Robert_Moptimize3

Sat, 11/6 3:19PM 18:59

SUMMARY KEYWORDS

function, differential calculus, endpoint, interval, differentiable, equal, continuous function, draw, calculus, continuous, discontinuous, derivative, interior, problems, point, left, includes, tangent line, optimization problems, discontinuous function

SPEAKERS

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Hello, everyone. Welcome back to this video on optimization, we're now going to introduce calculus. We're going to use differential calculus to help us solve more complicated optimization problems, to solve more complicated models that reflect more complicated real life situations in the social sciences and business. And to do that, you're going to want to have more sophisticated tools. And one of those tools is differential calculus. So you've already learned something about differential calculus in previous videos, now you're going to apply it to optimization problems. Taking a look at the slides in front of you, you can say the very first line tells you just what I told you. Yeah, differential calculus can help you solve more complicated sophisticated problems, it's going to help you find the most effective use of a resource, maybe that resources time. Or maybe that resource is a quantity of production, maybe that resources and money. Those are the kinds of problems that differential calculus can help you solve. And before we jump right into using calculus, to solve these optimization problems, we want to talk about the conditions under which optimization with calculus, differential calculus can help you. And the situation that is going to be able to help you is one where you're working with a function or a relation that is both continuous and differentiable. And before we go into the calculus, I want to define and explain to you exactly what I mean, when I say continuous and what I mean when I say differentiable.

What we, what we mean by a continuous function is, well, there could be very complicated mathematical definitions of a continuous function and what exactly that would be. But we can use a very simple one from high school. And that simple definition is, a function is continuous if you can draw from one endpoint to the other without lifting your pen from the paper. So I could draw a graph like so X over here and Y over here. Now, if I want to draw a continuous function, I've got a continuous function. Now I'm not, I can draw the function, but I'm not allowed to lift my pen from the paper. So if I drew something like this, that would be a continuous function. If I just drew a straight line, that could be a continuous function. It could be, you know, something very squiggly. And maybe I'm just going to redraw that one, make sure that I make sure that it is a function, or it's clearly a function. And you can have function or continue, I should say, you can have continuous relations which might be something like, say a circle. But notice that's not a function.

Typically, in the social sciences, we like to work with functions, because then for one X there's one Y, but you can see here that this circle is continuous according to our definition that we used up here. Now continuing with this idea of a continuous function, what is a not continuous or what is a discontinuous function? What does it look like? Well, a function is not continuous if it has gaps in it, like holes, or if it has jumps. So again, I can just maybe draw this for us. We've got a diagram here. And I'm going to draw gaps, holes, so I've got a discontinuous function if, say, there's a hole in it like that. And in this case, the function is discontinuous at this point. So if I called this A, function is discontinuous at point A. Now what about jumps, I can draw a diagram for jumps. We might have something that goes like this, right, so this is a jump.

And if I label this B, if we're at B, we're here. But if we're a little bit less than B, as soon as we go below B, we end up down there. And so we say the function is discontinuous at point B. Here is an example of another function that has a discontinuity. So here we have the function one over X. Notice that if we have one over zero, this is undefined. We don't know the answer to one divided by zero. And you can see that as we're approaching from the negative space, as we're going from left to right, our function F of X is going to negative infinity. But when we're going from right to left, as we get closer to X is equal to zero, our expression is going to positive infinity. And so this function is also discontinuous. And it's not even clear, in the slightest degree, how we can reconcile this function around X equals zero, we've got one case it's going to negative infinity at the other, it's going to infinity. So when we have situations like this, this discontinuity, calculus might not be able to find you the most effective use of a resource, might not be able to find you the optimal allocation. It might not be the tool that you want to use.

Now we might have a function that is continuous, but it's not differentiable, or at least it's not differentiable, a point on the interval that we're considering. Now look here, we've got this function F of X, it's an absolute function, there's a Y axis intercept of three, and we take the absolute value, or we're going to add the actual value of X minus two. And so maybe I'll just add in a two here, and three there. Now the problem arises when we are at X two. So just to clear this up a little bit, I'll sort of remove some of these dots. Now, if we are directly, here, we are at the I guess it's a vertex, we are at the bottom of this triangle, sort of triangle like shape, we're at a corner. And we can't be sure if we're going to go up into the right or upward to the left. And depending on whether you go to left or right, we're going to have different slopes, notice that if we go to the right, the slope is going to be one. And if we go to the left, slope is going to be negative one. Another way to illustrate this graphically, is as follows. If you think of a tangent line, if I try to draw a tangent line here, I can do it. So I've drawn a line that is just touching the bottom of this V shape, or what I called previously called the triangle. But there's lots of, there's no unique tangent, there's lots of possible lines that I could draw here, which you're seeing me do again and again, that would satisfy the definition of a tangent which is a straight line that touches the function just once. And since there's no unique tangent line, the derivative does not exist at this point. We can't be sure if you're going to be going up to the right, up to the left. So these sort of, I should say this sort of kink in the function, this kink right here, this kink in our function there, where it changes direction, this function is not differentiable at that point, even though it is continuous. So it this is a continuous function, but it's not differentiable when X is equal to two.

Here is another example, you're looking at this function, it looks continuous, it is continuous, notice I can draw this without lifting my pen. But it's not differentiable. And it might not be obvious to you looking at this, that it's not differentiable. It seems like wherever we draw our pen, we could, we looking at this, that it's not differentiable. It seems like wherever we draw our pen, we could, we could, or I should say, it looks like we can draw a tangent line at any point on here, at least there's no kink like we saw in the previous example. But if we consider the point, X is equal to three, notice that the function is looking very vertical at this point. And if we take the derivative, right, if G of X is equal to X minus three to the power of 1/3, then this implies that G prime X is equal to 1/3 X minus three to the power of negative two over three times one if we take the derivative of the inside. And where does the problem come in? Well, I could rewrite this as X minus three to the power of two thirds, three in front of there and one up there. And then if we evaluate this derivative, whoops, if we evaluate it, when X is equal to three, we end up with one over three times zero, two over three, and this is just going to be equal to one over zero. And that of course is undefined. So G prime three is undefined.

You may have heard me mention an interval. An interval is something that you've already seen before, we saw that at the beginning of this course, all it is, is a continuous set of real numbers with two endpoints and an interior. Everything between those two endpoints is on the interior of the interval. Interval notation includes the following general forms that you've seen using, well, as it says interval notation, there's really just four possibilities. Remember, this is open. This is an open set. And this is a closed set. And then there's variations of the two where one side it's open, the other side it's closed. Now remember with a closed set, a closed set includes A and B. Remember, A and B are just some real numbers, could be the square root of two, could be square root of four, with the condition that oh, it looks like there's a little bit of a typo here. This should be A is less than B, where A is less than B, B should be greater than a on this notation. You probably noticed that but it's written wrong. Now the closed set with the square brackets that includes A and B, while the open set excludes or does not include A and B.

Now why is this important at this stage in the course for optimization? Well, it turns out that extreme points are often located on or inside the interior of an interval. That's probably not much of a surprise to you. And it's, it's important, because we want to consider the endpoints. But it would be strange if all of our problems were solved at endpoints. Now, when we're talking about the interior of the interval, so we might have a closed interval sum, like this one here, this would be closed. But the interior of that interval is, in some sense, always open, right? So it can't include A is the endpoint, B is an endpoint, A is a endpoint, B is a endpoint. And the interior, the interval, are all the values between A and B, but not including A and B itself. So here is an example with a diagram. We've got two endpoints, we've got an endpoint A, and we've got an endpoint B. Notice I don't differentiate between a starting and ending point, we just call them both endpoints. It just depends whether you're going from the right to the left or left to the right, or whether you're increasing or decreasing. And so we've got two endpoints here. Everything else along this curve are everything else in there, that is the interior of the interval. So anything greater than A and less than B, falls on that interval, and the interval itself is always going to be continuous by definition.

Here's an example for you just at the end of this video. The question is asking you to label the endpoints and label the extreme points. So take a look at this, where are the endpoints? And where are the extreme points and label them. Now we've got one endpoint here. So I'll call that an end point. We've got another endpoint over here. And that solves the first part of the question. The next part, label the extreme points. While we've got one extreme point here, we've got a minimum, and we've got maximum. So here we have two extreme points. We should always have some sort of a, two extreme points of some kind on the interval, a maximum and a minimum at the extreme point.

So this question is really just about terminology. And make sure that you understand what these words that I'm using. Next video, we're going to get into calculus, actually using calculus to help us find these extreme points.