

# PfaffModule7L13

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## SUMMARY KEYWORDS

x axis, equals, integral, area, curve, function, interval, subtract, negative, integrating, orange, positive, discontinuities, values, rectangle, crosses, greater, multiplying, horizontal axis, green

## SPEAKERS

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So welcome. So today I'm going to talk about integrals in net area. And this is going to, it's going to be very related to something we've already talked about in terms of the integral being the area underneath the curve. It's just what do you do if a function is not always above the X axis, as in it's not always positive. Okay? So, and let me remind you a little bit of what we had learned already. So we have learned, right, that for our function  $F$ , so if I have a function  $F$ , right, so then I know that. So there are two things I would need to know, we would need that  $F$  of  $X$  is, we would always want that that's greater than or equal to zero for each  $X$  in a particular interval. So for each  $X$  in and then we had some kind of interval,  $A, B$ . I feel like my interval was orange before, but I can't entirely remember that, so, okay. And the second thing that we wanted was, well, if we want to be talking about we know that we can integrate if there are jump discontinuities, like a finite number of them. But for the sake of looking at the area under the curve, we're assuming that  $F$  is actually continuous on our interval.

So it's also continuous on this interval  $A, B$ . So if we have both of these things, right, then we know that this integral, so for a function like this, we know that if we take the integral from this  $A$  to this  $B$ , and then we have our function  $F$  of  $X$ . And then this  $DX$  tells us that we're integrating with respect to  $X$ , it's kind of like our  $X$  axis, or horizontal axis, is the area under the curve. So this is going to be the area under the curve  $Y$  equals  $F$  of  $X$ . Okay, between  $X$  equals  $A$  and  $X$  equals  $B$ , so we could have this could have been an integral with respect to  $T$  and then this would be from  $T$  equals  $A$  to  $T$  equals  $B$ , but this is what we have for now. So between  $X$  equals  $A$  and  $X$  equals  $B$ . Let me step to this side, because I think that orange a little bit blends in with, not that I have orange skin, but it does blend in a little bit with my skin. Okay, so we have the picture to have in mind. So this is like the  $X$  axis. Sometimes when I try to go too fast, it ends up just disappearing I've noticed. Okay, and then we have, so we're going from  $X$  equals  $A$  to  $X$  equals  $B$ .

Okay, and then we have the curve  $Y$  equals  $F$  of  $X$ . And then we're looking at this area underneath it, and that's going to give us this area here is actually going to be this integral, we'll have  $F$  of  $X$  from  $A$  to  $B$ . Because it goes from  $X$  equals  $A$  to  $X$  equals  $B$ . So we're looking at this kind of area right here. So we start this is the  $X$  equals  $B$  hitting up to the curve  $X$  equals  $A$  hitting up to the curve. So we're looking at this area between the  $X$  axis and these two lines and then the curve, okay. Now, but what

happens if we don't know that that function is always at least zero? Okay, so that's kind of what we had before but if we don't have, so if we don't have that this  $F$  of  $X$  is always greater than or equal to zero. So if you don't always have that  $F$  of  $X$  is greater than or equal to zero.

Okay, then what we ended up having having is that, we have the integral from  $A$  to  $B$ . Of, oh and this might be where and until a little bit of problems with the fact that we don't have quite as many colors as I'd like of  $F$  of  $X$   $DX$ , is going to equal the net area, which is equal to. Okay, so we're going to have two areas here. So the first one is going to be the area of the region above the  $X$  axis. So this is the one that you're kind of used to. So  $A$  one this is going to be the area of the region above the  $X$  axis, but below curve, right. Then we have to subtract off another color. Oh, dear, I'm going to subtract up, I'm going to, I think you can't actually see yellow. I mean, I think it looks the same as green. And I'm anyway, just going to have to deal with that. So this is going to be  $A$  two, which is going to be area of the region below the  $X$  axis, but above the curve. This is the area, the region and I'll draw a picture for you. Nope, you can't read that.

Of the region below the  $X$  axis and above curve. That says axis there, I think it might have gotten a little bit chopped off, okay? So we have our  $X$  axis. And we have some kind of function that's going to be going up and down and up and down. Maybe it goes like this, maybe it even touches and then goes like this and then goes down. Okay, let's say it was like this. So this is our  $Y$  equals  $F$  of  $X$ . Okay, and then I was making this right above. So this is our  $A$ . oops, we need to have our bounds here. So this is like our  $A$  and our  $B$ . Okay, and then we can kind of, it's like  $X$  equals  $A$  and  $X$  equals  $B$  there. And then we're looking at what's above. So all together and this forms  $A$  one. Okay? And then we want so this all has like positive, I don't know, can you see if I put a big plus on that? Kind of, a big plus, it's all gives me positive area in there. And then what falls below looks a little like green but see, I think it's actually going to be okay, because of, this gives me minus area.

The green somehow isn't playing another disruptive role, or it'd get confusing. Okay. So that's kind of the minus area. And this is like my  $A$  two. This is not the kind of rectangle  $A$  one and  $A$  two. Okay? And so that's kind of the idea is that if my curve doesn't always stay above the  $X$  axis, then I take the area from what's above the  $X$  axis and I subtract out the area from what's below, how do we, we have to think for a moment, well how do we know when it crosses between above and below is we have to look at every time you know, you need to look at every time the function equals zero. So when  $Y$  equals zero, and those are the potential places it crosses over. So I would have to break these into these intervals, where I would focus in on what happens on this interval. And then I would focus in on what happens on this interval. And this one, and I know this is like an artificial zero, but that's actually good. I put it there on purpose. So you could see, well, I could still focus on this interval, but  $C$  is positive, this one  $C$  is positive, and then look at the last one, and then see it's negative. So you can just kind of subtract when it was actually negative. And you would get. So if you had this area, and then you subtracted that you'd actually get the integral. So when you're integrating, you could imagine, so what happens when it's below the  $X$  axis, it means that  $F$  of  $X$  is negative. Well, if you were integrating a function that was negative, all of these values are negative. And then you're basically kind of infinitesimally multiplying negative values by width, so you're getting something negative. So you can think of it as like these really should be negative rectangles because you're multiplying a negative  $F$  value by your  $\Delta X$ . Okay? So this really would still kind of give the integral. And so it can kind of you can remember it in that way in your mind, is that the  $F$  values when you're below the, the  $X$  axis are actually negative. So you should actually be counting this as

negative, you shouldn't just be counting the area. Okay, great. So that just, I wanted to fill that in for you, so you kind of understood how that worked. And I hope it makes sense, and then of course, I'll see you in the next lecture.