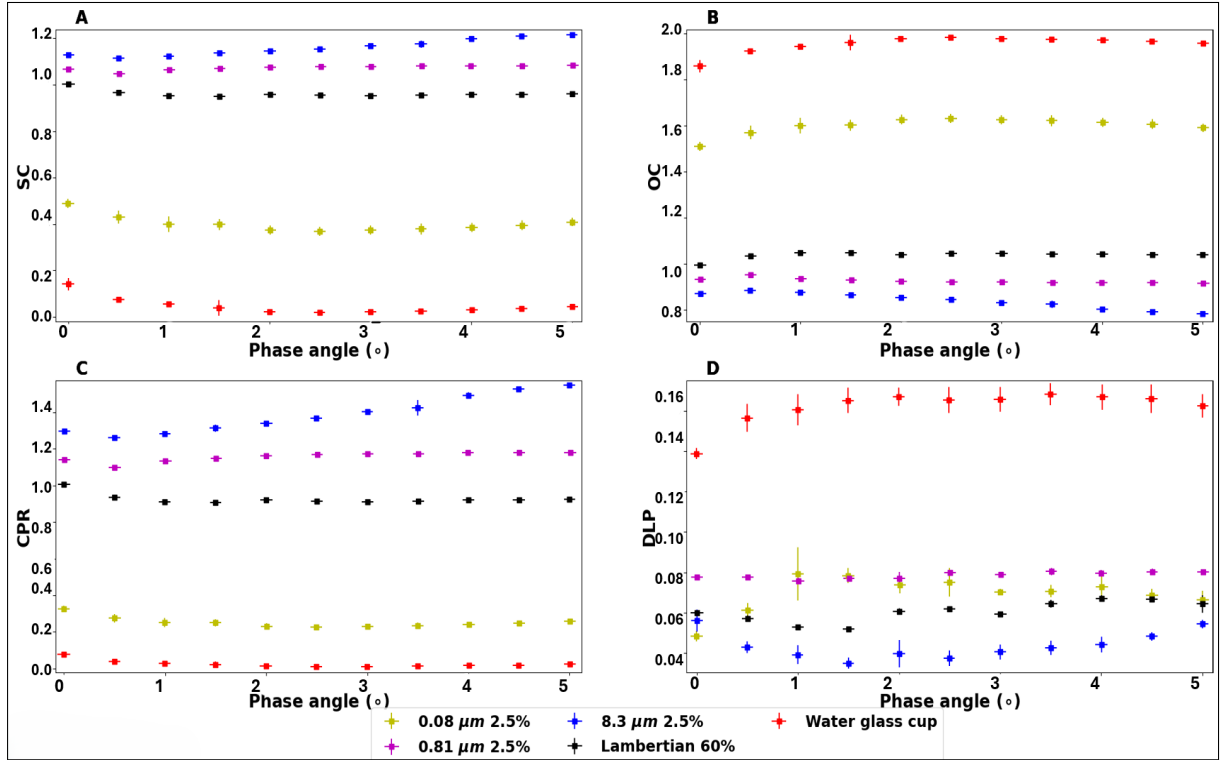


Figure 6.3: Backscatter of latex beads of varying sizes at low phase angles. (A) Latex bead *SC* radar backscatter. (B) Latex bead *OC* radar backscatter. (C) Latex bead *CPR*. (D) Latex bead *DLP*.

In Figure 6.3 we study the latex bead solutions at a constant 2.5% v/v, altering only the bead size. The solutions present a smooth specular surface, thus the dominant scattering process at ($i=15^\circ$) is multiple bounces scattering from the beads, or in the case of high transparency, subsurface single bounce scattering from the sample platform. For beads in the Rayleigh scattering regime ($0.08\mu\text{m}$, yellow curves in Figure 6.3), their small size compared to the illumination wavelength results in isotropic scattering, scattering the incoming signal uniformly in all directions. This causes the *SC*, *OC*, and the *CPR* to have a nearly flat response to phase angle, confirmed by observations of the yellow curves in Figure 6.3. Rayleigh beads have a reduced scattering cross section (Table 2 in .3 and Figure 3 B) relative to Mie, and geometric beads (Tables 3-4 in .4-.5 and Figure 3 A, C-D) that makes the scattering isotropic (uniform in all directions). The reduced scattering cross section of Rayleigh beads allows significant signal transmission to subsequent layers, the signal bouncing off the sample platform flips *SC* to *OC*, leading to a low *CPR*

characterized by $OC \gg SC$ with $CPR \ll 1$, and moderate degree of linear polarization (DLP) in the $0 - 5^\circ$ phase angle range that is characteristic of single bounce reflection. Even if the surface specular reflection is negligible due to the high ($i = 15^\circ$) incident angle, single bounce scattering from the sample platform cannot be avoided for highly transparent samples. Classical Mie scattering beads ($0.81 \mu m$, magenta curves in Figure 6.3) scatter incident radiation mainly in the forward direction, an anisotropic behavior. While their DLP resembles that of the Rayleigh beads, the response to phase angle is flatter. Large Mie scattering beads ($8.3 \mu m$, blue curves in Figure 6.3), although bigger than the illumination wavelength, aren't large enough to fall within the geometric scattering regime. Their SC , OC , CPR , and DLP responses show more variation compared to the classical Mie case, aligning with Mie scattering theory's prediction of stronger forward direction scattering. These beads exhibit high CPR and moderate DLP , akin to case C in Figure 53.13 in Neish and Carter [2014].

6.3 Scatterers of Constant Sizes with Varying Total Volume of Scatterer

6.3.1 Rayleigh Non-Dye Latex Beads

In Figure 6.4, we show the impact of varying the volume concentration of Rayleigh scatterers on SC , OC , CPR , and DLP curves across a $0^\circ - 5^\circ$ phase angle range. Specifically, the Rayleigh beads, depicted by yellow, red, magenta, and blue curves in Figure 6.4, demonstrate isotropic scattering uniformly dispersing the scattered field in all directions due to the small scattering cross-section relative to classical Mie and large Mie beads. This isotropic scattering characteristic of Rayleigh beads results in a higher transparency compared to classical and larger Mie beads. The transparency generates interaction of the signal with the sample platform that leads to a significant SC to OC flip, characterized by OC being substantially greater than SC ($OC \gg SC$). This results in a CPR value less than 1, and a DLP that inversely correlates with the beads' volume concentration. A reduction in the volumetric concentration of Rayleigh beads corresponds to an increase in

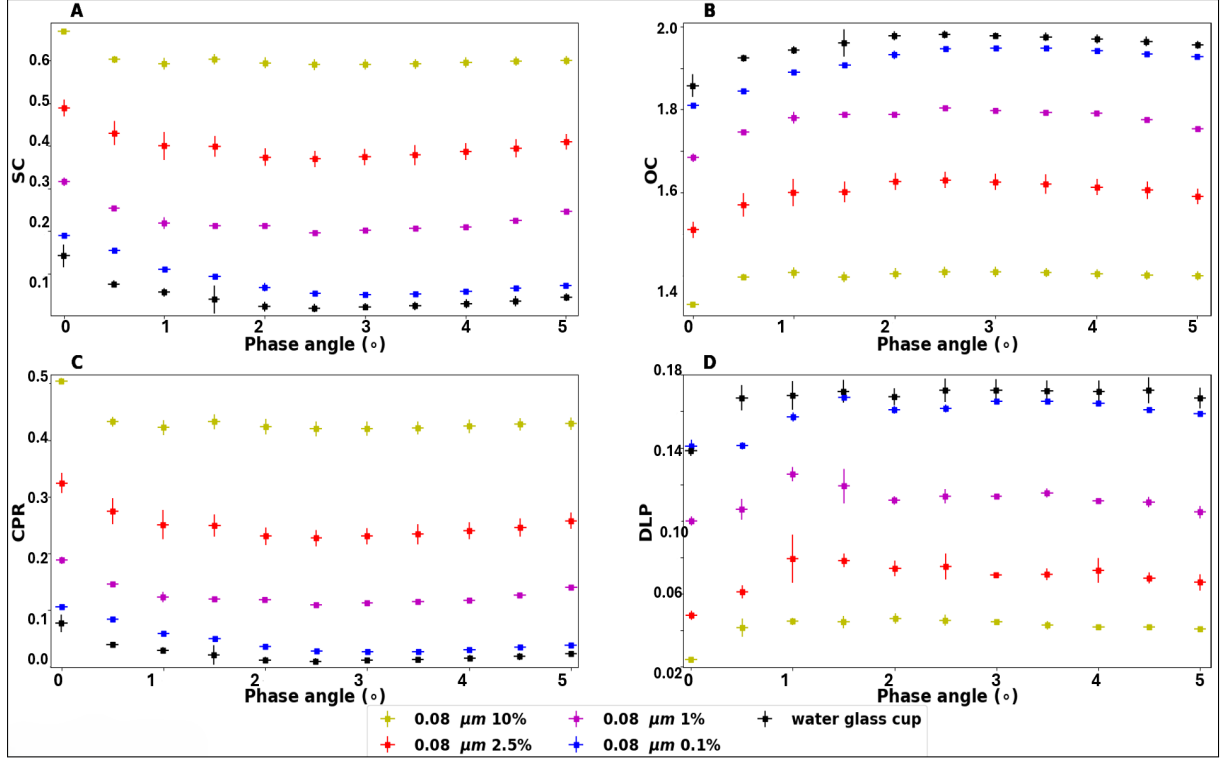


Figure 6.4: Backscatter of Rayleigh latex beads at low phase angles. (A) Rayleigh latex bead SC radar backscatter. (B) Rayleigh latex bead OC radar backscatter. (C) Rayleigh latex bead CPR . (D) Rayleigh latex bead DLP .

transmission, further promoting the SC to OC flip due to the sample platform. This effect mirrors the decrease in CPR and the increase in DLP , akin to the scenario presented in case B of Figure 53.13 in Neish and Carter [2014], where low CPR and high DLP are attributed to subsurface specular scattering. Further evidence of this phenomenon is observed by comparing the black curves in Figure 6.4, representing water without beads, to the other curves. As the Rayleigh beads' concentration diminishes, the disparity between the black curves and other curves in Figure 6.4 decreases. Notably, the 0.1% volume concentration (blue curves in Figure 6.4) aligns within one standard deviation of the water sample (black curves in Figure 6.4), underscoring the influence of Rayleigh beads concentration on radar backscatter characteristics.

6.3.2 Mie Non-Dye Latex Beads

In Figure 6.5 we explore the effect that changing volume concentration of Mie $0.81 \mu m$ scatters has on SC , OC , CPR and DLP curves. For all v/v, the curves have stronger phase angle dependence than Rayleigh beads. According to Mie scattering theory, it is due to the

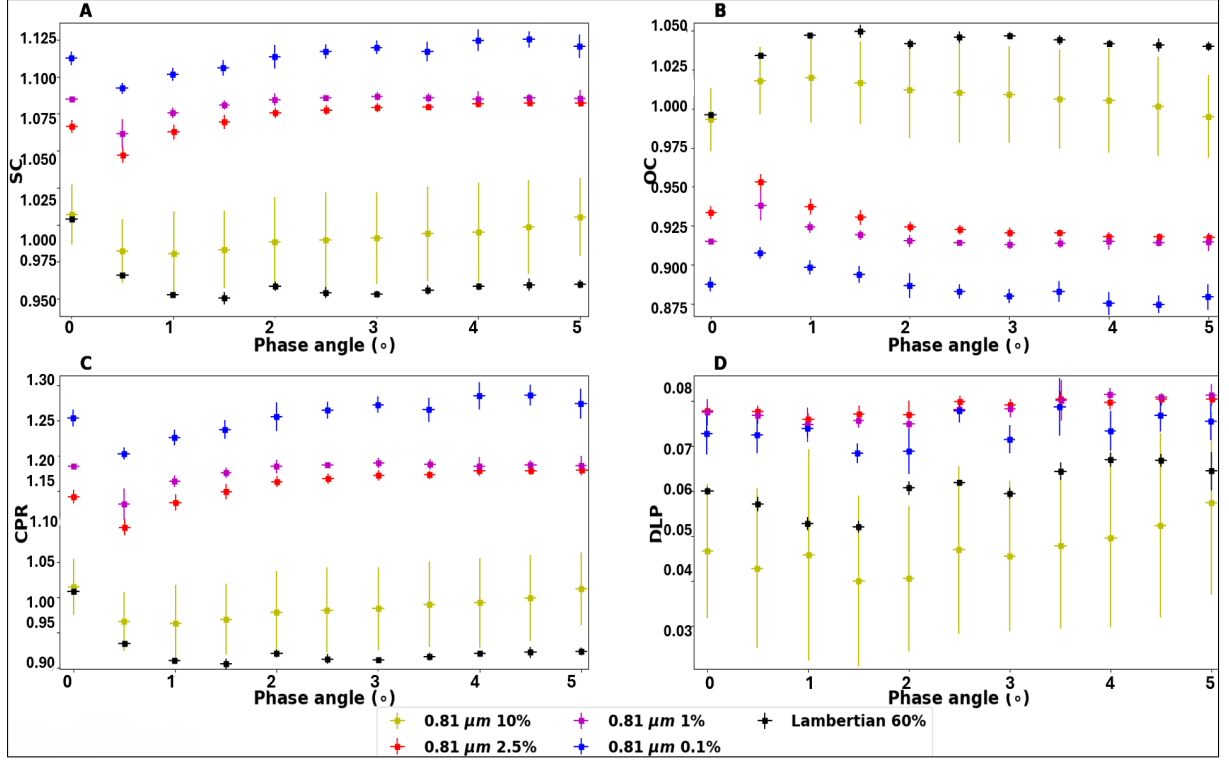


Figure 6.5: Backscatter of classical Mie latex beads at low phase angles. (A) Classical Mie latex bead SC radar backscatter. (B) Classical Mie latex bead OC radar backscatter. (C) Classical Mie latex bead CPR . (D) Classical Mie latex bead DLP .

incident beam being scattered with greater intensity in the forward direction rather than uniformly in all directions. To confirm the forward scattering, we plotted the scattering amplitude A_1, A_2 over scattering angle in a polar plot (Figure 2, and Figure 3), we see that the scattering phase function is non-uniform and forward (Figures 2 B, C, D, and Figure 3 A, C, and D). For both Lambertian 60% and Mie 10% volume concentration, the CPR is moderate ($\sim 1 - 0.9$) and the DLP is low-mod (0.04-0.06) from $0 - 5^\circ$ phase angle that is characteristic of diffuse scattering with weak to no subsurface scattering. The $0.81 \mu m$ samples form a continuous surface similar to the Lambertian 60% preventing the wave from penetrating into the sample, randomizing $SC \sim OC$ with $CPR \sim 1$ and low-mod DLP . When the volume concentration of beads decreases, transmission increases, thus more subsurface scattering occurs and increases the DLP but less than in the Rayleigh case, where the beam bounced off the sample platform. As subsurface scattering increases further, there are more paths for the photons to be scattered off the beads in the forward direction that conserves the SC polarization, with high CPR (> 1) inversely proportional to the volume concentration of Mie scatterers.

6.3.3 Geometric Non-Dye Latex Beads

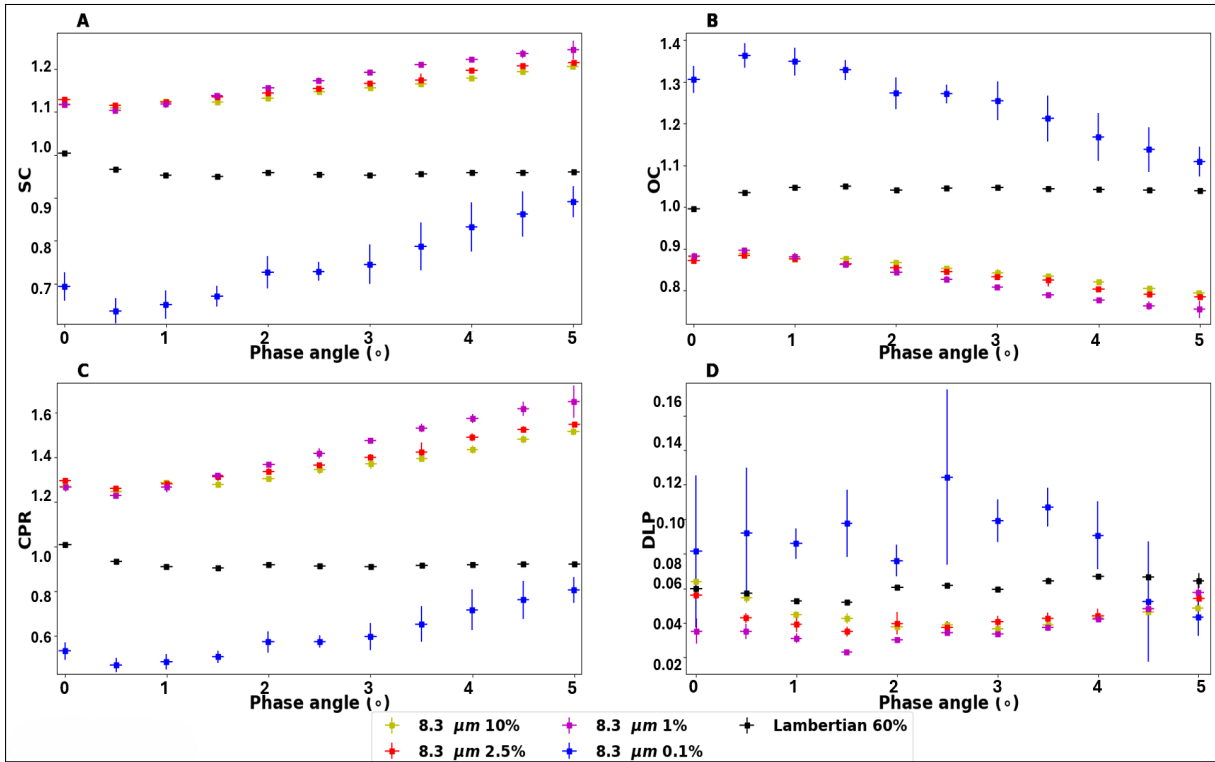


Figure 6.6: Backscatter of large Mie latex beads at low phase angles. (A) Large Mie latex bead *SC* radar backscatter. (B) Large Mie latex bead *OC* radar backscatter. (C) Large Mie latex bead *CPR*. (D) Large Mie latex bead *DLP*.

In an extended exploration that includes Figures 6.3-6.5, we observe the scattering behavior of large Mie beads at different volume concentrations as depicted in Figure 6.6. These large beads share similar scattering properties with their classical counterparts but tend to scatter incident light more in the forward direction (Table 4, and Figure 3 D in .5). The study finds that the absolute difference in same sense (*SC*), opposite sense (*OC*) circular polarization, circular polarization ratio (*CPR*), and degree of linear polarization (*DLP*) curves fall within the margin of error (0.5-1 sigma) for volume concentrations from 10% to 1%. This observation is attributed to the substantial bead size that makes the solution appear more like a continuous surface. Drawing on insights from Figure 1 in Nelson et al. [2018], the larger beads result in less "void space" and thus less subsurface scattering relative to classical Mie and Rayleigh beads within the same volume concentration range. This is confirmed by the low-mod (0.02-0.06) *DLP* noted for the large Mie beads, indicating lesser subsurface scattering than their counterparts.

Interestingly, despite exhibiting scattering behavior similar to a classical Lambertian 60% surface, the large Mie beads differ in that they preserve the polarization properties of the incident field (SC) during forward scattering events, resulting in $SC > OC$ and a high (> 1) CPR . However, at a 0.1% volume concentration, the signal penetrated the solution and reflected off the black metallic background, thereby producing $OC > SC$ with $CPR < 1$, suggesting single bounce scattering from the metallic background.

6.4 Scatterers of Constant sizes, with Varying Absorption Properties

6.4.1 Dye latex beads

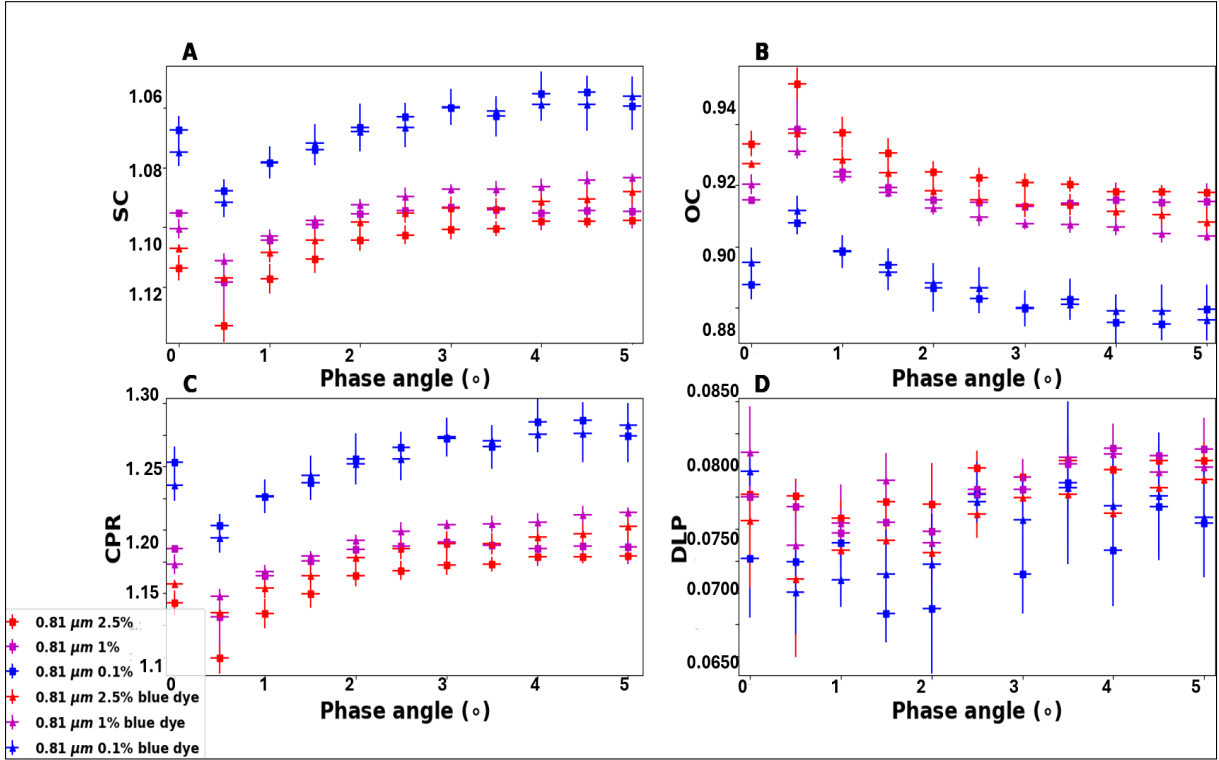


Figure 6.7: Top left: Mie latex bead SC radar backscatter (A). Top right: Mie latex bead OC radar backscatter (B). Bottom left: Mie latex bead CPR (C). Bottom right: Mie latex bead DLP (D).

In Figure 6.7 we explore the effect that changing the dye of the beads to 'blue' has on SC, OC CPR and DLP curves, compared to the Mie non-dye bead (Figure 6.5) at 2.5% – 0.5% volume concentration (red, magenta, blue squares). Mie "blue" dye beads have 0.1%, 1%, 2.5% volume concentration (blue, magenta and red triangles in Figure 6.7) and are within the margin of error (0.5-1 sigma) of the non-dye beads at equal volume concentration. This is to be expected, because our experiment is performed under $1.064\mu\text{m}$ RCP which is in the NIR and if we were to shine a blue surface with a NIR source, there should not be any blue component to be reflected. Consequently, we should expect the same absorption from the non-dye Mie beads and the dye Mie beads which is exactly what is happening in Figure 6.7 where the SC, OC, CPR and DLP curves of Mie dye beads are

within the margin of the non-dye Mie beads at equal volume concentration.

6.5 Simulation Results

Modeling SC , OC , CPR , and DLP using coherent backscattering theory is useful in characterizing the interaction between RCP beam and imperfections like impurities in a weakly absorbing medium. As the scattered signal undergoes multiple forward scattering events, it retains its polarization properties. This scattering process can result in a coherent backscatter effect, amplifying the SC radar backscatter and leading to CPR values exceeding unity [Neish and Carter, 2014].

6.5.1 Effect of particle size simulation

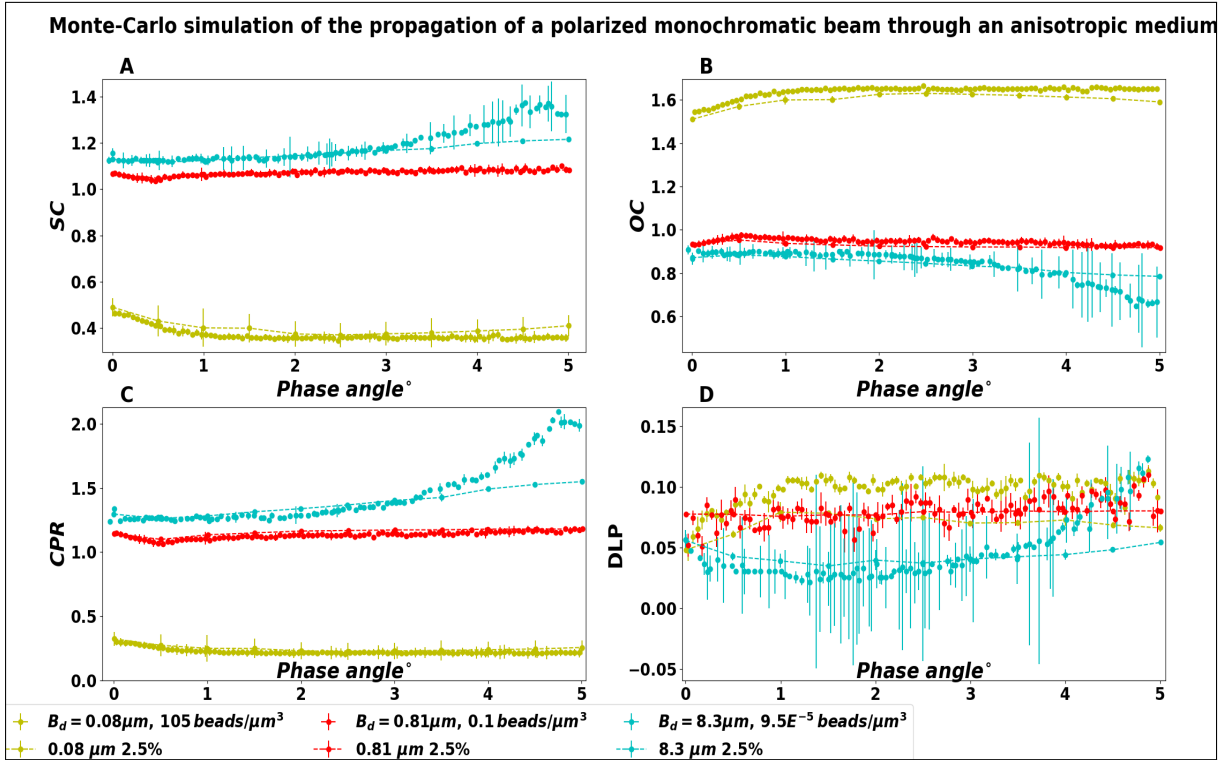


Figure 6.8: Backscatter of latex beads of varying sizes at low phase angles, dots are the model, dashed lines are the experimental data. (A) is the latex bead SC backscattered signal. (B) is the latex bead OC backscattered signal. (C) is the latex bead CPR return. (D) is the latex bead DLP return.

In Figure 6.8 we compare our modeled SC , OC , CPR , and DLP to our experimental SC , OC , CPR , and DLP (yellow, red, and cyan dots and dashed lines in Figure 6.8) of

latex bead solutions at a constant 2.5% v/v, altering only the bead size. Modeled Rayleigh beads (yellow dots in Figure 6.8) show increased transparency relative to experimental Rayleigh beads (yellow dashed line in Figure 6.8). The increased transparency of modeled Rayleigh from the reduced scattering cross section allows for more photons to meet the boundary condition ($z \geq z_{max}$) which flips the polarization state from SC to OC with $OC \gg SC$ with lower $CPR \ll 1$ (yellow dots in Figure 6.8 C) that remain within the margin of error of experimental CPR of Rayleigh beads (yellow dashed line in Figure 6.8 C) with a moderate-high degree of linear polarization (DLP) that is characteristic of single bounce reflection [Bourget et al., 2024b]. Modeled DLP values of Rayleigh beads (yellow dots in Figure 6.8 D) follow the same trend but are high (0.05-0.1) and are 1.5-2 sigma of the experimental Rayleigh DLP values (yellow dashed line in Figure 6.8 D), being an indication of single bounce from the increased transparency. Modeled Mie beads ($0.81\mu m$ red dots in Figure 6.8) have SC , OC , CPR , and DLP features similar to experimental Mie beads SC , OC , CPR , and DLP curves. Modeled Mie beads do exhibit the randomization of $SC \sim OC$, with $CPR \sim 1$ that is associated with diffuse surface scattering from our experimental Mie beads (red dashed lines in Figure 6.8). The surface diffuse scattering results in similar modeled transparency, allowing for the photons to be scattered off the beads in the forward direction that conserves SC polarization resulting in $CPR (> 1)$, and DLP that are within the margin of error of our experimental Mie beads (red dashed line in Figure 6.8). Modeled large Mie beads (cyan dots in Figure 6.8) have similar features to the modeled and experimental classical Mie beads (red dots and dashed lines in Figure 6.8) but more exaggerated, showing stronger forward scattering behavior that is confirmed by the scattering amplitude A_1 , A_2 in Figure 3 D relative to their classical counterpart. The strong forward scattering behavior is more pronounced for our modeled large Mie beads that result in more SC conservation with higher (> 1) CPR between $4 - 5^\circ$ phase angle (cyan dots in Figure 6.8 C) relative to experimental large Mie beads (cyan dashed lines in Figure 6.8 C).

6.5.2 Effect of Particle Concentration in the Rayleigh Scattering Regime Simulation

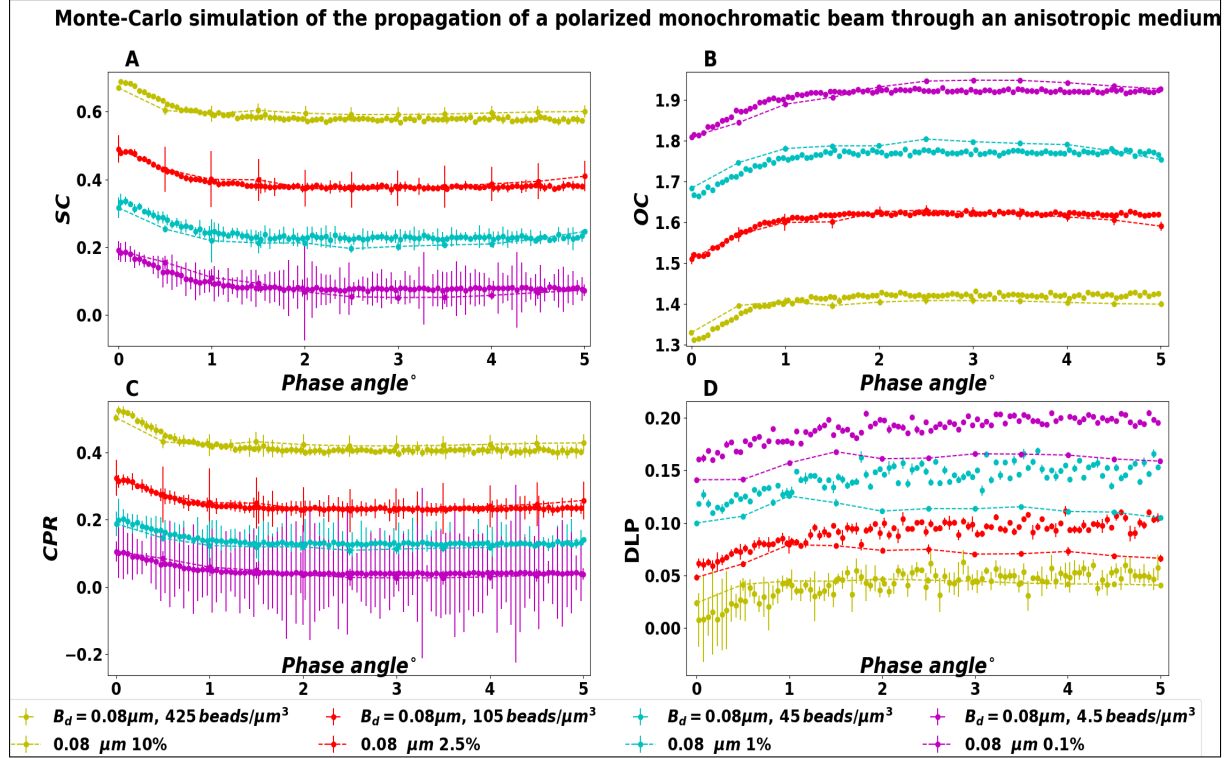


Figure 6.9: Backscatter of Rayleigh latex beads at low phase angles, dots are the model, dashed lines are the experimental data. (A) is the Rayleigh latex bead SC backscattered signal. (B) is the Rayleigh latex bead OC backscattered signal. (C) is the Rayleigh latex bead CPR return. (D) is the Rayleigh latex bead DLP return.

In Figure 6.9, we show the impact of varying the volume concentration of Rayleigh scatterers; we compare our SC , OC , CPR , and DLP for model and experiment (yellow, red, cyan, and magenta dots and dashed lines in Figure 6.9). Rayleigh beads generate isotropic scattering (Figure 3 B) uniformly disperse the scattered field in all directions due to the small scattering cross-section relative to classical Mie and large Mie beads for both model and experiment. This isotropic scattering results in a uniform Henyey-Greenstein scattering phase function with more transmission to subsequent layers that allows photons to bounce off the boundary, resulting in $SC \gg OC$ and $CPR \ll 1$ for both model and experiment (dots and dashed lines in Figure 6.9). Modeled and experimental SC , OC , CPR , and DLP in Figure 6.9 follow the same trend, where the SC and CPR curves are proportional to volume concentration, while OC and DLP curves are inversely proportional

to v/v . However, modeled Rayleigh beads show increased transparency characterized by a higher (1.5 sigma) DLP (dots in Figure 6.9 D) relative to experiment (dashed lines in Figure 6.9 D). The increased transparency of Rayleigh beads should allow more photons to meet the boundary condition $z \geq z_{max}$ and bounce in the opposite direction with their circular polarization flipped from SC to OC with $CPR \ll 1$. This is confirmed by a lower modeled CPR that is still within the margin of error of the experimental CPR (dots in Figure 6.9 C) for all v/v . Overall, our modeled SC , OC , CPR , and DLP capture the experimental features of the Rayleigh beads with a slight discrepancy in modeled DLP (1.5 sigma).

6.5.3 Effect of Particle Concentration in the Classical Mie Scattering Regime Simulation

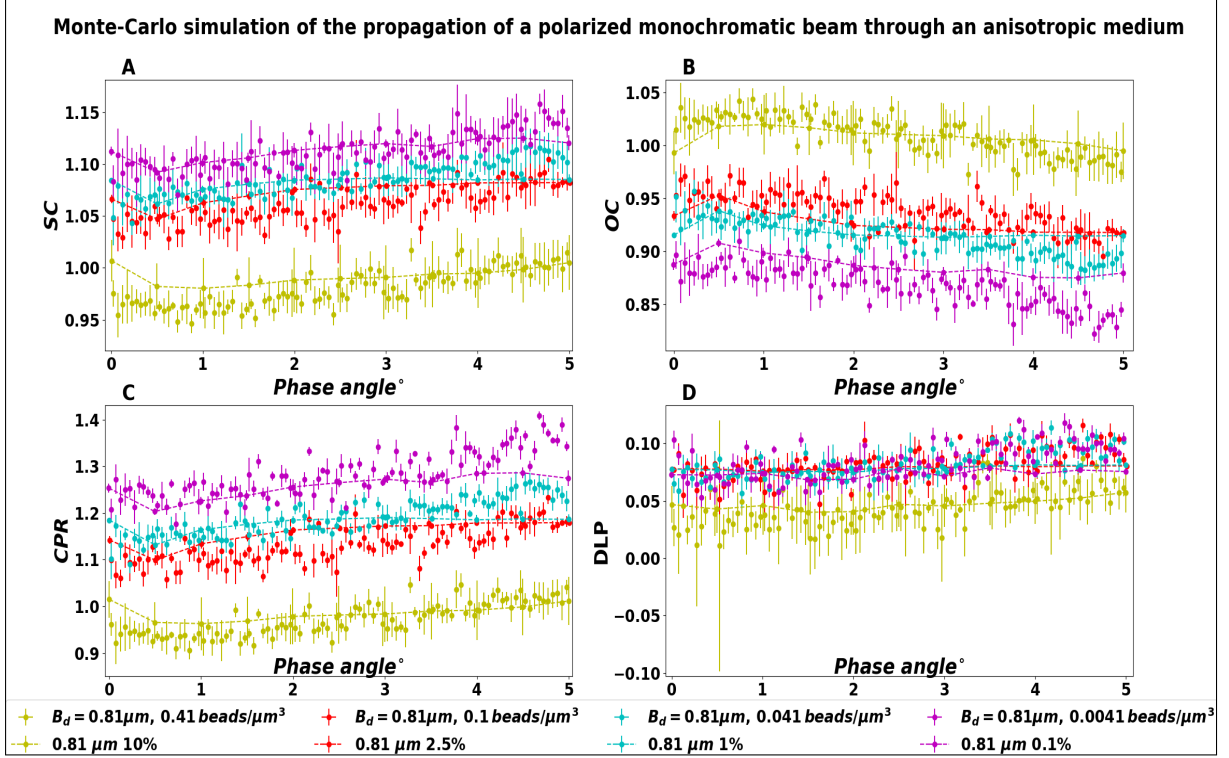


Figure 6.10: Backscatter of classical Mie latex beads at low phase angles, dots are the model, dashed lines are the experimental data. (A) is the classical Mie latex bead SC backscattered signal. (B) is the classical Mie latex bead OC backscattered signal. (C) is the classical Mie latex bead CPR return. (D) is the classical Mie latex bead DLP return.

In Figure 6.10 we explore the effect that changing volume concentration of modeled and experimental classical Mie $0.81 \mu m$ scatters has on SC , OC , CPR , and DLP curves (yellow, red, cyan and magenta dots and dashed lines in Figure 6.10). For both model and experiment, the curves have stronger phase angle dependence than Rayleigh beads due to the incident beam being scattered with greater intensity in the forward direction (Figure 3 A, C) rather than uniformly in all directions for all v/v [Bourget et al., 2024b]. Modeled classical Mie beads (dots in Figure 6.10) have similar SC , OC , CPR curve features that are more pronounced between $4.5^\circ - 5^\circ$ relative to experimental classical Mie beads (dashed lines in Figure 6.10). In both experiment and model the high ($10\%v/v$) Mie beads showed $SC \sim OC$ with $CPR \sim 1$ and a low-mod DLP, that is indicative of surface diffuse scattering that randomizes SC and OC circular polarization with little

to no subsurface scattering [Bourget et al., 2024b]. All modeled and experimental SC , CPR , and DLP (dots and dashed lines in Figure 6.10 A, C, D) curves have the same inverse relationship to volume concentration. When the volume concentration of beads decreases, transmission increases thus more subsurface scattering occurs and increases the DLP (dots and dashed lines in Figure 6.10 D). As subsurface scattering increases further, there are more paths for the photons to be scattered off the beads in the forward direction that conserves the SC polarization, with high CPR (> 1) inversely proportional to volume concentration of Mie scatterers Bourget et al. [2024b] for model and experiment (dots and dashed lines in Figure 6.10 A, B).

6.5.4 Effect of Particle Concentration in the Large Mie Scattering Regime Simulation

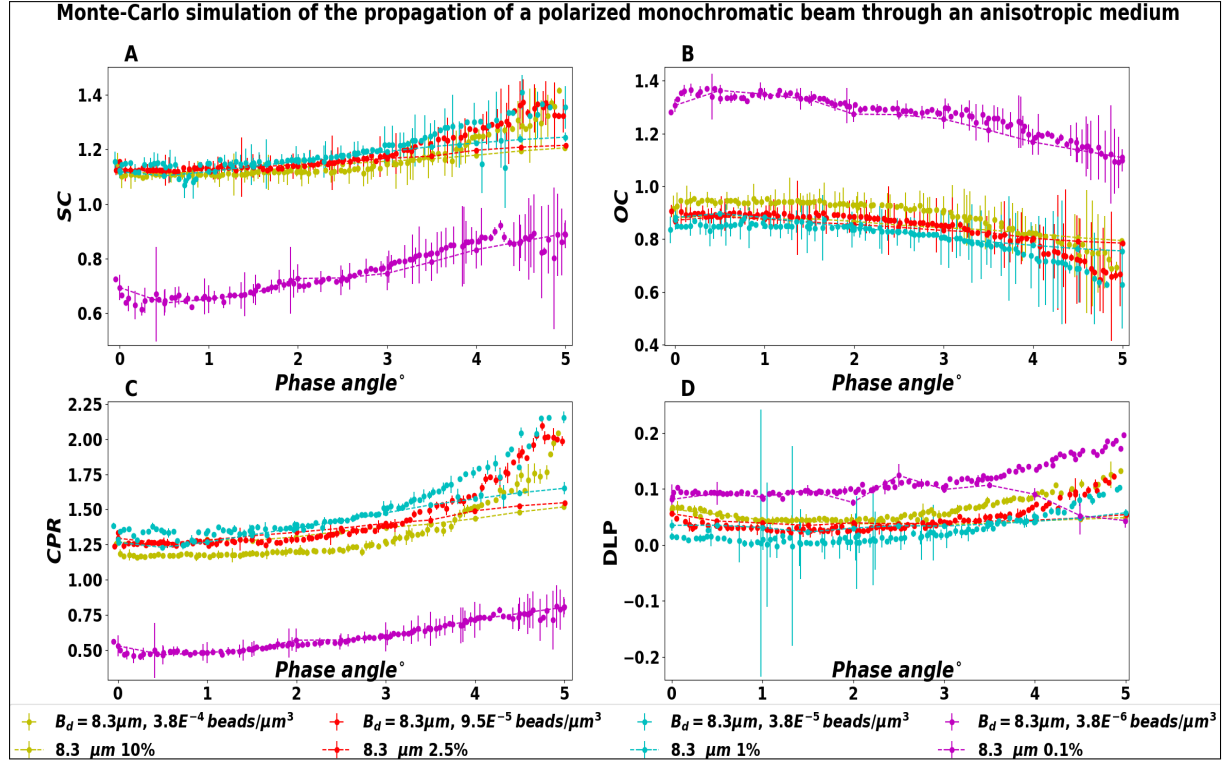


Figure 6.11: Backscatter of large Mie latex beads at low phase angles, dots are the model, dashed lines are the experimental data. (A) is the large Mie latex bead SC backscattered signal. (B) is the large Mie latex bead OC backscattered signal. (C) is the large Mie latex bead CPR return. (D) is the large Mie latex bead DLP return.

In an extended exploration that includes Figures 6.8-6.11, we observe the scattering behavior of modeled and experimental large Mie beads at different volume concentrations as depicted in Figure 6.11. Both modeled and experimental large Mie beads share similar scattering properties with their classical counterparts but scatter incident light more in the forward direction (Figure 3 D), preserving the polarization properties of the incident field (SC) during forward scattering events, resulting in $SC > OC$ and a high (> 1) CPR at 0° phase angle for both model and experiment (dots and dashed lines in Figure 6.11) from 10% – 1% v/v. Both modeled and experimental large Mie beads have an absolute difference in (SC), (OC) circular polarization, (CPR), and (DLP) within the margin of error (0.5-1 sigma) from 10% to 1% v/v. This observation is attributed to the larger beads resulting in less "void space" and thus less subsurface scattering relative to modeled

and experimental classical Mie and Rayleigh beads within the same volume concentration range. This is confirmed by the modeled and experimental low-mod DLP for the large Mie beads, indicating lesser subsurface scattering than their classical counterparts. However, at a 0.1% volume concentration, both model and experiment (magenta dots and dashed lines in Figure 6.11) have the photons meet the $z \geq z_{max}$ boundary condition which flips SC to OC , thereby producing $OC > SC$ with $CPR < 1$, suggesting single bounce scattering with mod-high DLP . The strong forward scattering behavior is more pronounced for our modeled large Mie beads that result in more SC conservation with higher (1.5-2 sigma) CPR and DLP uptake between $4 - 5^\circ$ phase angle relative to experimental large Mie beads. Overall, our modeled SC , OC , CPR , and DLP capture the experimental features of forward scattering of the large Mie beads with a slight discrepancy in modeled curves between $4^\circ - 5^\circ$ phase angle.

Chapter 7

Discussion and Conclusion

7.1 Discussion

7.1.1 Incident Angle

This study investigates the effect of incident angle, surface structures, and the presence of impurities on diffuse (volume) scattering and single bounce (specular) scattering at surface and subsurface levels through the study of *CPR*, *DLP*, *SC* and *OC* circular polarizations using a $1.064\mu m$ RCP beam. The metallic black sample platform displayed low < 0.5 *CPR* and high > 0.1 *DLP* at both 0° (yellow squares in Figure 6.1) and 15° incidence angles (yellow triangles in Figure 6.1). This is due to the free electrons surrounding the metal ions in metallic bonds, which, when interacting with incident polarized radiation, excite and emit photons in *OC* polarization, resulting in single bounce (specular scattering). The *CPR* and *DLP* curves at $i = 0^\circ$ are within the margin of error (0.5-1.5 sigma) relative to the *CPR* and *DLP* curves at $i = 15^\circ$ incident angle for flat surfaces (sample platform, 60% Lambertian alone), revealing a constant response to the incidence angle. Specular smooth surfaces (water glass cup with and without 60% Lambertian (bottom)) scatter energy along the specular direction at low phase angle. It does not mean that at $i = 0^\circ$ there is no diffuse scattering, but if specular scattering is present, it will hide any another scattering process such that diffuse scattering, because the portion of specular photon reaching the detector is much greater than the portion of diffuse scattered photon. At

$i = 15^\circ$ incidence angles, the specular reflection does not return to the detector, making it possible to detect diffuse scattering from Lambertian 60% surfaces beneath water (magenta triangle curves in Figure 6.1). This matches the expected outcomes in previous studies and is attributed to the surface’s microfiber structures, which are randomly oriented, causing a randomization of $SC \sim OC$ with $CPR \sim 1$ (black triangle curves in Figure 6.1).

7.1.2 Comparison to Other Work

This section outlines the testing of a Monte-Carlo simulation using several examples from 3.2.7. The first test replicates a case from Sawicki et al. [2008], while the second explores the behavior of circular and linear polarization ratios CPR , and LPR over varying incident angles from Mishchenko [1992b]. The third test attempts to recreate results from Hapke and Blewett [1991]. In revisiting Sawicki et al.’s work, the simulation is used to reproduce the backscattering enhancement of circular (Figure 3.3 A, B) and linear polarization (Figure 3.3 C, D). The results show that the enhancement decreases with increasing phase angle, which is typical in backscattering studies. The simulated data in Figure 3.3 closely matches the trends observed in the original study of Sawicki et al. [2008], indicating that the model is reliable. We also present a study on the circular polarization ratio (CPR) in Figure 3.4 B and linear polarization ratio (LPR) in Figure 3.4 A as functions of incident angle at a 0° phase angle, comparing the simulation results to those from Mishchenko [1992b]. The findings show that both CPR and LPR decrease with increasing incident angle, with LPR values in Figure 3.4 A, generally lower than CPR in Figure 3.4 B, indicating a less pronounced backscattering effect in linear polarization. As the refractive index increases, the peak of CPR decreases, demonstrating a strong dependence on both incident angle and refractive index. The trends observed in the simulation closely align with Mishchenko’s findings, confirming the model’s accuracy. Finally, the alignment of our work with studies by Hapke and Blewett [1991] underscores the versatility and robustness of EMC simulations in exploring the complex interplay between light and turbid media. Comparison between our work and the work of Hapke and Blewett [1991] shows the CPR , and the LPR (blue, and red dots in Figure 3.5 C, D) to be within margin of error (0.5-1.0 sigma). The CPR simulation (blue dots in Figure 3.5 C) had to be normalized by the

value at 3° in order for agreement with the Hapke and Blewett [1991] results. Modeled *CPR* has discrepancies (1.5-2 sigma) between $1.5^\circ - 2.75^\circ$ associated to experimental error from Hapke and Blewett [1991]. Comparison of our results to the work of Hapke and Blewett [1991] indicates a robust integration of the EMC Monte-Carlo simulation (Figure 3.2 B) and its integration into a python workflow (Figure 3.2 A).

7.1.3 Alumina Powders and Planetary Icy Regoliths

Alumina samples, with a purity exceeding 99.20% AL_2O_3 [MICROAbrasives, 2022] have a hexagonal crystal structure similar to that of water ice, were studied in 6.2.1. We measure the *SC*, *OC*, *CPR*, and *DLP* of alumina powders, and compare the results to Nelson et al. [2000, 2002] for identical size parameters (yellow curves in Figure 6.2 C). For similar size parameter, MAGI, when compared to Nelson et al. [2000, 2002], is within the margin of error for *CPR* from $1.5^\circ - 5^\circ$ (Figure 6.2 C). The Mie ($1.0\ \mu\text{m}$, $2.1\ \mu\text{m}$, and $4.0\ \mu\text{m}$) samples, according to Mie scattering modelling, scatter light in the forward direction relative to incident light (Figure 2 C), due to the high anisotropy g (Table 1 in .2) relative to smaller size parameter. Using Figure 1 in Nelson et al. [2018], the samples with high porosity, at $2.1\ \mu\text{m}$; and $4.0\ \mu\text{m}$ exhibited high (> 1) *CPR* (yellow and blue curves in Figure 6.2 c), attributed to increased subsurface scattering from the platlets particles and high void space. Conversely, samples with lower porosity $30\ \mu\text{m}$ showed fewer paths available for photons to scatter off, reducing the number of scattered *SC* photons, and therefore resembling a classical 99% Lambertian surface, randomizing $SC \sim OC$ with $CPR \sim 1$ (magenta curve Figure 6.2 c). The combination of forward scattering relative to smaller size parameter, and high void space of the Mie ($2.1\ \mu\text{m}$, and $4.0\ \mu\text{m}$) relative to larger size parameter, allows for subsurface forward scattering events that conserve the *SC* circular polarization, resulting in $SC > OC$ with $CPR > 1$. Hence, our alumina samples ($2.1\ \mu\text{m}$, and $4.0\ \mu\text{m}$) may represent fine grain water ice with different porosity. High porosity fine grained water ice ($2.1\ \mu\text{m}$, $4.0\ \mu\text{m}$) has a higher *CPR* and *DLP* due to more subsurface forward scattering that conserves the *SC* polarization more than lower porosity fine grain water-ice ($30\ \mu\text{m}$).

7.1.4 Non-Dye Latex Beads and Planetary Icy Regoliths

The study also compares the impact of impurities in a water-ice scattering medium, using a latex bead solution analogue. For equally sized impurities, the absolute difference between the 10% – 1% v/v curves decreases as parameter size increases (yellow, red, magenta curves in Figures 6.3-6.6). Rayleigh ($0.08\ \mu\text{m}$) impurities (Figure 6.4) scatter light uniformly in all directions. Due to isotropic scattering and a relatively small scattering cross-section of Rayleigh samples, the RCP incident beam bounces off the black metallic sample platform that flips SC to OC with $OC \gg SC$ (Figure 6.4 a, b) and yields a $CPR \ll 1$ (Figure 6.4 c) due to high transparency relative to larger impurities. It's worth noting that an experiment involving the study of CO_2 ice under different pressure and temperature regimes using a custom Goniometer in Isen and Smith [2022, 2023] found that an increase in CO_2 ice transparency resulted in an increase in basal reflections from their sample platforms [Isen and Smith, 2024]. Model for the propagation of the RCP beam through a latex beads solution, and the measurements of SC , OC , CPR , and DLP in Figures 6.8-6.11 enable the drawing of analogies to radar scattering in water ice that enhances the understanding of backscattering phenomena. Modeled Rayleigh beads show similar SC , OC , CPR , and DLP (dots in Figure 6.9) trend to phase angle relative to experimental Rayleigh beads (dashed lines in Figure 6.9). Modeled Rayleigh beads show identical SC , OC , CPR , and DLP trend to phase angle relative to experimental Rayleigh beads (dots and dashed lines in Figure 6.9). However, modeled Rayleigh beads show a slight discrepancy in DLP characterized by a higher (1.5 sigma) modeled DLP (Figure 6.9 D) due to increased modeled transparency relative to experimental Rayleigh beads. The increased modeled transparency results in more photons meeting the boundary $z \geq z_{max}$, which flips the polarization state of the photons from SC to OC more than in our experiment, resulting in a lower (0.5 sigma) modeled CPR (dots in Figure 6.9 C) still within the margin of error of the experimental CPR (dashed lines in Figure 6.9 C). Overall, the simulation captures well the backscattering properties of Rayleigh beads along with the boundary conditions ($z \geq z_{max}$) from the metallic platform in Bourget et al. [2024b] interacting with the photons.

Mie impurities show anisotropic scattering, where the field is scattered in the forward

direction and has lower void space. As v/v of Mie impurities decreases, more subsurface scattering occurs with higher DLP (yellow and red curves in Figure 6.5 d), therefore more paths are available for the photon to forward scatter off the impurities which conserve SC polarization with $SC > OC$ and increase CPR . Modeled classical Mie beads have similar SC , OC , and CPR forward scattering curve features that are more pronounced, characterized by a higher (1-1.5 sigma) modeled SC , and CPR , but lower (1-1.5 sigma) modeled OC from $4.5^\circ - 5^\circ$ relative to experimental SC , OC and CPR (dots and dashed lines in Figure 6.10 A, B, C). Modeled and experimental SC , CPR , and DLP (dots and dashed lines in Figure 6.10 A, C, D) curves have the same inverse relationship to volume concentration. As volume concentration of beads decreases, transmission increases and more subsurface scattering events occur that increase the DLP (dots and dashed lines in Figure 6.10 D). As subsurface scattering increases further, there are more paths for the photons to be scattered off the beads in the forward direction (Figure 3 C) that conserves the SC polarization, with high CPR (> 1) inversely proportional to the volume concentration of Mie scatterers Bourget et al. [2024b] for model and experiment (dots and dashed lines in Figure 6.10 A, B).

Experimental large Mie impurities behaved similarly to a continuous surface with weaker DLP than classical Mie and Rayleigh impurities at equal v/v . However, these large Mie beads scatter the incident field strongly in the forward direction, preserving the polarization properties of the signal with $SC > OC$ (Figure 6.6 a, b) and high (> 1) CPR (Figure 6.6 c). At 0.1% volumetric concentration the signal transmits through the medium and bounces off the sample platform generating $OC > SC$ with $CPR < 1$. Large Mie beads share similar scattering properties with their classical counterparts but scatter incident light more in the forward direction, preserving the polarization properties of the incident field (SC) during forward scattering events, resulting in $SC > OC$ and a high (> 1) CPR at 0° phase angle for both model and experiment (dots and dashed lines in Figure 6.11) for 10% – 1% v/v . However, as with the modeled classical Mie beads, the forward scattering features are more pronounced for the modeled curves, characterized by a higher (1.5-2 sigma) modeled SC , CPR , and DLP , but lower (0.5-1 sigma) modeled OC from $4.0^\circ - 5^\circ$ relative to experimental classical Mie SC , OC , CPR , and DLP

(dots, and dashed lines in Figure 6.10 A, B C, D). The transparency of the modeled and experimental Mie large beads at (0.1%) v/v allows the forward-scattered photons to reach boundary conditions ($z \geq z_{max}$) and undergo single-bounce scattering events, resulting in polarization flipping from SC to OC , with $CPR \ll 1$, akin to the behavior of Rayleigh beads (magenta dots and dashed lines in Figure 6.11). Both modeled Mie classical and large Mie beads show forward scattering features that are more pronounced than their experimental counterparts between $4.5^\circ - 5^\circ$ (Figure 6.10) and $4^\circ - 5^\circ$ phase angle (Figure 6.11). The forward scattering behavior of classical and large Mie beads is clear in Figure 3 C, D relative to the more uniform scattering behavior of Rayleigh beads in Figure 3 B.

The discrepancy observed between the model and experiment between $4^\circ - 5^\circ$ is more pronounced with the large Mie beads (Figure 6.11) relative to classical Mie beads (Figure 6.10). The lower (1-2 sigma) classical and large Mie beads experimental DLP relative to model (Figures 6.10-6.11 D), is characteristic of surface diffuse scattering that randomizes SC and OC circular polarization with little subsurface scattering for all v/v . Hence, our experimental classical and large Mie beads are displaying more surface diffuse scattering events, and less subsurface forward scattering between $4^\circ - 5^\circ$, randomizing SC , and OC circular polarization, leading to lower CPR , and DLP relative to model (dots and dashed lines in Figures 6.10-6.11 C, D). The increased surface diffuse scattering and lower subsurface forward scattering events observed in our experiment could be due to the classical and large Mie beads forming a continuous surface that generates more surface diffuse scattering events, and less subsurface forward scattering events relative to our model. At equal v/v , drawing on insights from Figure 1 in Nelson et al. [2018], the larger beads result in less "void space" relative to Rayleigh and classical Mie beads, making the surface diffuse, and the discrepancy between $4^\circ - 5^\circ$ more pronounced for the large Mie beads than the classical Mie beads.

7.1.5 Dye Latex Beads of Identical Sizes and Planetary Icy Regoliths

Mie "blue" dye beads have 0.1%, 1%, 2.5% volume concentration (red, magenta and blue triangles Figure 6.7) are within the margin of error (0.5-1 sigma) of the non-dye beads

at equal volume concentration (red, magenta and blue squares Figure 6.7). This is to be expected, because our experiment is performed under $1.064\mu\text{m}$ *RCP* which is in the *NIR* and if we were to shine a blue surface with a NIR source, there should not be any blue component to be reflected. Consequently, the absorption of a water-ice medium with Mie 'blue dye' buried impurities is identical to the absorption of a water-ice medium with Mie non-dye buried impurities, such that the *SC*, *OC*, *CPR* and *DLP* curves in Figure 6.5 are within the margin of error (1-1.5 sigma) of the *SC*, *OC*, *CPR* and *DLP* curves in Figure 6.7.

7.1.6 Strengths and Weaknesses

Our study presents several strengths. Firstly, conducting a similar experiment using radar wavelengths would be costly and complex, whereas employing $1.064\mu\text{m}$ near-infrared (NIR) optics is comparatively affordable and reliable. Secondly, our beam splitter technique facilitates highly accurate measurements of back-scattering conditions at a 0° phase angle. This precision is not achievable with other goniometers, which can only approximate these conditions by minimizing the angle between the source and detector without causing overlap. Additionally, our usage of alumina particles and beads, which follow a Gamma-distribution identical to that described by Nelson et al. [Nelson et al., 2000, 2002, 2018] and Hapke et al. [Hapke and Blewett, 1991], enhances the reliability of our testing. These Gamma-distributed particles and beads effectively replicate the unique circular polarization ratio (*CPR*) of icy bodies. One advantage of using beads is their azimuthal symmetry, making them ideal for studying the impact of impurities in radar scattering. Our simulations have revealed intricate details about the interaction of RCP light with various scattering mediums, emphasizing the importance of particle size (Figure 6.8), and concentration (Figures 6.9, 6.10, 6.11) in the backscattering and polarization of light. The results highlight the critical role of coherent backscatter in enhancing the radar backscatter signal, a phenomenon that is crucial for interpreting radar observations of icy bodies in the solar system.

However, our study does have certain limitations. We assume that the system is time-isotropic, meaning there is no time dependence, and that the distribution of scatterers

(beads) remains constant over time, avoiding clumping. We also assume uniformity in the composition and density of the impurities. Despite our efforts to minimize them, back-reflections from the sample platform are still present, particularly for Rayleigh impurities, due to their reduced scattering cross-section. This phenomenon could inadvertently convert our (*SC*) signal into an overestimated (*OC*) signal, leading to an underestimation of the *CPR*. The study faced discrepancies between experiment and model, especially with large Mie beads that are characterized by less subsurface forward scattering events, and increased diffuse surface scattering behavior between $4.5^\circ - 5^\circ$ for the classical Mie beads (Figure 6.10), and $4^\circ - 5^\circ$ for the large Mie beads (Figure 6.11). This highlights the need for a continual refinement of our experiment and model. We recommend that future work should focus on studying the effect that void space has on surface and subsurface scattering moreover, the use of a dark diffuse platform rather than a metallic platform would scatter away the back-reflections for high transparency sample, making the data analysis and modeling more reliable. The fidelity of EMC simulations can be improved by incorporating more detailed beads characteristics (shapes) and interactions (clumping) and their effects on the statistical distribution of the scattering angle through a modified Henyey and Greenstein scattering function. Optical polarization studies, while excellent for simulating certain aspects of radar scattering (depolarization effects, phase angle changes, and angular scattering behavior), lack the temporal resolution characteristic of radar systems. Radar systems operate in the time domain, where the temporal characteristics of a pulse provide crucial information, such as distance measurement, where radar systems can determine the distances (range) to scatterers using the time delay between transmitted and received pulses. Using the velocity information through Doppler shift, radar can infer motion relative to the observer, providing data on dynamics within the medium. Radar pulses can resolve rapid, transient changes in scattering properties, due to small-scale dynamic processes like particle aggregation or melting [Van Zyl and Kim, 2011, Ulaby et al., 1986, Chen and Jin, 1993]. In the case of our experiment on latex beads in aqueous solution, the lack of time-resolved measurements means that dynamic phenomena (how scatterers evolve or interact over time) aren't captured. Moreover, we are observing a steady-state or ensemble-averaged behavior (scattering from the beads), whereas radar can separate contributions

from different scatterers over time. In the context of water ice mediums, this limitation might obscure certain transient or dynamic effects that are critical for interpreting radar data, especially when ice includes moving inclusions or time-varying interfaces [Bohren and Huffman, 1983, Mishchenko et al., 2000, Schmitt et al., 1998, Born and Wolf, 1999]. Suggesting that future work could incorporate time-resolved optical methods or complementary radar experiments to address this gap. The error bar from Figures 6.2-6.7 and from Figures 6.8-6.11 is the ratio of the standard deviation relative to the average measurement (5 trials). The large error bar of the large Mie beads in Figure 6.8, and 6.11 is due to the pronounced forward scattering and high angular variation of the scattering phase function (Figure 3 D) for model and experiment. In the case of the Rayleigh beads (Figure 6.9 D) it is due to high transparency resulting in a low *CPR* for model and experiment, which increases the relative error as the standard deviation stays relatively constant. The relatively low number of trials (5) might result in high variation (standard deviation), especially for the large Mie beads for model and experiment. These challenges have opened avenues for future research, particularly in refining our experiment in Bourget et al. [2024b], and EMC simulations to incorporate more realistic particle characteristics (shapes) and interactions (clumping). Furthermore, the study has potential in the field of planetary science, such that atmospheric physics, and remote sensing, where understanding light scattering properties is essential.

7.2 Conclusion

This work aims to elucidate radar scattering within icy regolith and water-ice medium using icy regolith analogs, ultimately helping to interpret data from radar observations of icy bodies in the solar system. The study began with an examination of the effect of incidence angles on scattering, comparing low 0° and high 15° incidence angles. Smooth, non-specular surfaces (including a background and a 60% Lambertian) exhibited similar *SC*, *OC*, *CPR*, and *DLP* curves within the margin of error for both low 0° and high 15° incidence angles. Specular surfaces, represented by water and a 60% Lambertian with water on top, were observed to flip $OC \gg CC$ with $CPR \ll 1$ at low 0° incidence angle due to specular reflection. At a 15° incidence angle, specular reflection became negligible,

enabling the detection of diffuse and dihedral (multiple bounces) scattering. This shift is evident in the comparative analysis of the Lambertian surface with and without a water glass cup at 15° incidence angle, and at 0° incidence angle.

The study also involved the evaluation of $2.1\mu m$, $4.0\mu m$, $30.0\mu m$ alumina samples, chosen for their similarities to fine grain water-ice. The high porosity samples ($2.1\mu m$, $4.0\mu m$) displayed higher void space, leading to increased subsurface scattering and higher *CPR* values relative to $30\mu m$. Conversely, the low porosity sample ($30\mu m$) had less subsurface scattering and fewer paths for the *SC* signal to scatter, yielding a *CPR* value similar to a Lambertian 99% surface. This leads to the conclusion that the high *CPR* detected on icy bodies like Europa and Ganymede (Table 53.2 and Figure 53.15 in Neish and Carter [2014]) may not be due to water-ice itself, but rather the ice's porosity, being also consistent with particulate scatterers (Figure 1 in Nelson et al. [2018]). Hence, *CPR* alone may not be a robust tool for determining the presence of water-ice.

The study also considered the impact of impurities, simulated with latex beads suspended in water to mimic buried impurities within transparent water-ice. Rayleigh impurities due to isotropic scattering and relatively high void space yielded $OC \gg SC$ and low *CPR* due to single bounce scattering from the metallic background, meaning that a transparent slab of water-ice with Rayleigh impurities would return the radar signature of the layer beneath the icy medium. Our modeled Rayleigh beads show increased transparency relative to experimental Rayleigh beads that generate more subsurface single bounce scattering events resulting in higher (1.5 sigma) modeled *DLP* and lower modeled *CPR* within the margin of error of the experimental Rayleigh beads. Transparent water-ice with classical or large Mie impurities displayed anisotropic scattering proportional to the size of the impurities and void space inversely proportional to the size of the impurities. This caused a higher *CPR* inversely proportional to the v/v of the classical Mie impurities. Our modeled classical and large Mie beads show more pronounced modeled forward scattering behavior between $4.5^\circ - 5^\circ$ for the classical Mie beads (Figure 6.10), and $4^\circ - 5^\circ$ for the large Mie beads (Figure 6.11). Finally, large Mie impurities acted like a continuous surface with conserved *SC* and $CPR > 1$ due to forward scattering. Modeling large Mie beads is challenging due to the narrowness of the scattering phase function [Bourget et al.,

2024b] and the multiplicity of the forward scattering events (Figure 3 D).

Dye impurities behaved like regular Mie impurities as the right circular polarized (*RCP*) signal was in the near infrared (*NIR*), with no blue components to be absorbed. In summary, this study of icy regolith analogs offers valuable insights into the complex interactions between radar waves and icy regolith. It revealed how the angle of incidence, surface porosity, and impurities influence radar scattering, informing the interpretation of radar observations of icy planetary bodies.

In summary, this study of icy regolith analogs offers insights into the complex interactions between radar waves and icy regolith. Our study contributes valuable insights into the mechanisms of light scattering in turbid media and sets the stage for future investigations that could extend these findings to a broader range of applications. It reveals how the angle of incidence, surface porosity, and impurities influence radar scattering, informing the interpretation of radar observations of icy planetary bodies and providing a dataset that can be used to test future scattering models. Our modeling work brings valuable insights into the mechanisms of light scattering in turbid media by being successfully able to recreate the work of Sawicki et al. [2008] in Figure 3.3, Mishchenko [1992b] in Figure 3.4, and Hapke and Blewett [1991] in Figure 3.5 with some discrepancies due to experimental error from Hapke and Blewett [1991]. Our modeled Rayleigh beads show increased transparency relative to experimental Rayleigh beads that generate more subsurface single bounce scattering events resulting in higher (1.5 sigma) modeled *DLP* and lower modeled *CPR* within the margin of error of the experimental Rayleigh beads. Modeling large Mie beads is challenging due to the narrowness of the scattering phase function [Bourget et al., 2024b] and the multiplicity of the forward scattering events (Figure 3 D). These large Mie beads, due to lower 'void space' relative to classical Mie and Rayleigh beads, form a continuous surface that generates more surface diffuse scattering, and less subsurface forward scattering events relative to our model. Our model was able to recreate the *SC*, *OC*, *CPR*, and *DLP* return of our Rayleigh, classical Mie, and large Mie beads with discrepancies between $4^\circ - 5^\circ$ for our classical Mie and large Mie beads due to higher surface diffuse scattering in the experiment. The successful integration of EMC simulations with experimental data validates this as a robust framework for future studies, promising

to enable future studies in optical and planetary sciences. Moreover, our experimental and modeling work also sets the stage for future investigations that could extend these findings to a broader range of applications including light scattering by planetary surfaces and biological tissues.

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Appendices

Samples Characteristics and Simulation Tables

.1 Reflectance at 0° phase angle

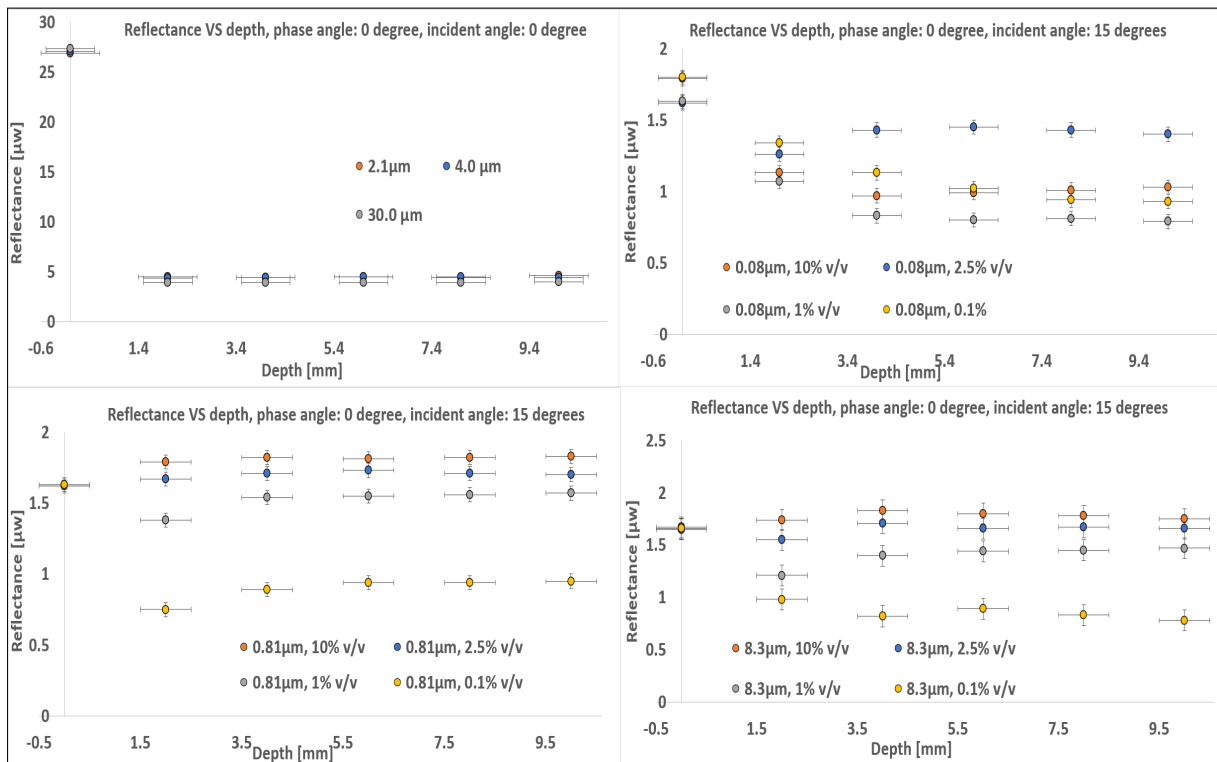


Figure 1: Depth versus reflectance

.2 Simulation Tables of Alumina Powder

Table 1: Alumina powder simulation $\rho_{AL_2O_3}=3.95\text{g/cm}^3$ Elements [2024], index of refraction $m = 1.75$ (relative to air) Polyanskiy [2018] at $1.064\mu\text{m}$ (relative to air). The medium refractive index is 1.0. The calculation of the coefficients is based on [Mätzler, 2002].

1

Samples	X_0	$\langle X_0 \rangle$	$\langle \beta_{sca} \rangle [m^{-1}]$	$\langle \beta_{abs} \rangle [m^{-1}]$	$\langle \beta_{bs} \rangle [m^{-1}]$	$\langle g \rangle$	$\frac{bead}{m^3}$
$0.1\mu m$	0.295	0.271	3.03×10^3	$1.95 * 10^{-11}$	4.27×10^3	8.0×10^{-3}	10^{20}
$1.0\mu m$	2.95	2.71	5.68×10^5	$4.96 * 10^{-11}$	3.59×10^5	0.562	2×10^{17}
$2.1\mu m$	6.20	5.71	3.82×10^5	$6.84 * 10^{-11}$	1.55×10^6	0.731	5×10^{16}
$4.0\mu m$	11.80	10.89	2.66×10^5	$5.41 * 10^{-11}$	1.73×10^6	0.806	10^{16}

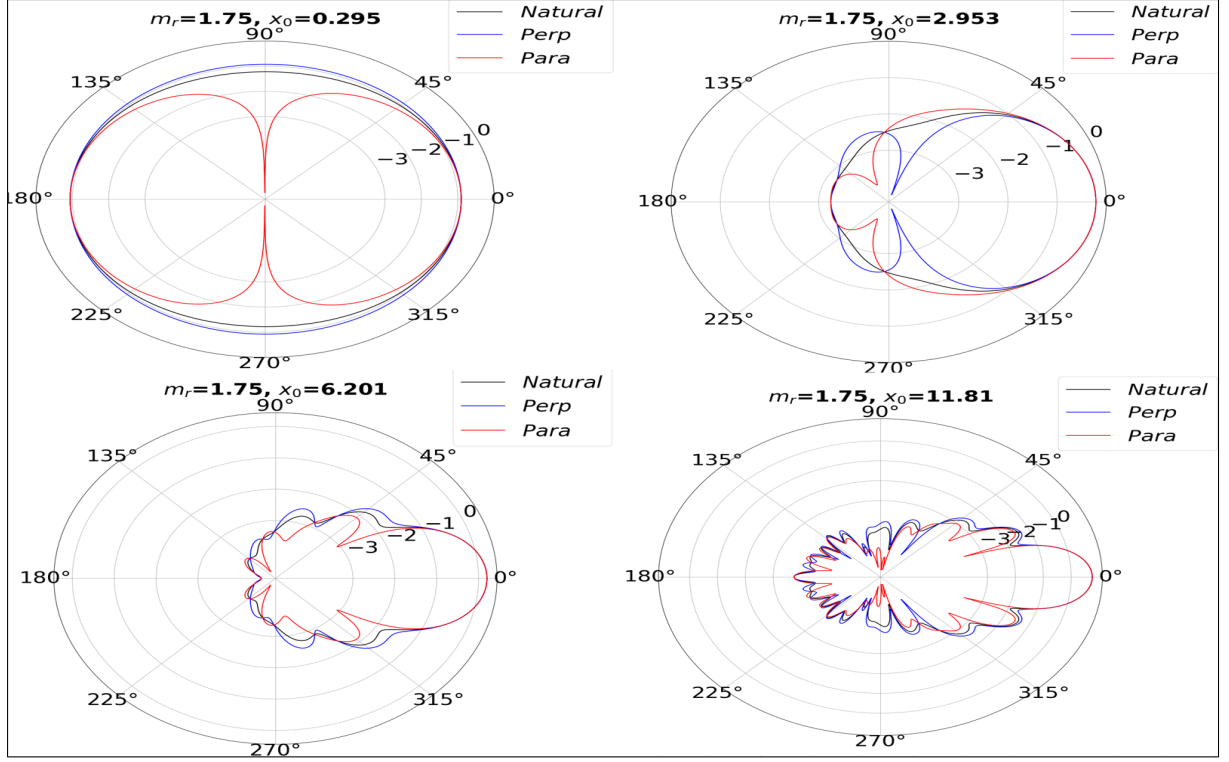


Figure 2: $Natural = \frac{|\langle \hat{F}_{11}(\theta) \rangle|^2 + |\langle \hat{F}_{22}(\theta) \rangle|^2}{2}$, $perpendicular = |\langle \hat{F}_{11}(\theta) \rangle|^2$, $parallel = |\langle \hat{F}_{22}(\theta) \rangle|^2$ scattered electric field (log-scale). (A) Natural, perpendicular, and parallel for Rayleigh ($0.1\mu m$). (B) Natural, perpendicular, and parallel for classical Mie ($1.0\mu m$). (C) Natural, perpendicular, and parallel for classical Mie ($2.1\mu m$). (D) Natural, perpendicular, and parallel for classical Mie ($4.0\mu m$). Incident light from the left-side (180°), the expected scattered fields, the calculation of the scattering amplitudes is based on [Mätzler, 2002].

.3 Rayleigh Beads ($0.08\mu m$) Simulation Tables

Table 2: Rayleigh ($0.08\mu m$) latex beads Monte-Carlo simulation input parameters table, the size parameter is $X_0 = 0.235$, the expected size parameter is $\langle X_0 \rangle = 0.216$, the expected anisotropy is $\langle g \rangle = 0.01$, the expected normalization factor is 5.92×10^{-05} , the incident wavelength is $1.064\mu m$ (relative to air), the medium refractive index is 1.33 (relative to air), the refractive index of the beads is 1.59 (relative to air) scientific [2015]. The calculation of the coefficients is based on [Mätzler, 2002]

$\langle \beta_{sca} \rangle [m^{-1}]$	$\langle \beta_{abs} \rangle [m^{-1}]$	$\langle \beta_{bs} \rangle [m^{-1}]$	$[\frac{bead}{m^3}]$	% solid	$\langle l_s \rangle [m]$
2.274×10^3	$2.68 * 10^{-11}$	3.284×10^3	4.25×10^{20}	10%	9.18×10^{-4}
5.62×10^2	$7.08 * 10^{-12}$	7.70×10^2	1.05×10^{20}	2.5%	3.71×10^{-3}
2.40×10^2	$3.03 * 10^{-12}$	3.47×10^2	4.5×10^{19}	1.0%	8.67×10^{-3}
2.41×10^1	$3.03 * 10^{-13}$	3.4×10^1	4.5×10^{18}	0.1%	8.67×10^{-2}

.4 Mie Beads ($0.81\mu m$) Simulation Table

Table 3: Mie ($0.81\mu m$) latex beads Monte-Carlo simulation input parameters table, the size parameter is $X_0 = 2.39$, the expected size parameter is $\langle X_0 \rangle = 2.20$, the expected anisotropy is $\langle g \rangle = 0.19$, the expected normalization factor is 1.53×10^1 , the incident wavelength is $1.064\mu m$ (relative to air), the medium refractive index is 1.33 (relative to air), the refractive index of the beads is 1.59 (relative to air) scientific [2015]. The calculation of the coefficients is based on [Mätzler, 2002]

$\langle \beta_{sca} \rangle [m^{-1}]$	$\langle \beta_{abs} \rangle [m^{-1}]$	$\langle \beta_{bs} \rangle [m^{-1}]$	$[\frac{bead}{m^3}]$	% solid	$\langle l_s \rangle [m]$
4.94605×10^5	$5.58 * 10^{-11}$	8.82542×10^5	4.1×10^{17}	10%	2.33×10^{-6}
1.23345×10^5	$1.39 * 10^{-11}$	2.20090×10^5	1×10^{17}	2.5%	9.34×10^{-6}
4.9460×10^4	$5.58 * 10^{-12}$	8.8254×10^4	4.1×10^{16}	1.0%	2.33×10^{-5}
4.946×10^3	$5.59 * 10^{-13}$	8.825×10^3	4.1×10^{15}	0.1%	2.33×10^{-4}

.5 Simulation Table of Large Mie ($8.3\mu m$) Suspended Latex Beads in Water

Table 4: Large Mie ($8.3\mu m$) latex beads simulation table, the size parameter is $X_0 = 24.51$, the expected size parameter is $\langle X_0 \rangle = 22.59$, the expected anisotropy is $\langle g \rangle = 0.74$, the incident wavelength is $1.064\mu m$ (relative to air), the medium refractive index is 1.33 (relative to air), the refractive index of the beads is 1.59 (relative to air) scientific [2015]. The calculation of the coefficients is based on [Mätzler, 2002]. $N_{ph} = 10^7$, $N_{scat} = 361$, $N_{phase} = 100$, $R_{phase} = 5^\circ$, $I = 15^\circ$, $S_i = [1, 0, 0, -1]$, $\tau = 7mm$.

	$\langle \beta_{sca} \rangle [m^{-1}]$	$\langle \beta_{abs} \rangle [m^{-1}]$	$\langle \beta_{bs} \rangle [m^{-1}]$	$[\frac{bead}{m^3}]$	% solid	$\langle l_s \rangle [m]$
4	4.0609×10^4	$3.54 * 10^{-11}$	5.4439×10^4	$3.8 * 10^{14}$	10%	2.27×10^{-5}
	1.0152×10^4	$8.87 * 10^{-12}$	1.3609×10^4	$9.5 * 10^{13}$	2.5%	9.11×10^{-5}
	4.069×10^3	$3.54 * 10^{-12}$	5.4439×10^3	$3.8 * 10^{13}$	1%	2.27×10^{-4}
	4.14×10^2	$3.62 * 10^{-13}$	5.55×10^2	$3.8 * 10^{12}$	0.1%	2.23×10^{-3}

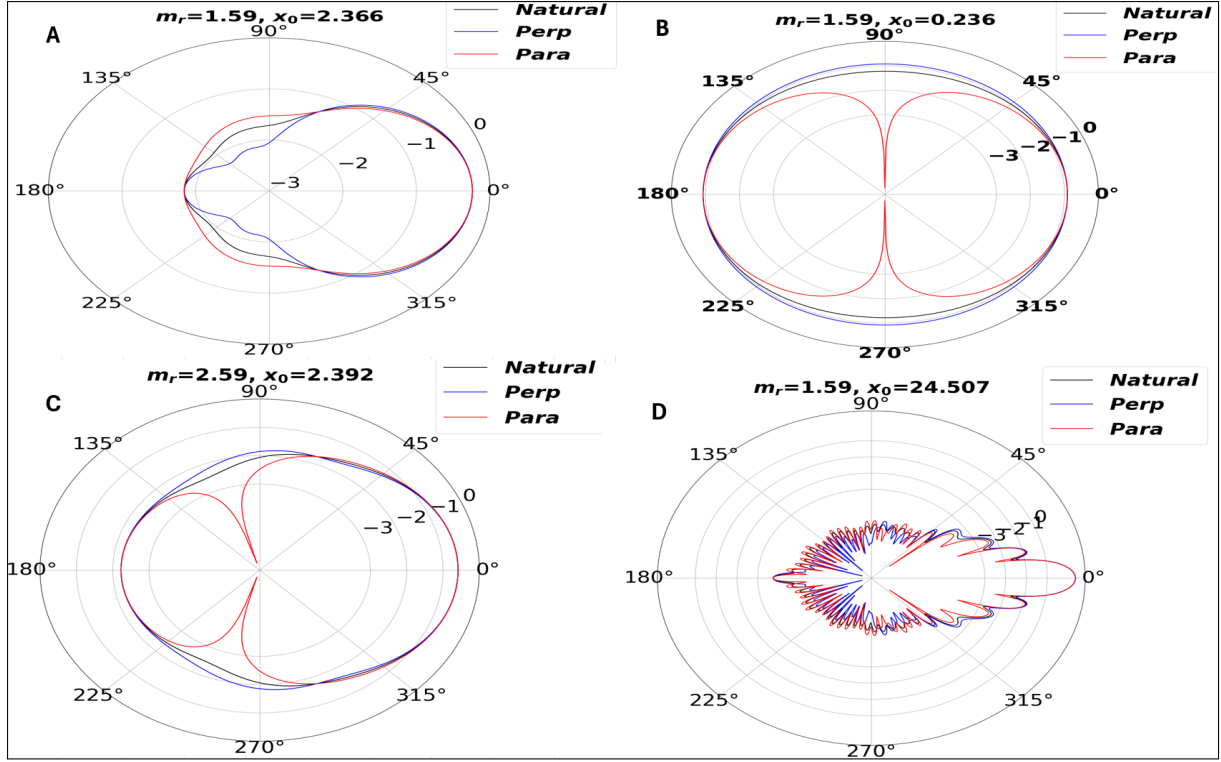


Figure 3: $Natural = \frac{|\langle \hat{F}_{11}(\theta) \rangle|^2 + |\langle \hat{F}_{22}(\theta) \rangle|^2}{2}$, $perpendicular = |\langle \hat{F}_{11}(\theta) \rangle|^2$, $parallel = |\langle \hat{F}_{22}(\theta) \rangle|^2$ scattered electric field (log-scale). (A) Natural, perpendicular, and parallel for Mie ($0.497\mu m$) from 3.2.7. (B) Natural, perpendicular, and parallel for Rayleigh ($0.08\mu m$) from 6.5.2. (C) Natural, perpendicular, and parallel for classical Mie ($0.81\mu m$) from 6.5.3. (D) Natural, perpendicular, and parallel for large Mie ($8.3\mu m$) from 6.5.4. Incident light from the left-side (180°), the calculation of the scattering amplitudes is based on [Mätzler, 2002].

.6 Mie Beads ($0.497\mu m$) Simulation Tables

Table 5: Mie ($0.497\mu m$) latex beads Monte-Carlo simulation compared to experimental data in Fig.1 from [Hapke and Blewett, 1991]. The concentration is $5.5 \times 10^{17} \frac{beads}{m^3}$, the size parameter is $X_0 = 2.36$, the expected size parameter is $\langle X_0 \rangle = 2.18$, the expected scattering coefficient is $\langle \beta_{sca} \rangle = 2.84 \times 10^5$, the expected absorption coefficient is $\langle \beta_{abs} \rangle = 2.42 \times 10^{-11}$, the expected backscattering coefficient is $\beta_{bs} = 5.07 \times 10^4$, the expected anisotropy is $\langle g \rangle = 0.65$, the expected normalization factor is 1.49×10^1 , the expected mean free path is $\langle l_s \rangle = 1.31 \times 10^{-6}$, the amount of solid is 10% matching that of [Hapke and Blewett, 1991]. The incident wavelength is $0.66\mu m$ (relative to air), the medium refractive index is 1.33 (relative to air), the refractive index of the beads is 1.59 (relative to air) scientific [2015]. The calculation of the coefficients is based on [Mätzler, 2002]

5

Item	Peak area ($0 - 2^\circ$)	HWHM	$2^\circ - 3^\circ$ avg	$2^\circ - 3^\circ$ STD	$2^\circ - 3^\circ$ var
<i>SC</i> sim	1.16	($0.4^\circ, 3.59$)	2.53	0.044	0.002
<i>OC</i> sim	0.26	($0.52^\circ, 1.86$)	1.69	0.026	0.0006
<i>CPR</i> sim	0.25	($0.39^\circ, 1.27$)	1.02	0.02	0.0005
<i>CPR</i> Hapke 1991	0.22	($0.41^\circ, 1.28$)	1.10	0.05	0.002
<i>LPR</i> sim	0.094	($0.36, 0.81$)	0.9	0.016	0.0003
<i>LPR</i> Hapke 1991	0.098	($0.37, 0.84$)	0.93	0.021	0.0004

Multi-Axis-Goniometer-Instrument, EMC Simulation, and Acronyms

.7 Multi-Axis-Goniometer-Instrument Assembly Dimension

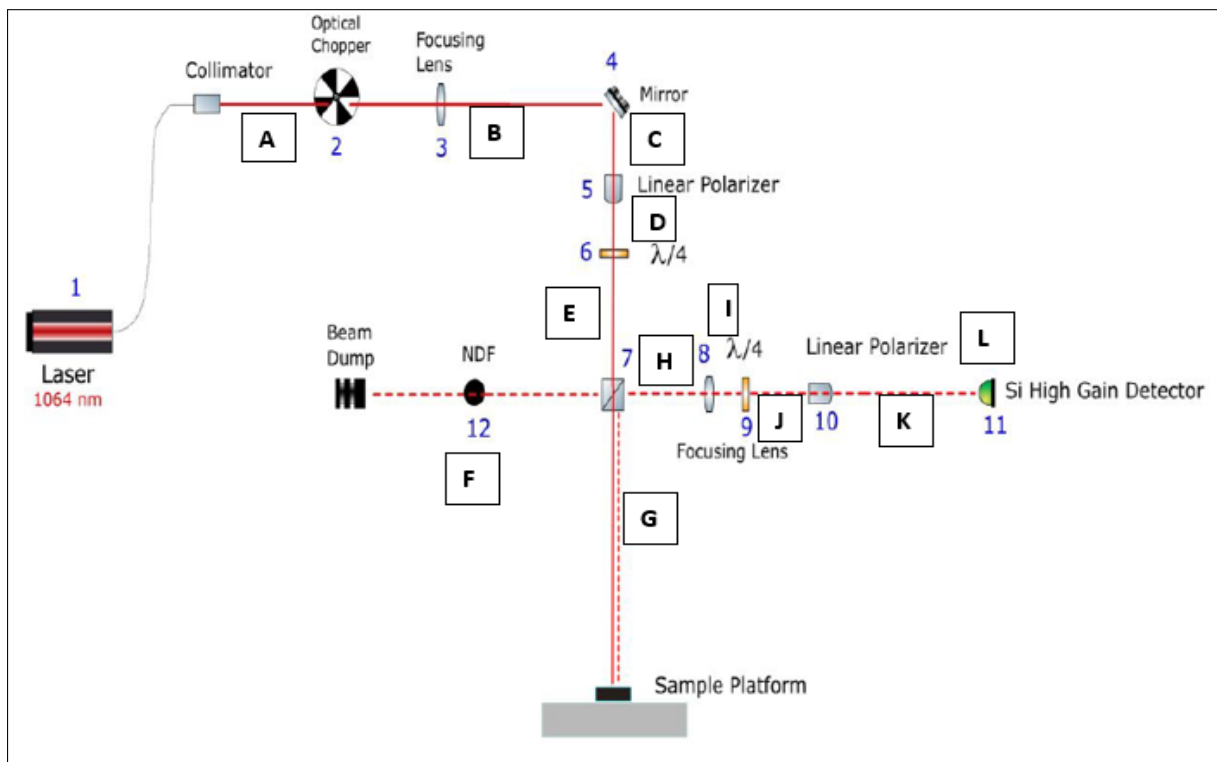


Figure 4: MAGI schematic Blewett et al. [2018], optical components 1-6 generate right circular polarization and are installed on a caddy. Components 8-11 are installed on a polarimetric arm and serve as an analyser for the scattered light from sample. Both caddy and arm can move independently. NDF (12) is our beam dump

Table 6: MAGI distance table

Distance	Value
A (collimator to chopper)	86mm+/-1mm
B (chopper to 45° mirror)	280mm+/-1mm
C (45° deg mirror to LP 1)	100mm+/-1mm
D (LP1 to QWP 1)	56mm+/-1mm
E (QWP1 to BS)	225mm+/-1mm
F (BS to ND)	100mm+/-1mm
G (BS to platform)	639 mm+/-1mm
H (BS to focusing lens 1)	80mm+/-1mm
I (lens 1 to QWP 2)	34mm+/-1mm
J (QWP 2 to LP2)	56mm+/-1mm
K (LP 2 to lens 2)	46.5mm+/-1mm
L (lens 2 to photodiode detector)	18mm+/-1mm

.8 EMC Simulation Integration into a Python Workflow

Table 7: Integration of the EMC simulation in a python workflow (A in Figure 3.2 from 3.2.4)

Steps	Description
A1	Define N_{ph} , N_{scat} , N_{phase} , R_{inc} , R_{phase} , S_i , R_0 , λ , m_1 , m_2 , τ , C
A2	Calculate the size parameter X_0 using λ and B_d
A3	From S_i , get initial E_{para} , E_{perp} .
A4	Get the gamma distribution $n(x)$ centered at X_0
A5	From Mie scattering and $n(x)$, returns $\langle A_1 \rangle$, $\langle A_2 \rangle$, $\langle Q_i \rangle$, $\langle l_s \rangle$, z_{max} .
A6	Run EMC, from E_{para} , E_{perp} get Stokes parameter 2.3.
A7	Generate polarization ratios 2.3 from Stokes parameter.
A8	Graph polarization ratios as a function of phase and incident angle R_{phase} , R_{inc} .
A9	Check if this is the last incident angle in R_{inc} .
A10	if A9 true, end the program.

Table 8: EMC simulation (B in Figure 3.2 from 3.2.4)

Steps	Description
B1	The EMC simulation is initiated.
B2	Input parameters from Mie scattering output.
B3	Initiate path using the anisotropy (g) and, Henyey-Greenstein scattering function.
B4	Calculate the distance (z) to boundary (z_{max}).
B5	Check if the photon hits the boundary ($z \geq z_{max}$).
B6	If B5 is true, bounce light and update the photon position, direction, and rotation matrix accordingly.
B7	Rotate E-field to align with the local scattering plane at each scattering event.
B8	Update and rotate E-field using rotation and scattering matrix.
B9	Check if the path is completed (last scatter reached).
B10	Store the E-field of the forward path (1st to last scattering event).
B11	Initiate reverse path (1st scattering event becomes last scattering event).
B12	Align E_{perp} with the normal of the scattering plane, rotate frame of reference.
B13	End of the EMC, return E_{para}, E_{perp} .

.9 Table of Acronyms

Table 9: Table of Acronyms

Acronym	Description
MAGI	Multi-Axis-Goniometer-Instrument.
S_i, S_{ini}	Incident Stokes vector.
NIR	Near infrared radiaton.
RCP	Right circular polarization.
LCP	Left circular polarization.
SC	Same sense circular polarization.
OC	Opposite sense circular polarization.
CPR	Circular polarization ratio.
DLP	Degree of linear polarization.
LP	Linear polarizer.
QWP	Quarter wave plate.
BS	Beam splitter.
NDF	Neutral density filter.
EMC	Electric-field Monte-Carlo.
E_{para}	Parallel electric-field.
E_{perp}	Perpendicular electric-field.
$\tau [mm]$	Sample thickness.
N_{ph}	Number of Photon.
N_{scat}	Number of scattering angle.
N_{phase}	Number of phase (exit) angle.
$I [^\circ]$	Incident angle.
$R_{inc} [^\circ]$	Incident angle range.
$R_{phase} [^\circ]$	Phase (exit) angle range.
$R_0 [m]$	Central radius of the scatterers.
$\lambda [m]$	Incident wavelength.
m_1	Refractive index of the scattering medium.
m_2	Refractive index of the scatterers.
m	Relative refractive index.
m_r	Real part of the relative refractive index.
κ	Imaginary part of the relative refractive index.
$C [\frac{beads}{m^3}]$	Concentration of the scatterers.
θ	Phase (exiting) angle.
Θ	QWP angle.