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SPEAKERS

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Hello everyone, in a previous video, you looked at the first order condition, the first derivative test for whether a extreme point is a max or a min. In this video, we're going to look at the second order condition, we can use the second derivative to determine whether a critical point represents a maximum or a minimum. As we're going forward, you know, these tests are valuable because, sure with a single variable, you know, sometimes you can figure these things out just by graphing the function. But when we go into multivariate calculus, we start getting more and more complicated functions, you can't just simply graph the function and find the max and the min. And so these tests become more and more important, as you advance further in this course, and as you advance further in your careers.

So we've got a graph down here, but let's take a look at the words. Before we look at the graph, we've got a function, F, and it's defined on an interval. And I guess, well, we can turn our attention to the graph, this will be our interval, we'll call it maybe A to B, doesn't really matter what we call it. And we've got a critical point C. So let's call this critical point C. We know it's a critical point, remember, because I could draw a tangent line at that point where the tangent line slope is going to be zero. Now let's take a look at the diagram, we've got a function and if we start at point A, the beginning of the interval, you can see that the slope is positive. So the function is increasing as we move from point A towards point B, notice that the function may be increasing, but it's increasing at a decreasing rate. So this point has a higher slope than the point here, than the point here. And then finally, the slope is zero when we reach that critical point. And then beyond the critical point, as we go past point C towards point B, as we move from point C to point B, we can see that the function is then decreasing and it's decreasing at a decreasing rate, that slope is becoming more and more negative, or the slope is becoming smaller and smaller. The concave function is increasing or decreasing. So a concave function could be either increasing, or it could be decreasing, just like we see in the diagram at a decreasing rate. So the function is whether it's increasing or decreasing, it's doing so at a decreasing rate. Now, if a critical point occurs, I can say therefore, if a critical point occurs on a concave interval of the function, critical point must be a maximum. So we can see here again that we're going up the hill as we move from point A towards point C, and then when we go past point C, towards point B, or going down. And that's why the second order condition or the second derivative test works.

Let's look at it again. This time, we're going to look at it with a function that is convex. So the function if we define our interval, this time, we'll say we've got a here and we'll go with B here. And we're going to have a critical point C again, and that of course curves when the slope of the tangent line is equal to zero, the slope there is zero. Now as we're at point A moving to C the function is decreasing. But as we get closer and closer to C it is decreasing at an increasing rate. And when we get past point C, and we start moving towards point B, the function actually becomes an increasing function. So we can say that a convex function is increasing or decreasing, notice that this is a convex function. At some points, it's increasing at other points it's decreasing, increasing or decreasing at a increasing rate great. So the slope is getting larger, might start off as very small, very negative. But as we move from left to right on the diagram here, from the beginning of the interval at a towards the ending at B, or one endpoint A towards endpoint B, the convex function is increase or it's whether it's increasing or decreasing at a increasing rate. Therefore, if critical point occurs on a convex interval of a function, the critical point must be a minimum. And we can see that again, thinking back to the Hill analogy, you know, you're going down hill, as we move from endpoint A towards C, then we're moving uphill. Or you're moving uphill, as we move from point C towards point B. And so point C must be the bottom of the valley, we must be going down the hill into the bottom of the valley and then up the other side. And so if the function is convex on an interval, and there's a critical point on that interval, then it must be a minimum.

Let's take a look at an example. First we're going to find the critical point. And then we're going to evaluate whether each critical point is a maximum or minimum using the second derivative test, or the second condition test. So to get started, let's take the derivative of H of X, you've got H prime of X is equal to well, we've got a negative here, so I'll leave the negative. And I've got to use the chain rule to take the derivative of this thing here. So the two is going to come down, and we have A minus X to the power of one, and then multiplied by the inside the derivative of the inside of the bracket. And that's just going to leave us with negative one. Now this whole thing, of course, is equal to zero. That's how we find the critical point or that's the definition of critical point H prime X is equal to zero. Now we can simplify this a little bit, and I can rewrite this as two A minus X. Since we've got the sort of two negatives, a negative times a negative is positive. Solving for X, we're just going to have X is equal to A. So we found one critical point and that critical point occurs when X is equal to A. So that's the first part of the question. Now what about the second part of the question?

Well, we want to use the second derivative test. I know that the first derivative is equal to two A minus X. Notice that I want to, I want to, you know, I don't necessarily want to set the first derivative equal to zero, if I'm trying to find what the second derivative is. And if we take the derivative of the first derivative, that will give us the second derivative. The second derivative is going to be equal to minus two. And since this is negative, that means that if we have, well what does this mean? This means that the second derivative is negative. So, the function is either increasing or decreasing at a decreasing rate. So the critical point X equals A is a maximum. Illustrating our previous example, you can see that on the slide and the screen in front of you, we have point A, remember point A, whatever value point A takes on it's an unknown, unknown variable. That's going to be, well that is a critical point. And as the second derivative test told us or the second order condition tells us it must be a maximum, why must it be a maximum? Because on the interval or, well, in this case, for this specific function, the entire function is concave. And since the entire function is concave, a critical point on that function must be a maximum.