Robert_Moptimize1_march27

Tue, 3/29 3:12PM 11:48

SUMMARY KEYWORDS

domain, function, maximum, extreme, differential calculus, equal, optimization, answer, unemployment, social sciences, infinity, maximize, dotted line, economics, module, oil extraction, defined, minimum wage, plug, squared

SPEAKERS

Robert McKeown

Hello and welcome. In this module, we're going to be looking at optimization. Optimization is a really valuable set of tools that have many applications to economics, finance, and the social sciences.

So how can we use optimization and differential calculus to answer interesting questions? Well, let's take a look at the kinds of questions that we can answer using optimization. If we're in business, we can use the techniques that you're going to learn in this module to find the profit maximizing rate of oil extraction from an oil well, or the maximum flow of oil from one location to another, given existing pipelines. Or maybe you're trying to evaluate whether to build a new pipeline or not. Or you might want to evaluate how much you can move on on rail if you don't like pipelines. In economics, you might ask yourself, would a higher minimum wage increase unemployment. If the minimum wage is very, very low, then one might expect that a increase in the minimum wage won't decrease, or I should say, won't increase unemployment very much, maybe not at all.

On the other hand, if the minimum wage is very high, you might expect that an increase in minimum wage is going to create more unemployment. And you might be interested in political science, their applications in political science. Here's an example that I found, which is, you might look at how many lawn signs it would take to maximize a candidate's chance to win an election.

So these are the kinds of problems that we want to deal with, the classic one is a to maximize a firm's profit, or to minimize a firm's cost. But there are many other applications in the social sciences as well. And I look forward to sharing some of them with you. As we work through this module on different optimization techniques. We're going to get started optimizing by defining what an extreme point is. Once we've completed defining what an extreme point is, we're going to look at some of its properties. And then in a future video, we will apply differential calculus to finding extreme points in different situations.

Now, what is an extreme point? An extreme point is a technical term for where a function reaches its largest value, or where it reaches its smallest value. That means an extreme point can be either a maximum, or it can be a minimum, right? That's, which is exactly what you would think, it'd be its largest value, or its smallest value.

Now, in order to find this maximum or minimum, this extreme point, you first need to know the domain of the function that you're working with. Now, here's a function. And as a review, why don't you find the domain D of this function F of X. Assume that X can take on any, any value between negative infinity and infinity.

Which of those X's is going to be in the domain? For X to be in the domain D of this function F of X, it must be the case that if we sub in a value for X, we choose a value for X, F of X has to be equal to some real number.

Or we can say that F of X must be defined, cannot be undefined. So for example, here, suppose we say well let X be equal to negative one, we get F of negative one is equal to the square root of negative one plus one over negative one. Well, no problem dividing, but the square root of negative one is undefined.

So that doesn't help us very much. We can see here that X equals negative one. This is not in the domain. Let's try X equals zero.

If we say F of zero, that's going to be equal to the square root of zero plus one over zero, which is just square root of zero is just zero, zero times zero equals zero. So we have one divided by zero, and this is also undefined. We don't know what the answer is to one divided by zero, we don't know. So we can say here that the domain capital D, of F of X is zero, must be less than X. So X has to be positive. And X can be any large number you want anything up to, but not including infinity. Remember, infinity is not a number. It's a concept - a concept of a very large number.

Now, if you plug in a number here that obeys this range, you're going to be okay, you're going to get a defined result, you're going to get end up with a real number, F of X is going to be equal to a real number. You can try that with 0.1, you can plug in 2, you can plug in 5. And if you plug that into your calculator properly, according to this expression formula up here, you should get an answer, you'll get a real number for the answer.

Now, here's our definition of an extreme point. Once again, an extreme point is the maximum or minimum value of F of X over the domain D. So I could draw a function X over here. And if I drew some function like that, and I called that F of X, you can see that there is an extreme point maximum up there. And there's another extreme point, this time a minimum down here.

And here, we would have the domain going from this extreme point there, to this extreme point up there. Now, here's a function for you. Can you just by looking at this function? Can you tell me? Does it have a maximum? Does it have a minimum? Now you might be able to tell whether there's a maximum or minimum just by looking at the function itself. Maybe in your head, you're plugging in some values, and you're noticing what happens to the value of that function for different X values. And that's great. Another way to answer this question would be to use a graph, a diagram. Now I've gone ahead and graphed the function. And you can see it here on the slide in front of you. Notice that when X is equal to two, if I were to just do this dotted line I'm walking straight up a vertical dotted line, that there is a maximum. We've got an extreme point, this is a max. And the maximum is occurring when X is equal to two. Notice when X is equal to two, we've got three minus zero squared, which is just equal to three. If we had F of four, we would get three minus four minus two squared, we get three minus four, we end up with negative one.

Now in general, we can write the following, if C is an element of the domain. So if C is in D and C is a maximum point for the function F, then this implies F of X is less than or equal to F of C for all X in the domain.

Now, all this is really saying is if C is a max. So let's draw this on the diagram, C is a maximum, and there it is, that's a maximum, then any other X must give us an F of X less than F of C. So if I pick an X, say here, pick X one here, and I walk up to the function, I get F of X one and notice that it's less than FC. If I pick some X square over here, wherever I pick it, wherever I pick an X, it's never going to be as high as this FC up here.