PfaffModule7L12

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SPEAKERS

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Welcome. This lecture is a little bit different, because what I'm going to talk about is we talked about integrability last time, like when those limits of the Riemann Sums existed as you like, chopped up the interval finer and finer. And I feel like it's important when somebody gives something like that, that you know that this is not always true. You usually need to cook up something pretty complicated for it not to be true. But I feel like for completion, I want to show you this, also because it's a really cool example that is kind of nice when you have in your wheelhouse or whatever. Okay, so here's the point of today, which is that not everything is intergrable everywhere so not everything is integrable everywhere, intergrable, everywhere.

Okay, so here is this example, I'm just going to use up a lot of marker doing this, so I did one that was almost gone. But near the end, it got a little too gone. Everywhere, okay, hopefully you can read that now. Great. So I wanted to give an example, or examples. So the first, the first example is not as, you know fun, it's just a good thing to know. So here's not integrable.

Right, so the first problem that you can have is actually that the function is not defined somewhere. Okay? So we have that F of X equals 1 over X. So F of X equals 1 over X is not integrable on this, not integrable on, and we can just look at an interval like, you know, I could just choose zero and then I could choose any number, but let's just even say 0, 1 For example, but you could choose any number there. Since it's not defined at zero, right? And zeros included in there. Okay? So since it's not defined, it's zero.

Okay, so that's just an example of a function that's not integrable at a particular point. But then, here's kind of the really cool example. So we have the function F of X. And this is equal to, so we have zero if the our X is rational, right? It can be written as a fraction, like an integer over an integer, right? So this is like, it looks like integer divided by integer. Okay, it's 1 if X is irrational.

Okay, which means it doesn't look like that. Okay, so we could define a function like this, what does it kind of look like? It's a little tricky. It's like, you know, you've got points all over the place there and points all over the place there. Okay? And this is not, this is integral nowhere. Okay? So this is integrable nowhere. So if somebody asks you for a function, and it's not integrable anywhere, there you go. You can be the hit of every nerd party. Okay? So, but many functions are integrable okay. So the first moral was that not everything is integrable. In fact, you could have things that are integral nowhere, but many of the functions that we actually like are. So many functions are integrable. So many functions actually are integrable.

And this is where we have this kind of important theorem. Okay, so what does this tell me? It says that if, so we're going to have two things. If F is continuous on the interval, so F is continuous on this interval A, B. Or we're allowed to have just a finite number of of bad points, or F has only a finitely many jump discontinuities. So F has only finitely many jump discontinuities. Right, so what does that look like? Right, so I have like a function where only finitely many times does it get funny? So it goes like here, and then maybe it jumps there, maybe actually this was open, and then it goes like this. And then maybe it's open there, and then like this.

Okay, so it's continuous except for at some finite number of points, okay? Then we have that. So if both of those things are true, or one of those things is true, then F is integrable, then F is integrable, right, on this interval. Right, which is to say that that integral exists, right, i.e., the integral from A to B exists of this function exists, and we can talk about it. So F of X DX exists. Okay? So, this is kind of, basically most of the functions that we like and we talk about actually are integrable so, we'll be able to take the integral of that, and that's really kind of pointed this. While, at the same time, right, we have examples that are not intergrable. So most of the time, everything's good, we can talk about it. But you should know that, you know there are these weird circumstances where things are like non-integral anywhere. And this is actually an entire advanced theory, which is, for some people, it would be very cool and interesting. It was one of my favorites, okay? So, I hope this made some sense and I will see you in the next lecture.