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SPEAKERS

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Hello, everyone. Welcome to our video looking at optimization problems, we're going to solve some optimization problems together by looking at or by finding the critical points. Once we found those critical points, we're going to evaluate each of them using the second order condition or the second derivative test to determine whether each is a maximum or a minimum.

Here's our first optimization problem, we're looking at a firm that produces microchips. And their profit is determined by how many microchips they produce. And you can see that right here, our profit function, pi, that represents profit. Now, like the optimization problems we've been looking at, in this series of videos, we're gonna start by finding the critical points, and then evaluate those critical points with a second derivative test. Why the second derivative test? Because the question is telling us, that's the test that we have to use. If you are going to take a class in economics, for whatever reason, economists prefer the second derivative test instead of the first derivative test. And then finally, we'll sketch the graph. And I'll show you what that looks like when the time comes. So first off, let us find all the critical points. So I'm going to take the derivative of the profit function, and I'm going to set it equal to zero. And we find that we have negative 3X squared over 100 plus 2X, plus 20.

Now we have kind of an ugly answer here, this thing is equal to zero. And you could try and factor it in your mind, I don't think you're gonna get very far. But we can use the quadratic formula to make this easier on ourselves and solve the X values that satisfy this expression right here. The formula is X is going to be equal to negative B plus minus the square root of B squared minus 4AC, all divided by two A.

Now, let's not go through the the details of performing this calculation, you can actually find websites online that will do it for you. Notice that I will write down that our A value is going to be equal to negative three over 100. Our B value is going to be equal to two, and C is equal to 20. So these are the coefficients that are right here. Oops, that's our A. I did it again, there's A, there's B. And there's C right there. Notice that there is a plus minus here. So that means we should get two answers for X, should be two answers for X. And also keep in mind that sometimes it's not possible to, there is no values for X that satisfy this expression right here. And in that situation, the quadratic formula won't work. And when I say it won't work, this is usually the thing that breaks down. We don't want any negative numbers in there. So it must be the case that B squared is larger than four times A times C.

Now using the quadratic formula, we have X is equal to negative 8.9. And X is equal to 75.5. Or maybe, yes, let's just say 75.5. I took the liberty of copying our answers from the first part of question and representing them here on this new slide so that we can answer question two. Question two is asking us to evaluate both of our critical points using the second order condition to determine if they are maximums or minimums. And remember that we're, it's a profit maximization problem. So we would like to find the level of X, the level of output that maximizes the firm's profit. The first thing that we want to do here is we want to take the derivative of our first derivative, which will give us the second derivative. So to get the second derivative, we're going to have negative six over X plus two, plus zero. And maybe we'll just remove that plus zero, so it doesn't distract us. Now we've got our second derivative. So our next step, why don't we simplify it just a little bit, we can do a few things. One, we can see that six over 100 is equal to three over 50. So I could write this as 3X over 50. And why don't we give these two terms a common denominator. And we can do that by multiplying the two in the numerator by 50 and the denominator by 50, like so. And if you prefer, we could even write it like this. And the reason why we want to simplify this is because we have to evaluate two different critical points. And how are we going to do that? Well, we're going to take our second derivative, and we're going to plug in that X value that gives us, that is associated with a critical point. And when we do that, we're gonna get something here like negative three multiplied by negative 8.89, plus 100, over 50. All we really are interested in here is whether the second derivative is positive or negative, because if it's positive, the function is going to be convex, and therefore we must have found a minimum. And specifically, the function is convex on the interval that we're, where the critical point occurs. If the second derivative is negative, then the critical point on that interval of the function where the critical point lies is going to be concave and therefore must be a maximum.

So if you look at this number, if we do the calculations, we can see that basically, we've got negative three times negative eight, so it's going to be positive. So this thing is going to be a positive value. So X equals negative 8.9 is a minimum. According to the second condition, second order condition, excuse me. What about our other critical point right here, 75.5, this is going to be equal to 100 minus three times 75.5. And of course, I'm using this expression right up there. And we can see pretty clearly, and we don't even really need to do nominator, but we can see that this is going to be a negative value. So X is equal to 75.5 is a maximum. That is according to the second order condition. Now the third part of the question is asking us to sketch it to sketch this. So let's take a look at the sketch and see how we did. We worked through the optimization problem together, found the critical points, applied the second order condition. Here's what the sketch of the diagram looks like. You can see that there are two critical points critical point being where the slope is equal to zero and they occur here and here. Notice that one of them is a minimum, which seems to correspond with that negative X value that we calculated as one of the critical points, and there is a maximum, and the maximum seems to be happening around 75.5. So to solve the manufacturers problem, they should set X equal to 75.5, they should produce 75.5, say, million or 100 million microchip, microprocessors, microchips.

And that's how we can use the second order condition to solve optimization problems. As you go forward, as we introduce more than one variable, these solution techniques are going to become more important, because we won't always be able to simply sketch out the graph that can give us a guide to the solution. Also, keep in mind, this expression was not closed. It wasn't closed. So X, we allowed X to go from negative infinity all the way up to positive infinity. And so it'd be pretty, you need a computer if we wanted to sketch out all the possibilities there. And so these mathematical techniques are really valuable if we have sorts of issues around trying to sketch out everything by hand, or even using a computer when we get into multivariate calculus.