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SUMMARY KEYWORDS

delta x, xj, sub intervals, equal, height, endpoints, xn, delta, sigma notation, width, point, curve, interval, area, rectangles, greens, snuff, formula, compute, circumstance

SPEAKERS

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Welcome. In this lecture we're going to go through approximating the area under a curve using right endpoints. We were using left endpoints before, these are the endpoints of the interval, we were using left endpoints before, now we're going to use right endpoints. So here's our task. So our task is going to be, we want the area under the curve. First, we'll do it with a specific example. And then we'll, we'll move to, something general, so under the curve, F of X equals, and we're going to do X squared plus 1, and we're going to go on the interval. So on the interval where we have, so we have minus 1 to 1.5. And we're going to want to have, so this is going to be with 5, right, 5 sub intervals. 5 sub intervals, with 5 sub intervals. Sometimes one just says intervals here, some intervals helps to kind of do like little intervals sitting inside the big interval. Okay, so here's our picture. Oops, I want this to be orange.

I want it to be orange, and I want it to be a fresh orange, so that you can see it really nicely. Okay. So this is our X. And we have our curve, right? We have our curve. Well, we want our Y, that was what I meant to use his for. So we have our Y, our Y, and we then we have our curve, right we, we take Y equals that. So that's our curve. Again, I'm making it a little more flat than maybe realistic, but it makes a picture easier. I'm also making it less symmetric than it ought to be. But that's okay. So this is the curve, Y equals X squared plus 1, just like over there. And then we know how to do the width, right? Because we've done that several times now. So it'd right be, well, first off, let's put on our endpoints, so we have minus 1 and then we have 1.5. Okay, and then if we want the width of each of these sub intervals, right, we need that. So we can kind of figure out where everything goes, we know that our width, and our width, we call delta X, right? That this is going to equal, right, we need our entire length. So we take 1.5 minus minus 1, and then we divide it by the number of sub intervals, which is 5.

And we get point 5, so this is going to equal point 5. Great. So that's going to kind of indicate to me that we have, so these are, this would be like minus point 5, this would be zero. Right, I'm just adding my width each time. So I'm adding point 5, so I get minus point 5, I get zero, I get point 5, and then I get 1, and then I go up to 1.5, okay? And then we know that these areas that we're looking at, we're going to get by going, well now and we were going up, and over now we're going up and over right, we're looking at the right endpoints. So the right endpoint of this intervals minus point 5, so I go up, and then over and down. For this one, the right endpoint is zero. So I go up, over and down. The right endpoint of this interval is point 5. I take it, right I go like that for this sub interval. It's 1 so I go up. And for this one is 1.5. Okay, and then these are the areas that we look at. So this is area one, area two, area three, this is area four, and this is area five. Right, so this area that we're approximating is going to be the sum of these. So right we're going to take we're using these rectangles to approximate the area under the curve. So we need to take the sum of their areas. So I want the area of A one plus the area of A two plus the area A three, A four, and A five. Okay? And then we kind of remember this width is delta X, right? For each of these this width, right down, here's this with this delta X, right? And then the height now though, this is kind of the, one of the key points of what's going on. So now, my height is going to come from minus point 5, F of minus point 5, right, because the height is happening on the right hand side. Okay? And, again, I feel like this is hard to see. There's some reason none of the greens I want to use today are up to snuff, it might be hard to see. There we go. Hopefully, that's visible now. And then, so we're going to go all the way up. And on this side, right, this was where you can see it the best. So this point here is going to be the point is going to be, well, I take 1.5, and then I stick that in right to my function, F of 1.5, right?

And this is actually, make this big here. And this is actually this height here is this F of 1.5. Okay, so this height here is actually F of 1.5. Okay, so that gives me the next part of this, which is that I can go through and I have, so this is going to be my width, times my height, which in this circumstance is F of point 5, so F of point 5. And then I need to add, so then I have my delta X again. And now I've got to do it for F of zero for this height here. Oops, that should be a delta X. So X, and then I have my F of zero, that's really difficult to see. Just reaching over and getting something to clean that. So I have delta X times F of zero plus delta X times F of point 5, oh, this should be a minus point 5 did I write point 5? I wanted minus point five there. And then this should be F of point 5. And then I, right, because I'm looking at this rectangle. So I'm looking at this height, which is the F of point 5, and then this one's going to be F of 1. So delta X times F of 1. So I have plus, and then I have delta X times F of 1. And then I have one left, right, I look looking at this area. So this height I've already told you is F of 1.5. And that, again, is delta X. So had delta X, why don't we write that each of these is delta X, I think that kind of fits there. And then you can kind of see what we're doing here a little bit better, because I know these are confusing.

The more clear we can be the better, because these are historically one of the most intimidating aspects of calculus, okay, 1.5. Great. So then we just kind of compute whatever these F values are by sticking them into the function, right. And we likely pull out our delta X, because that's kind of a lot to have it in each term, but we can use our distributive property nicely here. So we pull out our delta X, which we know is point 5, so from over there. So we have point 5, and then we're going to multiply it by each of these. So let's go ahead and do that. So F of minus point 5, so we should get five fourths plus and then we should get one plus five fourths again, from this here. And now we need to do this one, and that should be two. And then we need to do this one, and that should be 13 fourths. And so all together but we're going to get as an answer, is going to be 35 eights.

Okay, but you can kind of see, again, we're just adding up the areas of these rectangles. Our width is our delta X, which you compute over there, a height comes from the Y value, which is actually sticking our right endpoints into the functions. Okay? And then this is where and I know that this gets a little bit tricky. But here's here's the formula. So more generally, so what is our more general formula. So more generally we're going to have that if I have RN, well, this is going to equal it's going to equal it's going to look a lot like this, we just have to put in our XJ's, remembering that actually in this circumstance, right, and let's kind of write these in, so maybe that'll help a little bit here. This is actually X zero, X one, X two, X three, X four and X five. So you can see, instead of starting with X zero, we actually start with X one. And we go all the way because we're looking at the right endpoints, the right endpoints. The right endpoint of the first interval, right, is that X one, the right endpoint of the second interval is that X two, the right endpoint of the third interval is that X three, the right endpoint of the Nth interval is at XN. Okay, so the formula is going to actually look a little bit different this time. So this is actually going to equal right, where we multiply in each circumstance, we take our delta X, and we multiply it by our F. And this time, we actually start with X one, right? And then we do we do this, we keep doing this, we go all the way up to we do our delta X.

And then we go F of XN. And this is the, right, that we're, Nth interval, the right endpoint is actually so this is a fifth, it's five. So it's actually the Nth, right? We do XJ, sorry, XN, so this is an XN. Okay? And in sigma notation, so this is the, the formula here. So this is our final, general formula. Okay, and then how do we write that in sigma notation? So like this, right. And so in sigma notation, kind of as we did with the left, we want to do sigma notation. And for that, right, we're going to take, so we have the RN is going to equal, and now notice that we start we go from 1 to N, so we don't need to go XJ minus 1, we can just do XJ, right? So we can go the sum from J equals 1 up to N of, and then we have this is, this delta X times F of XJ.

Okay, and so this is just the other notation for the same thing. Right? With a big difference being, and it's good to notice differences. I tell students to kind of stare at that for a moment to see the differences. And what you're going to see there is it actually, the only thing that's really different is that we have XJ instead of XJ minus 1. And that's because everything's kind of shifted over when we're looking at the right end points. We start with 1 and we actually go up to N, right? Go up to XN. Okay, so this is like when we're doing our heights, right, because we do our heights on the right. Okay? So, I hope that made some sense, and I will see you in the next lecture.