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SPEAKERS

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Greetings, everyone. Welcome to another video on optimization. In this video, we're going to be using Excel to solve an optimization problem. And we're going to solve this optimization problem using two different techniques, both with Excel. Now taking a look at the slide or the Excel window in front of you, you can see our problem off to the side. Now, of course, if you don't have access to Excel, you could be using Google Spreadsheets, and there are other similar spreadsheet programs available to you for free. Now back to the problem at hand, you can see we have a profit maximizing problem classic optimization problem from business and economics. And we have a revenue function, the revenue function is 400 Q, and we have a cost function, and the cost function is 50 plus Q squared divided by 10. In both cases, Q represents the amount of quantity produced by the firm as the quantity produced increases, revenue will go up. But as you can see, by the functions, it looks like cost is going to be going up faster, because even though the Q squared is divided by 10, Q squared grows really quickly. So if you're ready, let's go ahead and let's solve these that solve this maximization problem using two different methods.

Now, the first method we're going to use is just a graphical analysis, we're just going to graph the revenue and cost function and look for profit. Or I should say specifically, where profit is maximized. The second technique we're going to use is calculus. And we're also going to demonstrate graphically what that solution will look like if we solve this problem using calculus like we have in previous videos. Now let's start by formatting the screen the way we want, I'm going to press Ctrl A, or if you don't want to press Ctrl A, you can actually just, at least on Excel, if I tap this little section of the screen right here, this little triangle, these two triangles, highlight the whole space. And if I right click, I can go to Format Cells, if for some reason you can't right click, you can find your way there, using the menus at the top. I'll click on Format Cells. And maybe we'll just have, maybe I'll just focus on the font size first. And we'll select a font size of 16. That's good. So let's make sure we've got that. Now, for method one, we're just going to graph the revenue function, the cost function, so I'm going to have revenue here. Actually, maybe I'll just move it over here to F. And I've got cost here. And then I'm going to double click right at this part where the two columns F and G meet. And if I do that, it will automatically resize the columns to fit the titles. And then I'm going to highlight the first row, go to View. And I'm going to select Freeze Panes, and I'm going to freeze the top row. So now if we scroll down, we're still going to have that top row titled, so you see the title of each column. Now I've got the revenue function here, and the cost function here as well notice that both of them are a function

of Q, so we need to have quantity. So I'm going to create a column for quantity. And I want that column to appear to the left of revenue and cost. And you'll see, you could, if you remember why, it's because we're going to graph these two cost functions and take a look at the graph and analyze it, we always want to start with the X axis variable as the far left or the first column. So since we were to want quantity on the X axis, it's the independent variable, we'd like to have the independent variable on the X axis, I want to have quantity here in column E. Now I could put, say, a value here quantity is equal to one or maybe actually, we'll why don't we go with quantity equal to zero. And I'm going to move to the revenue column and I'm going to press the equal sign. That means that I'm going to be inputting a function or performing a calculation. Excel knows I want to perform a calculation. And for revenue, it's 400 times, and I use that star to represent times, the quantity in this case quantity is zero. So the revenue is equal to zero. Now before I go too much further, I want to I'm going to format this revenue square or cell, I'm going to format the cell. And this time, I'm going to select a currency with no decimal places.

Like so. And in fact, I'll copy and paste that format into the cost cell, right here, G2. And the reason why I want to do that is because they're in dollars, quantity, it's just a number, but the revenue and dollars, it's also going to be useful because Excel will add in a comma about every 1000 values. We'll see that and it makes it a little clearer on the eyes and easier to look at. And that will be an advantage to us soon. Now I copied the revenue cell over to the cost cell. But of course, the cost function is not equal to the revenue function, the cost function is, and I'll press the equal sign, is equal to 50 plus Q squared. Where's quantity square? Well, quantity is right here. And that E column to the power, that's a superscript, to the power of two squared, and the whole thing will be divided by 10. And the way Excel will read that they're gonna, they read that as E squared, or Q squared divided by 10. And we get $50, which is what we would expect.

Now for the next step, I could choose quantities equal to one, and then I could just highlight cells, F2 G2, and click this little square in the bottom right, and drag it down, and the cells would populate. Now, it looks like there's a bit of a problem, but I guess it's probably a rounding error. If I were to, you know, the cost should be 50, 50.2. So let's not worry about that. But just to check and make sure we're doing this properly, maybe let's see what happens if we go to three, and then four. And there's 51. So it seems like it's adding correctly. Now notice a bit of a problem we have here if we keep going with this. And I'll select that three and just, I'm selecting cell E5 there and I'm adding one. And then I bring this down, like so. And then I highlight all these and bring them down, you can see that revenue is growing much more quickly than cost. In fact, if I add in a little profit column here, you can see that profits increasing quite quickly. And we can also see that cost is increasing rather slowly. So I could just highlight all these and just go down as far as we want to. Now, notice we've got some issues here. So I'm going to double click, make sure these are legible. And we could just keep going down like this. But this is a little cumbersome. Notice that we've got 350, almost 350 calculations, and we're using almost 350 rows. So I'm going to press Ctrl Z, and I'm going to undo what I just did there. And in fact, I'm going to keep pressing CTRL Z until I can go back to just having these 10 observations, 11 observations.

Now to play around with this and make it a little more succinct, manageable in terms of the number of calculations, why don't we do this, we'll start with starting value. So that's where quantity will begin at the first observation. And then we'll have a difference. And so maybe we want to start with quantities zero. And instead of having the quantity there, I'll put the zero there. And maybe we want

the difference to be, say 10. And how am I going to change? How am I going to change the sort of the way that we written the commands in Excel? Well, zero is just going to be zero the first observation will just be whatever the starting value is. The second one, here we are in cell E3. I'm going to select cell D to the starting value cell. But I'm going to add these dollar signs. So that means that as I populate additional cells, it's not going to change that cell reference. That cell reference is going to be stuck. It's going to be stuck at D2 and I'm going to add this difference D3. So notice now we have zero plus 10, essentially.

Now, I could do it that way. What's another way of doing this? I could have said, well, let's do equal the previous cell, whatever the previous one is plus this difference here. And then I'll add in these dollar signs to fix the cell, add D3. And I press enter. And now as I, if I select D10, choose that little square, drag it down, I'm going to have quantities changing by 10 instead of by one. And I'm just going to highlight these cells here. And I'm going to double click on this little part of the screen here, just between the two columns, I and J, just to make it a little easier to read. In fact maybe I could well, we'll just leave it like that for now. Now notice that even with quantity changing by 10, revenue is growing much more quickly than cost. And we can see more clearly now that profit is growing quickly. And it doesn't, not really showing much signs of slowing down yet is it? So maybe we want to play around with our starting values and the difference between each row. So we could start instead of with zero, maybe we want to start with 100. So now we've jumped to 100. And maybe we want the difference to be quite a bit more, why don't we try 100. Now, again, I'm just gonna highlight these cells, I guess I could have just made them a bit bigger, I could have just done this, and tried to guess how big they need to be. Do that as well. And now when we see quantities are changing by 100. Things are still, you know, revenue still growing, or still quite a bit bigger than cost. But cost is getting closer, it's getting close to revenue. Profit is increasing, but it's not increasing quite as quickly. Now I could change this again, maybe what I'll do is I'll have a starting value of zero, and this time, the difference will be 200. And now we can see, cost is really starting to catch up with revenue. I'll highlight these cells. And I'll keep going maybe to say 4000. And I'll just highlight this row, I'll right click on it. And I'll choose the option Clear Contents to get rid of it. And so we can see that by the time quantities get to 4000 profits are negative. If we graph this expression, or graph these functions, we're going to be able to clearly see where profits lie now. Kind of as an exercise, let's just graph revenue and costs the revenue and cost function. And we can create a scatterplot. So let's create a scatterplot right here. This is how I get the commands, there might be other ways to find your way, navigate your way here. But you can see where I found it. Just going to Insert on the menu at the top and then navigating over to charts and choosing the scatterplot one. Now, you can choose the format, maybe, oh, maybe we'll go with these line ones. I'll take the line one. And I'm going to move the chart where am I gonna move the chart?

Now I'm going to try and make this chart look as square as possible. Why do I want it to be square? Well, I want an aspect ratio of one to one. And then that'll just make it a little clearer to look at. And I'm going to put it right down here. And if we look carefully at this graph now how do we find the profit maximizing quantity of production? Remember, we've got Q over here, well I just looking at this graph and I want to find the largest difference between cost and revenue. And if we look at it, you can see the difference from here to here is about one and a half grid, grid points. Here it's about one and a half. But here right at 2000, we can see that we've got two grid points. And so this is the profit maximizing quantity of production. And if we looked at our, if we double check our Excel spreadsheet, you can see that is occurring right here, where quantity is 2000. It seems to be at its highest now,

maybe, you know, it's possible the actual maximizing quantity was 2001. But I think for our purposes, this is close enough. And we can be pretty satisfied that approximately profits are maximized approximately when quantity is equal to 2000.

So that's the graphical analysis, you'd really just graph revenue and cost, and then look at the graph and try and identify where profits are maximized. Now let's take a look at the second, the second method. The second method is asking us to use differential calculus. And we can do that in quite a straightforward way. Now I'm kind of run out of room on the spreadsheet here. So what I'm going to do is I'm going to highlight row E, because it's got the quantities, and we might want that later. So I'm going to populate the quantities into this column J that's currently empty. And I'm just going to populate the values, paste the values. So I won't paste the formatting, but I will paste the values. In this case, it probably didn't really matter very much. And to make room, I'm going to highlight columns I to E, and I'm going to right click, and I'm going to hide them. I'm going to use this hide button here. And when I click on this hide button, you can see that now we go column D and then to column J, but I didn't delete them. I could go and highlight columns D and J and click this unhide button, and they would reappear. So you see that they've reappeared. So I'm going to hide them again. And there's the hide button. And now we've got our quantities here, how are we going to find the solution using calculus? Well, we need to calculate marginal revenue. So how does revenue change as Q changes, and we need to find marginal cost. So we're going to find the first derivatives of the revenue function. And we're going to find the first derivative of the cost function. Now I like to, I'm an economist, so I think of them as marginal revenue, marginal cost. If you prefer, we could do RQ. And we could do C prime Q for those of you that maybe you're not majoring in economics, we'll keep it more general. So we're gonna find the derivative of the revenue function, we're gonna find the derivative of cost function. Now, what's the derivative of the revenue function? Well, we have 400 times Q, so that's just going to be equal to 400. And it's going to be equal to 400 for any quantity produced. So changes in quantity don't affect the change in revenue. Now let's take the derivative of the cost function. So you see it's over here, and we're good, the 50 just become zero, we don't have to worry about that. Or maybe I could just write zero plus, we have zero plus the quantity. What's gonna happen to that Q squared? Well, the two's gonna come down, the quantity is this column J. So I'm going to choose J2.

Oops, oh, I see, I forgot to have my equal sign. So make sure you add in the equal sign before you start trying to calculate anything. And I hope you saw what happened there, it gave me a little error message. It didn't like it, but it autocorrected it, it seems to be okay. If we look up here, we've got equals to zero plus two, that's what we want. And I want two times quantity, the quantity produced, and that's going to be represented by the cell J2 right here. And finally, we want to divide by 10. Now looking at this, you could probably say to yourself, oh hey, Professor, can't we simplify this expression? We can simplify it, but it's not necessary. So we'll just, we won't bother simplifying it. Just hit enter, and the marginal cost is equal to zero and quantity produces zero. Not much of a result. But let's see if we calculated properly as we populate these two expressions when quantity is not equal to zero, it's greater than zero. And if we do that, you can see that we've got this constant rate revenue and the marginal cost or this, the first derivative of the cost function is changing, and the change is linear. Now notice that our, our revenue or profit function would be equal to 400 Q minus 50 minus Q squared over 10. And if we take the derivative of this thing and set it equal to zero, in order to maximize it, we're going to end up with the marginal, well, we're going to end up with R prime Q is equal to C prime Q. And so profit is maximized when the first derivative of the revenue function is

equal to the first derivative of the cost function. Where does that occur? Well, that occurs right here, when Q is equal to 2000. The first derivative revenue is equal to first derivative cost, or what we like to say as economists is the marginal revenue is equal to the marginal cost.

Now I'd like to graph this expression. And I want to compare it to our first graph. So I notice here that our first graph doesn't look so good. Maybe that's because we're hiding the cells. So let's try unhiding the cells that we had before that was for the first method. So I've clicked on this cells D, or the columns D and J, highlighted them, and then right clicked and gone to this option here on hide. And I'm glad to see that this chart has come back. And now why don't we make a diagram, a scatterplot for our second method here, and maybe we don't need quite that much space. So I'm going to highlight these three cells quantity, first derivative of the revenue function, first derivative of the cost function. I'm going to click here and Insert, go to choose scatterplot. I'll choose it with a line again. I'm going to bring this chart all the way down over here. And if I can line it up properly, which is a very big if. Now, you could see that our profit maximizing value was here, the first derivative of revenue and of marginal cost intersect is also where profits are maximized and that occurs when Q is equal to 2000. There you have two methods of using Excel to find the profit maximizing quantity production.