PfaffModule7L06

Fri, 1/21 2:11AM 14:23

SUMMARY KEYWORDS

sub intervals, width, endpoint, rectangle, delta x, area, curve, approximate, equal, left, height, delta, point, function, interval, write, divide, picture, graph, important

SPEAKERS

Catherine Pfaff

Welcome. In this lecture, we're going to go through kind of building up to Riemann sums by doing this with an example of approximating the area under a curve, using in particular the left endpoints. This is going to be very important, it may seem like it's a very funny way to compute an integral when you later on see what an integral is. The reason why I still believe in teaching this is because this is giving you kind of, it's giving you multiple things. One is it's going to connect up to that picture we had before, of where you could take like, a distance times a velocity and get the change in position, or something like this. And we're going to like gradually move from there to this idea of getting close to the area under the curve, which is then going to be the integral, and so on. And so I think it's important to actually build it up in this way. So you actually kind of have a feeling for what's going on. So let's, let's go ahead and get started. So example.

Or maybe I'll just say, actually, so here's our task, instead of saying it as an example. So our task is to find the area under the curve, so area under the curve. And the curve that we're going to look at is F of X equals X squared plus 1, so F of X equals X squared plus 1. And we're going to do this on the interval, so on and then the interval. So we're looking at the interval minus 1 to 1.5. Right, and we're going to look at this with 5 sub intervals. So with, and then we have 5 sub intervals. And we'll kind of see what we mean by that. So 5 sub intervals. Great. Okay, so we have a situation, we have a curve here. So let's get the whole thing set up. I have a picture here. So I want to go like this. Oops. Okay, so this is like my X. And then I have my Y. And then I have my curve, I'm going to make it a little flatter than it actually is just to make it fit here on this picture well. Okay. So this is a curve Y equals X squared plus 1, just like the function. And then what else do I want here? I didn't label this as is my Y axis. This is my Y axis. Okay, so I want the area under this curve. And the other kind of important information on here is I'm on the interval minus 1 to 1.5. So let's say that this is minus 1 and 1.5 is here. Okay, so if we're approximating with left endpoints, what does that mean we're going to do? It means we're going to go, I'm going to start here, I'm going to go up, right? And then first, we need to figure out, so we need to figure out what is the width of this? I need to divide this into 5 pieces, because I have five sub intervals, okay? So how many pieces am I going to have? So if I take, right, so I know I need 5 sub intervals, and I, I'm on this interval here, right? So what is this length? Here is the first thing to kind of figure out and then I'm going to divide it into 5 pieces. Okay, so let's go ahead and figure that out. Right? I take this minus this. Okay, so we're just looking at the width of a piece or a rectangle. So how are we going to figure out the width of a rectangle? So we have a rectangle? Well, that's going to be the total width, right? So this is going to equal, and maybe I'll actually kind of, alright, so I have the total width. And then I want to divide that by the number of intervals.

Right? You kind of believe me on that, that will, that'll actually give me the width of each of my sub intervals. So this is going to equal, so I've got the total width which is going to be, so I've got 1.5 minus 1. So I have 1.5 minus minus 1. It's kind of important to put that in parentheses so we don't mess up when you have the minus minus there. And the number of subintervals is 5. Okay, so now this is going to equal. So on top, I'm going to end up with this turns into a plus. So I end up with 2.5 divided by 5. And that's just going to be point 5. Okay, so the width of each of these is going to be point 5. So that kind of gives me the values in between. So I have one here. So I have minus point 5, I have zero, I have point 5, and 1. And now I'm building my rectangles, right, on the left endpoint. So every time I hit a left endpoint, I'm going to go up, up, up, I hit that, and then I go over and down at the next endpoint, okay? I go here, I go up till I hit the graph, I go over, I go down, I go up till you hit the graph, I go over, I go down, I go up till I hit the graph, I go over, I go down. Just looking at this picture, you believe me that this is, right, If I if I added together the areas of these rectangles, it might not exactly be the same as the area under the curve on that interval. But it won't be like ridiculously far. Okay, so I'm going to want to maybe go like this with you so that you can kind of see what I'm talking about here. And this area, except I'm going to leave a little space for myself, so I can write in, then I'm going to call this area one, A one, I'm going to call this area A two, this area A three. This area A four. And this area A five.

So these are the areas that I'm going to want to be summing together. So to approximate the area, so the approximate area, a little bit of orange still mixed into it, that's okay. So my approximate area, right, is going to equal, well, I'm just going to add these up, right? So I'm going to go A one plus A two plus A three plus A four, and then ultimately plus A five. Okay?

So now I need to figure out what those areas actually are. So the width of a rectangle is, we're going to have a name for that. So that's going to be delta X. Okay. So delta X is the width of a rectangle. So I can kind of write that in here. So this is delta X equals point 5. Okay, so because if I wanted the area of a rectangle, I'm going to want to take, give myself a little room here, I'm going to want to take the width times the height, right? We know that's how we do the area of a rectangle, right? We do the width times the height, well, this is the thing where it's really important to remember how the world works. And by the world, I mean, graphs of functions. So what is this point here, this point is, well, I'm at minus 1. But then I need to plug minus 1, this is the graph of that function, so I need to actually plug that in, like this. Okay? And that would actually give me that F of minus 1 there is actually going to give me this height. Right? So I get that height there, and it's going to keep on working like that, right? This point here is going to be minus point 5, and then, right this is minus point 5, then F of minus point 5, okay, and this would keep on going. So you would always get the height on the left hand side by plugging that left endpoint into your function, right? So I want to multiply the width, which is this delta X, this is the width of a sub interval, which we know is point 5, well I multiply that by the height, which we know is sticking that left endpoint into the function. So let's just kind of go ahead and show this. So this is equal to.

So I want to take the delta X, and I want to multiply it by F of minus 1 and then I'm going to add, right so we have delta X, F of the next one. So F of minus point 5. We keep on going, plus delta X of F of zero, plus delta X of F of point 5, right? And keep going. So delta X F of one, and then we're done because that's the last left endpoint. Okay? So then we keep going here. And we want to look a,t so this is equal to, and I'll keep going actually with blue, because for some reason my green is running out, anyway. Okay, so this delta X we already know is point 5. But I'm just going to pull it out of the whole thing, right? The delta X shows up in every single one of these, so I'm just going to pull it out of all of them. So I have point 5, and then I'm going to multiply that, well, what's F of minus 1. I have to actually stick that into the function here. So minus 1 squared plus 1, so that's going to be 2. And then I have to stick minus point 5 into the function, and I should get five fourths. I'm going to keep going, I'm going to plug zero into the function, and I'm going to get 1.

I'm going to plug point 5 into the function, and I'm going to get the same thing as if I plugged that in, I'm going to get five fourths. And then I have one last one, which is the plug 1 into the function, and I'm going to get 2 again. Okay, so I can go ahead and compute this and I can get fifteen fourths. Okay, so what did we just compute here? So what did we do, we went and did, I took, I wanted to use 5 sub intervals to approximate the area under the curve using rectangles. So I took, first I had to figure out the width of one of those sub intervals so I could kind of figure out these values and so on. How did I do that? I took the entire length, so 1.5 minus minus 1, I divided that by 5. So that's what I did here. 1.5 minus minus 1, I divide by 5, I got point 5. So I know that the width of each of these is point 5, right?

And then once I did that, I was able to, I knew their width, and I just need their height. But I noticed that if I'm looking at this length here, so I'm looking at the Y value of this point, which is actually F of minus 1. Here, I'm looking at the Y value of this point, which is F of minus point 5. Here, I'm looking at the Y value of this point, which is F of zero, here, I'm looking at the Y value of this point, or this point, which is going to be F of point 5. And then at this point, to get this height, I'm going to get F of 1. Okay, so that's kind of what I did here, I took those widths, timed those heights. I pulled out the width, so and maybe I can just kind of go over these with, I'm going to switch this, this is like my, I'm double using red for two things. It's okay. X, X, and then this is that delta X so that it's clear where that came from. And then I had to put my, and actually had to plug these into my equation, right, for my function. And that's how I got those values, and I just added them up. Okay, so this is a specific thing. This is the approximation by left endpoints and is called L, this one is L 5. So the L comes from left and then the 5 you may guess is from 5 subintervals. Okay, so this is 5, right? And here we write left endpoints.

And here we write 5 sub intervals. And that's just kind of notation that we use, because we like notation, because it means we don't have to write nearly as much. Okay? So, I know this stuff is a little bit tricky. So be patient with yourself. Okay? But I hope this kind of made some sense, at least as a first step to understanding what's going on. And then, and you know, practice and so on. And I will see you in the next lecture.