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SPEAKERS

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Hello, everyone, and welcome back. In this video, we're going to look at extreme points, global extreme points. And then we're going to talk about some of the properties of these global extreme points. And based on those properties, we're going to develop two tests for determining whether a critical point is a maximum or a minimum. And those are known as the first derivative test and the second derivative test. But first, let's take a look at the definition or the steps involved, I should say, of how to find a global extreme point. Here are the steps for finding a global extrema. Now we need a couple conditions. So we've got this function F, and it needs to be differentiable. So the ,the, we must have a derivative on the interval that we're interested in. And that interval has to be closed. That's like the square brackets that designates closed, that means that the X have to be within the two endpoints. And we want it to be bounded, which means that there's an F of X for all the X on this interval, so that it's less than or equal to A, and there's an F of X where it's greater than or equal to say some capital B. And really, what this is saying is, it has to be finite. So when we say bounded, it has to be finite, rather than infinite. And when we are dealing with that kind of situation, we'll look at an example in a moment, we can follow these steps to find extreme points. The first step is to find all the critical points, in that interior of this closed set. Notice the round brackets that we have here, designating that it's on the interior of this closed set over here. What is a critical point? That's whenever the derivative is equal to zero. A critical point occurs when, on the interior, the derivative is equal to zero. Next, we want to evaluate the endpoints A and B, so we want to calculate, we want to calculate F of A and F of B.

Now that we've calculated F A and F B, we want to also calculate all the, or the largest value and the smallest value among the critical points that we found on the interior in step one. And so if C is a critical point, let's calculate F of C, and then we'll compare its value to the values of F A and F B, the two endpoints. Now, those are, that's the sort of recipe to follow for finding a global extrema. Now let's look at an example together. Here, we're being asked to find the maximum and the minimum values of some closed and bounded function. This is going to be closed and bounded, we've got a nicely defined function here. And X can take on any value between negative one and three. Now let's follow the recipe. So step one, find all critical points. And so the first thing we're going to do is we're going to take the derivative of F of X, we're going to set that equal to zero. And when we do that, we get X to the power of four, use the power rule, the four comes down, four multiplied by one over four is just going to leave us with X cubed minus 3X squared plus 2X, and the one, the derivative of a constant is just zero. So I could add in a plus zero if I wanted to.

Now we really want to solve this expression for X. You can see we've got three terms and there's an X in each term. So why don't I, I'm not going to be able to explicitly solve for X, but why don't we factor out one X term, so we're left with X multiplied by open bracket, X squared minus 3X, plus two, close bracket. Of course, this is all going to be equal to zero. Because we want to find these critical points, we know the critical points have F prime of X equal to zero. Let's take a closer look inside this bracket, it looks to me like I could factor that. So why don't we try factoring it, I'm going to write X, open bracket, close bracket, open bracket, close bracket. And if I write X minus 2X minus one, notice that negative two times negative one, that's going to give us the plus two, negative two minus one gives us negative three. And so we're matching the coefficients that we see here and here.

So we've got X minus 1, X minus two. And that means we're going to have, well, how many critical points are we, we're going to have a critical point wherever the X value satisfies this whole thing is equal to zero. And so our critical points are going to be X equals 0, X equal two, and X equal one. Whenever X takes on one of these values, zero, one or two, this whole expression here is going to be satisfied. The right hand side is going to be equal to the left hand side. Now step two, tells us to evaluate the endpoints. So we've got an endpoint here and an endpoint here. And we're going to want to calculate F of negative one and F of three. And let's do that on a fresh slide. Now, our two endpoints, were F negative one, and F of three.

So we want to calculate the value of f at the end at each of our endpoints and each of our critical points, and so we get F of negative one is equal to. And if we calculate this out, after all the calculations, I get 3.25. We've got, that's going to be a negative one, but it's a negative minus a negative, that's gonna be plus one. So we ended up with three plus this, this term here is going to be a one quarter, .25. Now we can just move on to the next critical point, or endpoint. Now let's look at our other endpoint, that's F three. And if we plug three in, we get three to the power four divided by four minus three to the power of three, plus three squared plus one, and that is going to be approximately equal to well, it's going to be equal to 3.25. To skip through the calculations, you can trust me or you can verify for yourself at home. Now we completed evaluating the endpoints. Now we can evaluate the critical points. And the first critical point to evaluate is this one right here. X is equal to zero and we get zero by by zero minus zero cubed plus zero squared plus one. That's just going to be equal to one.

The next, x is equal to two. Two to the power of four, two to the power of four over four minus two to the power of three, plus two squared plus one. And that is going to be equal to 1.00. Last critical point to evaluate is when X is equal to one. We have one to the power of four over four minus one cubed plus one squared plus one, and that is going to be equal to 1.25. Now taking a look at what we've calculated, we've got kind of an unusual result here, we've got two values that are maxes, they are the highest value that this function is going to take on over the interval negative one to three. And there are two, there are two minimums.

Now, why don't we take a look at a graph of this expression and see if the graph matches the work that we did with calculations. Looking at the diagram in front of you, it seems pretty clear to me that we were successful in our earlier work at identifying extreme points. You can see that the, we've got two maximums and the maximums are occurring here and here. And we also have two minimums, and the minimums are occurring at the bottom there, where X is equal to two and X is equal to zero. We even have that critical point, that is not a global maximum at X equal one, and that is right here. And it is known as something called a local maximum. And so local maximum, we haven't really talked about that yet. We'll talk about it a little bit more in a future video. But for now, we were successful, we identified the global extreme points in this example. And the maximum values occurred when X was equal to negative one and X was equal to three, which also happened to be the two endpoints. And we found the two minimum when X is equal to zero, X is equal to two. They were critical points on the interior that were also the global minimum.