

Geometry I

SUMMARY KEYWORDS

squared, circle, square root, equation, equal, answer, coordinates, economics, unknown variable, distance, radius, solve, y axis, expression, robert j, negative, number, center, question, click

SPEAKER

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Hello and welcome to ALEKS walkthrough video number eight. My name is Robert J McKeown. And I'm very happy to have you here watching my video today. Our very last topic is topic eight geometry. Geometry plays a role in economics. Triangles will appear a number of times in your undergraduate classes. Circles are not as often, but they can be very useful for understanding a few important concepts. And economics. For example, when we talked about functions, the equation of a circle is not a function. So if you're wondering what's a function was not a function, the circle equation you're about to see is a good example of that. Now, some of the shapes in the ALEKS precalculus module and topic aid. You know, I've never seen a parallelogram in my economics career. But it doesn't hurt to go through the exercise of learning an equation, and then manipulating that equation to solve for an unknown variable, which is something that as an economist, you're going to do all the time, maybe we'll define a way that stock investor chooses their stocks. And then we use that expression again, and again to solve for maybe stock prices or something like that. So the exercise is still very useful. Some of the shapes do show up econometrics statistics, you can study that in a geometric approach just using many of the equations, or some of the equations anyway, that you're going to see in this video. So without taking up any more of your time, have a pencil, paper, maybe even one of these beauties, and wheat. Let's go ahead, and let's start solving some problems together. Let's start off by talking about circles. Circles come up occasionally in economics. But this is particularly good practice for you. Because you're going to be given an equation, and then you're asked to manipulate that equation. And that's something that economists do all the time, both in theory. And when we're trying to make a statistical analysis. The form here is very important. Our x's and y's, these are actually x axis and y axis coordinates. So those are often unknown variables. They're placeholders whatever the coordinates of the circle is, or whatever the coordinates are. That's where we're going to find them. What else is very important here are these h and k letters. These h and k letters give us the coordinates of the center of the circle. And the center of the circle is what we're going to need to graph on ALEKS. So we always need to find the center of the circle to graph a circle on ALEKS. And probably, you know, in general, if we want to graph a circle, knowing where its center is, is very helpful. Now let's take a look at our first question, using this equation of a circle. So this is the information we've been given. It's a good idea to answer these questions to memorize or know what the equation of a circle is. So I'm going to rewrite the equation of a circle up here. And notice that it's quite a bit different than what we've been given. So the question implies that we should be able to graph a circle here. But the way that it's given us the information is not in the form that we want it, we want to take this thing here and put it into that form. So how are we going to do that? Well, the first thing we can do is take care of this negative 11. And we can

add 11 to both sides of the expression and then I'm going to collect, well, not so much collect like terms has moved like terms, or at least terms that are similar to each other closer together. So we're gonna have our X's together, $x^2 + 2x + y^2 - 4y$, or excuse me, that's minus minus four y. And that's going to be equal to 11. Because we've added 11 to both sides of the equation. Now, we still have work to do. Because we want this thing here to look like that. And we want this thing over here. To look like that. How are we going to do that? Well, we're going to use a technique called completing the square. Notice that this thing up here is a square. And so is this over here. It's a square. That's why they're squared squares. And so we're going to complete the square and turn this expression into something that looks like the equation of the circle that we want. How are we going to do this? Well take a look $x^2 + 2x$ plus something. If I write $2x$ plus, or I should say $x^2 + 2x + 1$, that's gonna look like $(x - 1)^2$. And now it's a square, we've completed the square. How did I do that? I had to add one. Now let's take a look at the Y side of the expression. We've got $y^2 - 4y$. And I like to complete the square and turn this into a squared expression. I'm going to add four. And if I add four, I can rewrite this thing as $(y - 2)^2$. And so what have we done, we've added a one, we've added a plus four, and now we've got $11 + 1 + 4$. And that's going to be equal to 16. And we've actually done it, I'll rewrite it. Notice how careful I am being about my signage on the left hand side. I'll just write 16. Over there. What do I know about this circle? Well, I know that a is equal to negative one. I know that k is equal to two. And so its center is negative one, two, there is this. Those are the coordinates of the center. And let me rewrite center in case you can read that. What's the radius? Well, it's equal to the square root of 16, which is four. So I'm capable now of graphing this circle. I know what its center coordinate is, and I know what its radius is. And those are the two things that ALEKS is going to ask us for. So let's go over to ALEKS. I'm going to start by clicking on the very circular circular, not the oval the circle. And we want to coordinate this at x negative one, y two. So I'm going to click the center of the circle there. And then we know the radius is four. And so I'm going to go 1234 spaces out, I could go to the left, I could go up doesn't matter. And I click there. And then now we've got our very nice circle on ALEKS. Let's take a look at our next circle question. We've been given the center of the circle. And we've been given one point along the perimeter of the circle. And the question is asking us to find the equation. So the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$. So we have our center so we have h here, and we have k here. And this three is our x , and this negative two is our y . So let's go ahead and plug those into the equation. When we do that. We get negative three. Oh, excuse me. We get three minus minus three squared plus negative two minus one is equal to r^2 . Now, let's simplify. So we're gonna have three minus negative three, that's going to give us six squared, and we've got negative two minus three, which is going to be, well negative three squared, that's going to be equal to the radius squared. Working out the exponents, we've got r^2 is equal to 45. Now that r^2 is the only part of this equation that we're missing, so if we want to write out the equation of the circle, it's going to be x . Well, I'll write it as $(x + 3)^2 + (y - 1)^2 = 45$. I didn't even have to take the square root to find the radius, because actually, the questions not really asking us that. Now let's go take a look at ALEKS and see if we've got the right answer. I went ahead and input our answer into ALEKS. So you can see it right there. Now, I'm going to click the check button. And let's see if we've got the right answer. And we do. We're asked to calculate the distance between the point H and the point F. And the trick here is that H and F don't have the same y axis value, and they don't have the same x axis value. So we've got a diagonal line here, it's not horizontal, it's not vertical, it was horizontal, the distance would be easy. Was vertical, the distance

would be easy. So how do we measure the distance when we're moving across the x and y axis? And the answer to that is going to be, we're going to use a distance formula. So let's take a look at the nodes. You're likely already familiar with this expression. And it is something like this. The X distance plus the distance along y, both of which are squared, and then we'll take the square root of the sum of both notice these two are not equal. So we can't do that that is wrong. Don't do that, we're going to have to deal with the squared terms first. And then once we take care of the squared terms, sum everything up, and then we can take the square root. So looking back at the question, let's call this observation x two, and that eight will be our y two. So H is going to be our second observation. And we'll let f be our first observation. So we'll call this x one. We'll call that x two. Now, all we have to do is plug in the numbers. So we've got minus nine minus minus two. And that whole thing squared plus eight minus four, that whole thing squared. And then not to forget, we're going to take the square root of the whole thing, and that's going to be equal to D. Now let's work through some of these numbers. So we've got negative seven squared plus four squared is equal to something that's going to be 49 plus 16, which is, Oh, don't forget my square roots. And we've got the square root of three 65 Now let's see if we've got the right answer on ALEKS. Here we are on ALEKS. I'm going to click this little symbol over here the square root sign in the 65. Notice that the question wants us to give an exact answer. We're not allowed to give a decimal approximation. So I'll leave it with the square root above it. And I'll click on the check button and we got the right answer. So the distance between point H and point F is the square root of 65.