# Robert_Moptimize4

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**SUMMARY KEYWORDS**

optimization problems, extreme, point, slope, equal, f prime, differential calculus, function, interior, interval, horizontal line, maximum, draw, derivative, critical, rewording, notice, minimum, horizontal tangent line, solution

**SPEAKERS**

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Hello, and welcome. I'm glad you're here to check out our video. Today we are continuing our discussion of using differential calculus to solve optimization problems. Previously, you learned that when the interval of a function or a relation is continuous and differentiable. Differential calculus can find the most effective use of a resource and help you figure out the best course of action in a situation that you have modeled. Often in optimization problems, we refer to the solution to that optimization problem as an optimal allocation. Going forward, let's restrict our analysis to intervals where the function of relation is continuous and differentiable. That's going to be when calculus can help us best at finding the solution, finding that optimal allocation to a optimization problem.

Now let's consider some points C. C is an interior point within our interval. Now C can only be an extreme point for the function F, if and only if F prime C, the derivative of F at C is equal to zero. Now what do we mean when we say that C is an interior extreme point for the function F, if and only if F prime C is equal to zero? Well, let me just sketch out a graph. Suppose this is our function. Now let's consider this point C. So C, if this is C here, C can only be an extreme point on this function, if F prime C is equal to zero. Now notice that at this maximum point, if I draw a straight line representing the tangent line at C, this tangent line is a horizontal line. Why is it horizontal? Because the slope of F prime C is equal to zero. So C can only be an exterior extreme point for the function F, if F prime C is equal to zero, let's look at the case where it's not.

Say this point is C here. Notice that the slope C is greater than zero. And so C cannot be an extreme point, slope is greater than zero, we're still going uphill, the, the function as we're going from left to right, is actually increasing in size. And so C cannot be an extreme point. That's what this wording is trying to tell you. Any X in the interior of an interval where F prime X is equal to zero is called a critical point. And so critical point is going to be an important definition for you to remember because we're going to be, I'm going to be asking you, and you're going to have questions that are asking you to find critical points, because critical points is going to lead us closer, or perhaps to the solution to our optimization problems.

Sort of rewording this top one again, if C is going to be an extreme point, it's necessary that the derivative of F at C is equal to zero. Otherwise C cannot be an extreme point. So here's an example that we can work on together. The question is asking us to find all critical points and find all extreme points. So a critical point is going to occur whenever F prime X equals zero. So let's call this function F of X. And we're going to look for points on this graph, or I should say points on this function, where the derivative is equal to zero, where we can draw a horizontal tangent line. And if I do that, notice that we can draw a horizontal line right here. So let's call this point A.

And we have another point right here, the slope is zero, we're at the bottom of a little hill here, maybe that's one way to think about it. Prime B is equal to zero. And we've got another one up here. F prime, well we'll call that C, F prime C is equal to zero. And we've got one more range over here. We'll call that point D, F point prime D is equal to zero. Notice the slope is important because when the slope is zero, it means if you move left to right, it's not changing. If we change D, if we added to D, we would go up. And if we decrease from D, we would also go up. So we must be at least what we'll learn later is called a local minimum. Now, where are the extreme points on this diagram? To find the extreme points, you want to evaluate all the critical points and evaluate the end points. And so the critical points are A, B, C, and D. Notice that the extreme points while we have a minimum here at B. And we have a maximum here at C. And so being a critical point is a necessary condition for being an interior extreme point. And so we have two extreme points here, B and C. Both are on the interior. Both are interior extreme points.

Here's an illustration of what I had drawn out for you earlier. This is looking at a minimum, if D is going to be a maximum or a minimum, and clearly looking at this diagram, it's a minimum. This slope of G must be zero at point D, we must have G prime D is equal to zero, which of course it is. Because right at this point, we can see that we've got a zero slope. And that's represented as zero slope is a horizontal line. Let's look at some examples together. We've got a question here that says is there an interior extreme point on this diagram? So when we've got X is inside the interval five close to infinity. Infinity always has a round bracket. Is there an interior extreme point here? Well, the answer is no. There is an extreme point at the end point X is equal to five, and that's the minimum, and there is no maximum. As X increases, this function just gets larger and larger. Yes, it's the square root of X, but it's still getting larger. And so there's no maximum extreme point. If you give me a number, I can just take that number and add one and I found a bigger number than you can, you can possibly give me and so there's no real maximum here. It kind of goes, the function sort of goes off to infinity.