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SPEAKERS

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Welcome. In this lecture, we go from this Riemann Sum that we've been talking about to the definite integral, something we've been excited to learn about for a little while, hopefully. So, recall. But this is kind of a recollection of what we've been doing so we can kind of contextualize it. That approximating the area under a curve, that approximate to approximate, right, the area under a curve, so area under, and then we have this, we have a curve F of F of X, or some other function, but we'll just call it F of X for the moment, on the interval A, B. So on this interval A, B, and we have right with N sub intervals, okay? So with N sub intervals.

Okay, so when we want to do this, right, we have the left Riemann Sum. Right, so we have the left Riemann sum. So we have, so we have the left Riemann sum. And the left Riemann Sum is, right, so this is L, and then this is an N right there. And this was equal to, right, so I wanted to take F of, I want to take F of X zero, and multiply it by we're double using pink, just for a lack of colors. But to emphasize, right, and this actually came from, so this part here actually came from the height, right, and this part here came from the width of the rectangles, right. We were really summing up over rectangles. And then we kept adding, so then we have F of X one.

And then we multiply that by delta X. We kept going because we had to sum up over all the rectangles until we got to, so we have F of, and then we have XN minus 1. And again, times delta X. Each time, we're just taking the height times the width, right? And this was equal to, right we, we can maybe pull out the delta X, sometimes we like to. So we had delta X times the sum. And I was going from J equals 1 up to this N. Okay, so I know I'm using pink twice in this one formula, but that's okay. And then we have F of, and then because we're taking the left endpoints, it started with zero and so on, that we actually had to take, right, we had to take F, and then we went XJ minus 1. Right? And here, and this is going to be true in both of these, I'm going to put this down for a second, or not. Okay, sorry, delta X, right? Where we had delta X was equal to, so we needed the width of the interval. So we needed B minus A, and then we divided by the number of sub intervals, because we're taking that whole length and dividing it into those, right? And then we were also able to figure out from there that my XJ, right, was equal to, well, I started at A and then I had to add J copies of delta X, right? If you recall that, too, we had A, and then we had plus J times this delta X.

Okay, so this was the the left Riemann Sum. We also had the right Riemann Sum. So we've got the right Riemann Sum, not the correct one just the right hand one, because it has to do with whether we're looking at the function at the right endpoint or left endpoint of the intervals right? So this is the right Riemann Sum. So this is R and then we have an N, and this is again, this is height times width, right? But this time, remember, because we're starting on the right side, right then we actually are going to start with X one.

And then we need to multiply that by delta X. This is also the height, this is also the width, right? So this is applying to both of those at the same time, okay, be efficient. And then we have F of, and now we're going to, up to X two, right? Again, because we're using now the right side of each of the sub intervals. And we want to go all the way up to, and this time because we're ending at the right side, we actually end at XN. Right, X and then this was equal to, and then we took the sum from J equals 1 to N, and it's going to look exactly the same, except that we're on the opposite side of the intervals. So we are actually looking at just XJ. Oops, we forgot our delta X here, so we need to put that in. So that equals right, because we had delta X in each term, but we can pull it out to the front, we kind of knew we can do that. Okay, perfect. So now, and then the last part that we got from this piece of what we're doing is that we got the, we had both of those, and then we knew that the limit as N went to infinity, right? The left hand Riemann Sum was equal to the area, which was equal to the limit of the right hand Riemann Sum, right. And our hope is to understand how that relates to the integral. Okay? So here's the new, so what is new? And I'll kind of put it here. So this should look a little bit similar. Actually. Maybe I'll put it here. Nope. Okay, so here's what's new.

Which is that we could look at this, it doesn't need to necessarily be the left or right hand point. So the area is equal to, right, so it's equal to these limits. It's actually equal to so we have the limit as N goes to infinity, so this is going to look like this is like another Riemann Sum. There's, it's going to be very, very similar, except that one thing is going to be like slightly different. So we're going to take the sum from J equals 1 to N. Okay, of, and then we have F of, and then what I'm allowed to do now is I'm allowed to choose any point, I'm going to put a star here, meaning I'm allowed to choose any point in that interval. I've chosen the left endpoints here, I've chosen the right endpoint here, and I could choose anything in the middle even if I want now. And it's good just to know this, this is not something we're really going to work with, but it's good to know. It would be incomplete not to have seen this, and this is still a circumstance where this is the width and that's the height.

Okay? and this whole thing is called the Riemann Sum. Okay? So this whole thing not with the limit, but this is the Riemann Sum. Okay? And this works for any, right, for any, right where you have XJ. So, you're always going to have it in that interval. So, maybe the way to write it is that we have XJ minus 1 is less than or equal to XJ star is less than or equal to XJ, right? The point really is that we're in the sub interval where we have XJ minus 1 and XJ, and this XJ star can be anywhere in there could be the left endpoint, it could be the right endpoint and it could be dead in the middle or anywhere. Okay? So when we have this, okay, um, and when this limit exists, so this limit here, so, okay, so when this limit exists I'm going to put it down here, okay, so when this limit exists, so you know which one I'm talking about. So when this limit exists and is the same for all choices, right? And the same for all choices of and then we're looking at these XJ star.

Okay, then we're going to say that F is integrable on that interval, okay? So we're going to say that F, our function F, is integrable. It's like a new word, we're going to say integrable on this interval A, B. Okay, so when we're in the circumstance where this limit exists, and right, this limit exists, I could have chosen any point within the interval, then we're going to say the function is actually integrable. Okay, and then we have notation. So then, so when all this happens and is integrable, then we can write this notation. So we have the integral from, we start at the bottom, we go up to the top, so A is on the bottom, B is on the top, and then we have, so we have F of, and then this is going to be our X and our DX. So this is put that here. So you can see that this now has to match up, that that X has to match up with this, okay? These are X values also. So this is going to be important down the line. Okay, and we have names for all these things. Okay, so this here is called the upper limit. This here is called the lower limit. This function here is called the integrand.

Okay, and this part here tells me what I'm integrating with respect to. So this tells what I'm integrating with respect to. And you could see how similar this actually looks to the Riemann Sum. This is like with differentiation, you need to know what it was with respect to, okay. And it actually is, so it's going to come exactly from there. So it's going to come from the limit as N goes to infinity, right, of the sum from J equals 1 to N. And it's that same function, so you can see how similar these look.

And then we have, instead of right, this was when we had and I was double using things, this was a delta X. So you can kind of see the integral comes from the sum, this integrand comes from that function there. The DX is like the infinitely small version of delta X, right, because why does it end up infinitely small? because N gets infinitely big, which means we chop it into infinitely many pieces. So each piece gets infinitely small. Okay, we've just kind of seen that kind of thing before. Okay, so this is kind of a new thing here, too. There's so much writing all over it. But we can manage to kind of box this brand new thing that we've probably been waiting a long time for. There we go. Okay, so we have this, and this has a special name. So this is called the definite integral. Okay, so called, and it's called the definite integral, definite integral.

Okay, of F So of F, and then from A to B. And so you can see, because we talked about that being like the area under the curve, that this is actually related to the area under the curve, although now so something that's no longer necessarily true is that our function always stays above the axis. Okay, so this is a very general thing. We took the limit of these Riemann Sums that we were doing as we chopped up into finer and finer and finer intervals. And what did we end up getting? We actually ended up getting the integral, right, and they're very closely related. Even in notationally, this sum turns into the integral, integrand, the delta X gets infinitely small, and it becomes a DX like an infinitesimal. And you can think of that as telling you what you're integrating with respect to, or basically what your X axis was labeled, and what kind of variable your A and your B have to be. Okay? So, I hope that makes some sense, and I will see you in the next lecture.