# PfaffModule7L07

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**SUMMARY KEYWORDS**

xj, sub intervals, xn, notation, interval, delta x, delta, dividing, equal, endpoint, work, width, sigma notation, remember, orange, sum, height, rectangles, subtract, tricky

**SPEAKERS**

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Welcome. In this lecture, we build on the last two lectures to give kind of a fancy notation. It's very useful to have kind of a more general way to symbolically represent something so that you can do things with it as we're going to want to do with these by taking limits. So bear with me with the notation, you can do it. So let's get started. So when we have, the first thing that we we have, so let's say we're dividing. So we're dividing the interval A, B. So dividing the interval A, B, and we're dividing it into N sub intervals. So into N sub intervals, so into N sub intervals.

And the first thing that we remember, right, well let's, let's actually start drawing the picture already over here. So I have X axis, right, and I have here my A and I have here my B. And I said I wanted to divide this into N sub intervals. And so how do I do that? I take the entire width, which is B minus A, I divide it into N sub intervals. So I have to divide it into N pieces. Okay? So, and remember that we call the width of each piece delta X. So let's go ahead and do that. So I write, so we're dividing this, so the first thing that we get is that delta X is going to equal, right, the width of the top interval. So we get B minus A, and then we divide that by the number of sub intervals we want. And then we can use that to figure out what each of the endpoints of these intervals is. So we get that XI, right. So this, I used, okay, we're going to call it XJ. And this is going to equal remember, what we do is we start with that A. And then we add the number of times, right, we add, N sorry, J, this J times the width, right? Because we have to move over we start at A and if I'm talking about the N, like XJ, here, let's, let's write this on here, because we kind of did before. This is A, so this is X zero, right? And then the next one was X one, and then we got X two. And we kept on going and this would be like XJ. Remember that like X one talks about the first sub interval, right. And if I want left endpoints I'm looking at here. And for the second sub interval, right, I've got, I've got to move over to get to X one, I had to move over one copy of delta X. To get to X two, I had to move over two copies of delta X. To get to X three I move over one, two, three copies of delta X. To get to XJ I have to move over J copies of delta X. So that's where that comes from, okay? And then we remembered that this one here, so another tricky thing to remember is that this is going to be XN, right? Okay, so we figured out this much. And then if I want to know the ice interval, so if I wanted to know the first interval, it's an interval that starts at zero and goes to one. The second interval starts at one and goes to two, and so on. So I'm going to write down the funny notation for this. Ready? So what are we going to do is that I'm going to get XJ minus 1 to XJ, right, is at Jth interval.

Right, and let's test this. Okay, so looking at this, what's the first interval? So the first interval is going to be X zero, X one, right? So we're testing this, so let's test so you kind of get comfortable with the formula. So the first interval right, well it is, it's X zero X one. Right? That looks right. Okay, because zero is, right, first interval. So my J equals one and then subtract off one I get this. I keep it and I get that, right? And then to do the second interval. Okay, so I know that my, right, so if I'm looking at the second interval, then my left endpoint is X, sorry, my right endpoint is X two, my left one is X one. So I have X one, X two. So this index really does match up with this. A little sloppy of me there. Okay, so this endpoint, this right hand endpoint really does match up with this. For my left end endpoint, I needed to subtract one. Okay, so we kind of believe this at this point. So now let's kind of keep going here. So I have the graph of my curve Y equals F of X.

Right? And then I have and then I do, to do these rectangles I go, right, I'm doing left hand, so I go like this. Like this. I'm going to do left hand, I'm going to have one here. And I'm going to have the one there is a little harder to, right, it goes like this. Okay, and that's XJ minus 1, Nminus sorry, NX minus 1, this is XN minus 1. Okay? Now, each of these, so this is area one. This is area two. We keep going. So I took it from this side. So actually, the easier one to do here is like this. Okay. Okay, so from here, this is AJ. So that's actually AJ plus 1. Right? And then this one is actually, right, so it comes from the side. So this is actually AN, my last one.

Which is good, because I should have N areas and I started at one. Okay, perfect. So if I wanted to get this left hand sum, right, that tells me the sum of the area these rectangles, then, what do I want to do? I just want to add those areas together. So I want to take that this is equal to, well, I'm just going get so this is equal to, so I have A one plus A two, plus all the way up to AJ, plus all the way up to AN, okay? And then we remember, right? I know that I've used N there, but we're going to use this also, this is like my delta X. And then we have this point here, right? This is going to be the point here is going to be, well, here, it's going to be X zero. And then it's going to be F of X zero, which tells me that this height here is F of X zero. And then maybe it's easiest instead of crowding that one, to look at what's, what's happening here. So I still have that this is delta X. And then here I have, so this is XJ minus one. So that's actually going to tell me that this height here, right is going to be F of XJ, right? Because it's a Y coordinate of that point there. Okay? And then I can keep on going. And notice that this height here is going to be F of XN minus one. And this width here is delta X.

Okay, so let's do the next step of this. Each of these has a, it has a delta X, and then it has the height. So I have, this is going to equal, so I have delta X times, and then my first one is F of X zero plus, and then I have this is a, I replace my green with another kind of green apparently. So then I have delta X times F of X one. I keep going. And I have delta X times F of, and then we, here, for that, the Jth interval I actually have XJ minus one, right. And then we keep going, and I get delta X. And then I have F of XN minus one. Okay, and we notice that when we're doing this, right, this is this N here. I'm double using red, there are only or so many colors. Okay, so there is a way to kind of write this in sigma notation. So let me show you. So this is going to equal the sum from, right, and we start with so our J here. We're going to start with one because we're going to start start with the first interval. So we're going to have to subtract off one to get that right, and then we're going to end with N. And I have this F.

I'm going to put the inside of that in orange, so this is X. So you can very well see what's on the

I'm going to put the inside of that in orange, so this is X. So you can very well see what's on the inside. In fact, why don't I do that for each of these, this is X zero, this is X one, because I think the green is a little hard to see, but the orange is really nice and crisp. And these are also a little tricky to work with, this is XN minus one. Okay, so here we go. So this is XJ minus 1, this is delta X. Okay, so, and we can just kind of pull out the delta X if we want, and we're going to get, so we have delta X times the sum. And we're going from J equals one up to that special N. And then we're taking this of F, F of XJ minus 1. Okay. Let's make sure that's really clear, because that's really important. So that's XJ minus 1. Okay, and this has a special name. So both of these, either of these has the same special name. And the special name for either of those is sigma notation. Sigma notation.

And one of the things that can really help if you're getting a little confused by summation notation, is to write out a few of the terms kind of like we have here and align those in a particular circumstance, and see if you kind of believe and understand how it works. I understand sigma notation takes a while to get used to, It's a little tricky as a notation. But one of the things that happens it's really useful when you use notation is that then you can do things like take limits, and, and like work in more complicated circumstances. Okay? I'm going to do the same thing with orange here before we finish off, this is X zero, this is XJ, and this is X, in case you couldn't read it very well, XN minus 1. Okay. Great, so I hope that made some sense, and I will see you in the next lecture.