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SPEAKERS

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Let's take a look at this question. We have seen it before, we're going to find all its critical points, and then we're going to evaluate whether those critical points are a minimum or maximum using the first derivative test and a sign diagram. And I'll show you how to create a sign diagram, which can be helpful for applying the first derivative test. Now, to find the critical points, we're going to take the derivative of CT, we're going to set it equal to zero. And, in this situation, I'm going to apply the quotient rule. So we have a quotient here, we have a function divided by another function. And so I'm going to take the derivative of the numerator and multiply it by the denominator, minus the derivative of the denominator, multiplied by the numerator, and this whole thing is going to be divided by the denominator squared. Now I can simplify the numerator, notice that we've got T squared plus four minus two T squared divided by that same denominator squared. And this is going to give us negative T squared plus four divided by T squared plus four squared.

Now looking at the numerator, that looks like a difference of squares equation to me, and so we could say zero is going to be equal to two minus T, T two plus T, divided by T square plus four squared. And what values of T are going to satisfy this equation right here? How are we going to satisfy? How are we going to satisfy this equation here? Well, it's going to be satisfied when T is equal to negative two, and when T is equal to two. And so these are our critical points. When T is equal to negative two, T is equal to our derivative of CT, C prime T is going to be equal to zero.

All right, so we found the two critical points from our original function C of T. And now we've got this first derivative, which we found right here. And, of course, we got the critical points when we set that equal to zero, and then found the T's that satisfy that equality. Now we want to draw a sign diagram, because we want to be able to apply this first derivative test, and I'll show you how to do it, and then remind you how I explained it earlier. So trying to sign that diagram, I usually start with writing the critical points. So I'll write a negative two here and a two there, you want to start with the lowest value further to the left, and then go upwards to the right, so writing from left to right, and I'm going to draw vertical lines below them. And then I'll draw another line here, and another line here. And I'll call this infinity and negative infinity. And now what I've got all the way along here in this top row, is this is a interval from negative infinity, to infinity. And I've marked the critical points, because I want to evaluate at those critical points if the first derivative changes signs. I want to evaluate whether this whole expression here is going to change signs when we go through one of these critical points.

Now the next thing I'm going to do is I'm going to write each of these factors in the first column, the first column, in row so we're going to write two minus T two plus T. And if you think of division, as just the reciprocal of multiplication, all the factors go in there, and you think of all the things that are multiplied by each other. So we're gonna write T squared plus four squared. I'll draw another line, across like that. Now, let's look and evaluate this first factor. If T is less than negative two, but greater than negative infinity, so anywhere below negative two, this expression, a negative minus a negative is going to be positive. So this whole thing has to be positive. If T, and remember, this is the interval here, for T interval of T, make sure to add the T there. Looking at the next interval from negative two to two, this expression is still going to be positive. As long as T is less than two but greater than negative two, this thing here is going to be positive. Now if T becomes larger than two, that's when the factor will turn negative. Next, let's evaluate two plus T, T is a very large negative number, this thing will be negative. But if T is greater than negative two, it'll be positive.

And it will remain positive as T gets larger and larger. This last factor is squared, and so a negative squared is a positive. So for any value of T, this thing is going to be positive. Positive, positive, positive. Now I'll draw a line like so. And we want to evaluate whether this entire expression up here, C prime of T is positive or negative. So when T is less than negative two, we've got a positive times a negative times a positive, that's going to be a negative value, the slope is going to be negative. And if T is greater than negative two, but less than two, we're going to have a positive times a positive times a positive, that's got to be equal to a positive. And in the last column, we've got a negative times a positive times a positive, which is going to be negative. And these little dots that I, a little dashed I drew here, that was just counting making sure that I took account of each of the factors. Now we're ready for the first derivative test.

We've got this, well the derivative this first derivative is negative here, and it's going down and down and down. But once it hits negative two, it becomes positive. And it's going to remain positive until T is equal to two. And then when T is equal to two, it's going to be negative. And you can see that we've got a minimum when T is equal to negative two, and we've got a maximum when T is equal to two.

And that's how we use the first derivative test to evaluate whether a critical point is a minimum or a maximum. We use the sign diagram. Remember, if it's a maximum, that means if you're to, to the left, if you're below the critical point, you're going to be going up the slope, you're going to be going up the hill. And then when you go past it, when you go beyond, you're going to go down the hill, that means you must have been at the top of the hill. Similarly, if you are going down into the valley, if you walk down into the valley, and then you start going up the valley, that means that you must have passed the minimum point in the valley. And that minimum point is these critical points that we've identified.

And here we are, we've graphed the, the function that we were working with. And you can see that there does appear to be a minimum at negative two, notice T is equal to negative two. And there

there does appear to be a minimum at negative two, notice T is equal to negative two. And there seems to be a maximum when T is equal to two. So you can see that the first derivative test worked. And we can verify that it worked in this case by just looking at a diagram since we're only working with one variable. Now we'll continue on we'll look at the second derivative test that economists like myself seem to prefer, and sometimes mathematicians make fun of us because of it.