PfaffModule7L05

Fri, 1/21 1:59AM 9:57

SUMMARY KEYWORDS

sub interval, notation, figure, width, delta x, pieces, arbitrary, nth, divide, interval, compute, tricky, delta, xn, lecture, xj, notational, give, equal, dividing

SPEAKERS

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Welcome. In this lecture, I want to focus in on one of the aspects that people find difficult of Riemann sums. I understand that the notation is difficult, so we tried to actually give you a lot of the notational aspects much earlier in the course, so that you get a chance to get more used to them before, all of a sudden, you're trying to understand very difficult concepts at the same time. It's kind of tricky notation. And I'm also doing this lecture as separate, to try to at least isolate a little bit of some of the tricky notation. So you can get comfortable with that before we pile some more on. Okay, so this is kind of the task of this lecture, which is that I have, so the task is to divide the interval, so I have an interval A, B. So the interval A, B, interval A, B. And I want to divide it into N sub intervals, so into N sub intervals. I don't know what this number is, so there's a lot of abstraction in here. We don't know what the endpoints are. And we don't actually know what this number N is. But that's okay, we're going to do it always the same way. But let's kind of start with the picture. So I'm on, we're just looking at an interval. Usually, this is the X axis here. And I have a value A, and I have a value B. Right? This is what I mean that I'm that interval here is right in here, right? And now I want to just cut it into N pieces. Right? And if I want to cut something into n pieces, how do I do that? I take, right, the first thing I need is the width, right? So what is the width of this entire interval? Well, I'm going to take, right, so if I want to figure out this width, right, so this width, width is going to equal, so I'm going to take this B minus this A.

So it's going to be the B minus the A. Whatever those values are, some of them might be negative. So a lot of times it helps to put in parentheses. Okay, now what if I wanted to divide this into N pieces, however many there are. Then I'm looking at taking, right I want to know the width of one of these pieces, so I can start figuring out where the next, right, the next dividing point is, the next dividing point, the next dividing point, right? So, you know, without kind of knowing how many of these there are. Okay, then what do I want to do? Okay, so I want to take, if I want to know the width of one of these, okay, so if I, that's the width of the whole thing, but I want the width of just one of these. Let's just see, right? So how do I figure out this width? And we're going to call that delta X. So this width, and we're going to call it delta X. Okay, so the width delta X, well, I want to take my entire width, which is B minus A, right? So I want to take B minus A, and then I want to divide it into, I want N sub intervals, so I'm dividing it into N pieces. So I have to divide this with by N. Okay, so the number of pieces I want. Okay, so that's how I figured out what this delta X is. Now, what if I wanted to figure out

okay, so I'm going to call this, so notationally the way that we usually do this, is we call this X zero. Okay? Now, if I have N of these sub intervals, so let's think about this for a moment. So I have N of these, right? Then this here, is I'm going to call this X one I call this X two, I keep going.

Now, here's the key point, which is that the number there tells me whether this tells me I'm, this is the first interval, right? This number tells me this is the second interval, this number tells me this is a third, and so on. So this number right, the subscript on this one should tell me that this is the Nth sub interval, so I put an N there. This N is the same N as the number of sub intervals.

Okay, so that's the first thing kind of notationally and I'm going to want to be able to write it like that, so that I can give you a formula for each of these. Okay, so the first thing that we had to figure out was that well, we had to figure out the width, but then we also had to figure out that there, this is going to go up to XN, because this index tells you which of these intervals it is. Which of these sub intervals is the first sub interval, the second sub interval, the third sub interval, there's N of them, so this should be the Nth, so that's an N. Okay. Now, the next piece of this, in all of our trickiness, is I want to actually figure out how to compute these with some arbitrary variable. Okay, so the next kind of tricky bit is how do I compute, so how do I compute these XJ?

Okay, so J is just an arbitrary index there, right, and then we'll, we'll do some examples. And then maybe we'll kind of see the pattern. Okay. So let's look at how we compute at zero. So X zero is just A, right, so you know that X zero equals A. How do we figure out whatX one is? This is the first one, so this is the the trickiest in some sense. So we have X one equals, well, we take A and then we add the width, right, to get the value there. So we take a the width is delta X, and then I get to X one. So I get A plus delta X. Okay. Now if I want to get to X two, I need to add a second copy of delta X, right? I started at A, I had one copy of delta X, I had two copies of delta X. So X two is going to be A plus two delta X. You may already see the pattern, if not, let's just do one more. So this is X three. So how many copies of delta X do I need to get to X three. Right, so one delta X, two delta X, three delta X. So X three is going to equal A plus three delta X. Ah, so now we can probably see the pattern, right? Which is that if I have some arbitrary XJ here, if this was one, I added one copy. If it was two, I added two copies. If it was three, I added three copies. If I want to get up to XJ, then I'm going to need to add J copies.

And so that's my answer. Okay? So these are a few pieces that are going to be useful what we're going to do in the future. I know it's a lot of notation. I know that's confusing. It's actually good to get used to like kind of handling notation. It's also getting, it's good to get used to doing what I'm doing here, which is just like, break it apart so that you can kind of understand it one piece at a time, don't get overwhelmed, you can understand it. You just need to wait, how does this end up being that, and how does this end up being that, and what does that mean? And just kind of break it apart until you kind of understand. I'll try to break it apart a lot. But don't be afraid to break it apart even more. Okay, so how did we do this? So the first thing we did was we wanted the width of a sub interval, right. And for that, we had to take this whole length, which was B minus A, and then divide it into the fact that we're looking at N pieces. So each one of these its width is B minus A divided by N. Okay, so then if I wanted to figure out what X one was, then I would take A and I would add it, that width, delta X, to X one one. If I wanted to get to X two, I'd have to add that width delta X twice. If I wanted to get to X three, I had to add that width delta X three times and so on. Okay? So if I wanted an XJ is like an

arbitrary J, so that I could look at all of them at once, I know that indices often like are intimidating and confusing to people. One of the nice things about them is I don't have to write down every single one of these, I can just write it in a general form. Okay. And so I would start with A and then I would put in the number right, the sub interval that I'm looking at that many times of delta X, okay. The the other thing we did was we had to figure out where to end here. And the way that we figured it out was it this was for the first sub interval, this was for the second, this was for the third, there are N of these, this is the Nth one. So this should be the N sub interval XN. Okay. So, I hope that made some sense and I will see you in the next lecture.