Robert_Moptimize5a

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SPEAKERS

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Hello, everyone, welcome back. This video, we're going to solve some optimization problems, we're going to look at some graphically and look for some extreme points and critical points. Then we'll actually solve for critical points, and for any interior extreme points, just using the function and what we know about calculus. Hope that sounds good to you. If you're ready, keep watching and we'll get started.

Let's continue with some graphical analysis. The question here says, is there an interior extreme point? The answer is yes. There is where's the extreme point while it's right here, this is the minimum. And it's on, on the interior we have an end point here. And we have another endpoint here, which is also the max. But it's not in the interior, it's at the endpoint. Another great graphical analysis says find all critical points and extreme points. So we could look for the extreme points First, we see that we have a maximum up here. And we have a minimum down here. So we could call this A. And we've got a critical point occurs wherever the slope is equal to zero, so that's going to be at the top of a hill, that's a hill right there, or at the bottom of a hill if that's, or the bottom of a valley, if that's the bottom of the valley. And so we've got a critical point here.

Got another one here. And I can write, there we've got F prime B is equal to zero. This F prime A is equal to zero up there, and I draw straight lines there, we've got F prime C is equal to zero. So critical points. We've got critical points at A, B and C. And we have extreme points at A, and if we call this end point over here D, at D. So we have one interior extreme point. But we also have an extreme point at an end point and point D. And we have three critical points, A, B, C, one of those critical points is an extreme point.

Here is an example from the real world. So this is a useful problem if you want to understand how long drugs stay in the system and also how drugs can affect a human body. And so we're looking at the concentration of drugs in a person's bloodstream, T minutes after injection. And we have a function that's going to capture this concentration in the bloodstream, and it's right here. Notice that

T is in minutes and we're being asked to find the time after injection at which the concentration is highest. So we want to maximize this expression or at least see if we can find the point at which this function is maximized, its extreme points. And if we want to find its extreme points, well we know if there's an interior extreme point, it must be the case that C prime T is going to be equal to zero for any interior point to be a extreme point.

Now let's find C prime T. And if I asked you which rule, which derivative rule do we have to use to solve this? Well, the most obvious one would be the quotient rule. So I'll apply the quotient rule. And I'm going to take the derivative of the numerator, which is just equal to one, and multiply it by the denominator minus the derivative of the denominator, which is 2T times the numerator T. And I'm going to divide this by the square of the denominator, like so. Now I want to make sure I set this equal to zero, because C prime T has to be equal to zero, in order for any T to be an extreme point. Now I'm going to simplify this expression.

And we have T squared plus four minus 2T squared divided by T squared plus four squared equals zero. Now if I multiply both sides of the expression by what's in the denominator, the denominator is just going to disappear and become zero. So we're going to have T squared plus four minus 2T squared equals zero. And simplifying this expression, we get negative T squared plus four is equal to zero, or simplifying, T squared is equal to four, and T is equal to the square root of four, which is just equal to two. And so when T is equal to two, after two minutes, the concentration of the blood in your bloodstream is going to be maximized. Sometimes we put a little star on here, maybe I could write this out.

After two minutes the drug in your bloodstream is at its maximum. Now, to some degree, you're gonna have to take my word for that, we'll look in the next video about a test to make sure to evaluate whether T star equals two as a maximum or a minimum. Since we haven't done that test, we can't prove it yet. But what we will do instead is I will show you a graph, I'll actually graph it. And you can see from this graph in front of you that we've got C prime to is equal to zero. That's when the derivative is equal to zero. And it's a maximum. And we can just see that when we look at the when we look at this expression, now if we want to know how much that concentration is worth, we could use the expression that we've been given. And we could evaluate the concentration at two minutes, or after two minutes. And we end up with one over four, a concentration of one over four. Not really sure what the unit is meant to represent. But if you were asked, what's the concentration in the blood, the value that we'd have here is one over four.