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SPEAKERS

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Hello, everyone, Professor Robert J McKeown here. Welcome. Today we're going to be looking at the properties of extreme points. And we're going to use those properties to derive something called the first derivative test. The first derivative test is going to tell us whether extreme point is a global maximum or a global minimum. If we're looking at local minimums and maximums, it'll tell us if some point is a local min or a local max, sort of a mini max, like a little hill, or a little valley. We'll talk more about that a little bit later. For now, let's get started. Let's look at the properties of these extreme points.

Now to understand how the first derivative test works, think about a function here with a maximum. So you can see that the maximum is right here. And maybe we could call this point C, if we wish, it doesn't really matter what we call it. And at that maximum, you, you, I hope can see that the slope is equal to zero. So the slope is equal to zero at the top of this hill, just think of it as a hill. In fact, let's continue with the hill analogy. And think of yourself as a hiker going up the hill. And there you are, and maybe we'll draw a little baseball hat with a little hat on you, and give you a walking stick. So you know that you're going up the hill. And each step you take up the hill takes you to a higher elevation, right, each step takes you up the hill, and each step up the hill takes you to a higher elevation we're moving higher in Y space, moving vertically. Now when you get to the top of the hill, when you get to the top of the hill, you are at the peak. And if you're at that top of the hill, imagine that it's just this tiny little top to the hill. If you take a step forward, you're going to end up going down the hill, right? And if we're looking for the top of the hill, we don't want to do that, you don't want to take a step off the top of the hill. And notice that if it's a maximum, you're going to be increasing when we are to the left of C, as we move closer to C from the left, say point A. We, our F value, or our Y value is increasing. We're going up the slope, the slope is positive. And when, if we go past this maximum point, say towards point D, we're going to be going down the slope there's going to be a downward slope.

So somewhat formally, but not too formally, if X is less than C, F prime X is going to be greater than zero. And if X is greater than D, but still in the interval, clearly I won't write that down but clearly in the interval, then the slope is going to be negative because you're going to be going down this hill. And if we see a situation like this, then we then we know that C is a max, it's a maximum. Now imagine that you're walking down into a valley, every step that takes you down the hill into the valley is going to take you to a lower altitude. So you're going to be moving downwards. When you reach the bottom of the valley, the lowest point in the valley, it's going to have a slope of zero, right? And that tangent line that just connects to the bottom of the valley is going to have a slope of zero. And why don't we call this point C again.

Similarly, if you keep going you go to the bottom of the valley and you keep walking you're going to start going up again and so, if we see a situation like this, where the slope is negative when X is less than C, but positive when X is greater than C, C must be a minimum. Now, in this specific situation we have a local minimum but the same test applies to a local minimum as it would to a global minimum. Why is that a local minimum? Well, we can see that the global minimum is over here, but notice that it is a end point, not a critical point.

This first derivative test, with this slope, it only applies to critical points to interiors. The interior of the interval. So I'm going to define this point here as point A and then any point over here can be B. And I can say that if, if for any X inside the interval A C, or I guess, I get A C, F prime X is, is less than zero, it's negative, and for any X in C F prime X is greater than zero, then F C point C is a minimum.