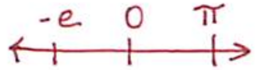
Euclidean (or Coordinate) Plane $\mathbb{R}^2$

Talked about:

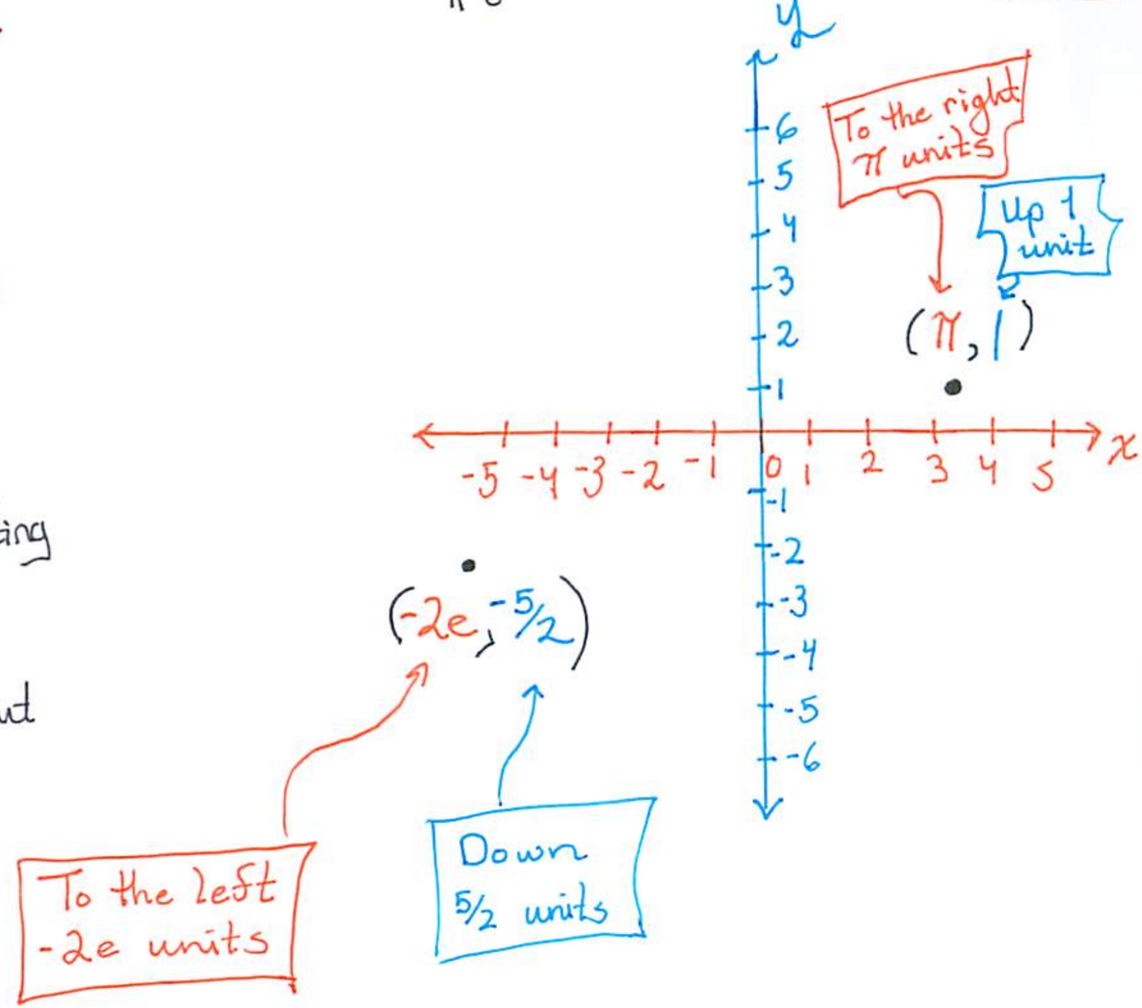
real numbers $\mathbb{R}$ and real line 

New

Sometimes want to keep track of 2 numbers at a time and the relationship between them

Euclidean (also called Coordinate) Plane

$\mathbb{R}^2$



* By drawing $(\pi, 1)$, we could be indicating "If $x=\pi$, then $y=1$"

This will be relevant when we talk about graphs of functions!

The Points of a Line Are a Set in $\mathbb{R}^2$

Can we write all points on the line $y = 3x - 1$, i.e. all solutions to the equation $y = 3x - 1$ as a set?

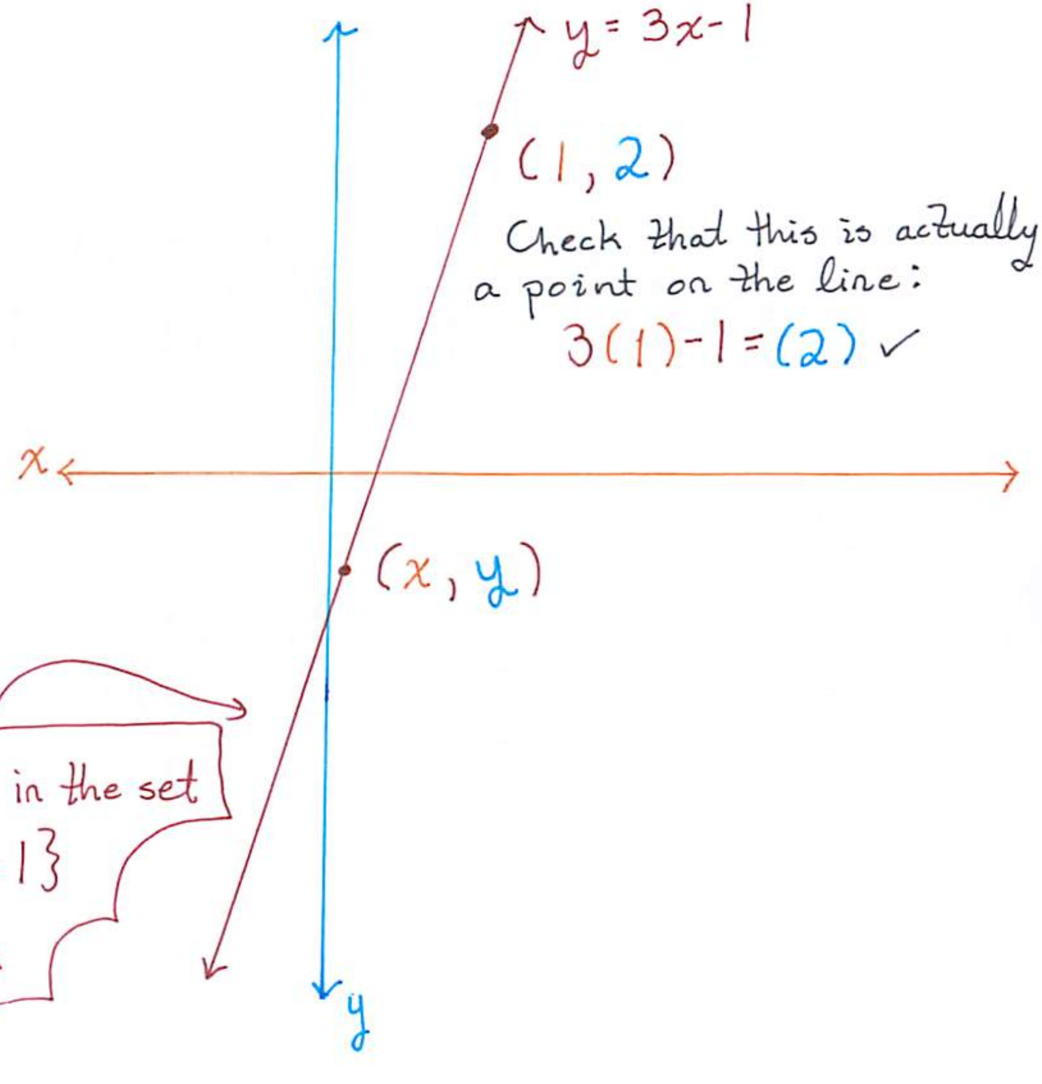
"set of all"

$$\{\underbrace{(x, y) \in \mathbb{R}^2}_{\text{points } (x,y) \text{ in } \mathbb{R}^2} \mid y = 3x - 1\}$$

"satisfying"

[the set of all points (x,y) in $\mathbb{R}^2$ satisfying $y = 3x - 1$]

What do these points really look like?



Check that this is actually a point on the line:

$3(1) - 1 = (2)$ ✓

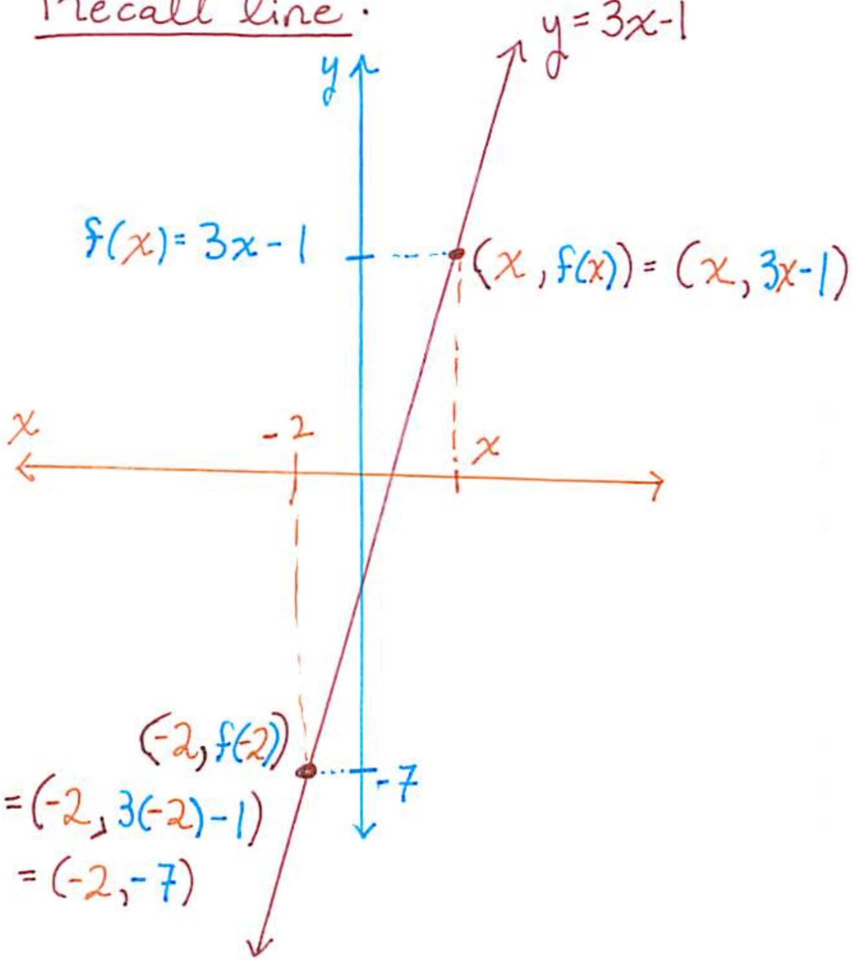
This line contains all points in the set

$$\{(x, y) \in \mathbb{R}^2 \mid y = 3x - 1\}$$

i.e. all points (x, y) satisfying the equation $y = 3x - 1$

Graphs of Lines Are Graphs of Linear Functions

Recall line:



Remark: This function $f(x)$ is a $\boxed{\text{linear function}}$ since it has the form

$$f(x) = ax + b$$

Linear because its graph looks like the line:

$$y = ax + b$$

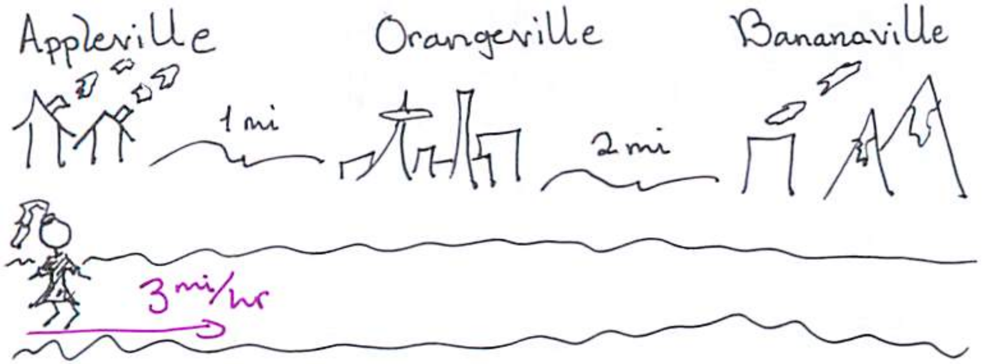
This line is also the $\boxed{\text{graph}}$ of the function $f(x) = 3x - 1$

(when x is the input, y is the function output $f(x)$)

Linear Word Problem Example

Set-Up:

Appleville, Orangeville, and Bananaville lie on the same road with distances as indicated in the image. Most mornings I walk the 2 miles from Orangeville to Bananaville. If I instead had to start 1 mile back in Appleville, and walk at a rate of 3 mph, how long will it take me to reach Bananaville?



Generalizable / Flexible Approach

The situation can be represented by

$\boxed{y = 3x - 1}$ where $\begin{cases} x = \text{time input (time passed)} \\ y = \text{distance past Orangeville} \end{cases}$

So if want to be 2 miles past Orangeville, substitute $\boxed{y=2}$ into $y = 3x - 1$:

$2 = 3x - 1$, now solve for time input x

Added 1 to each side

$2 + 1 = 3x$

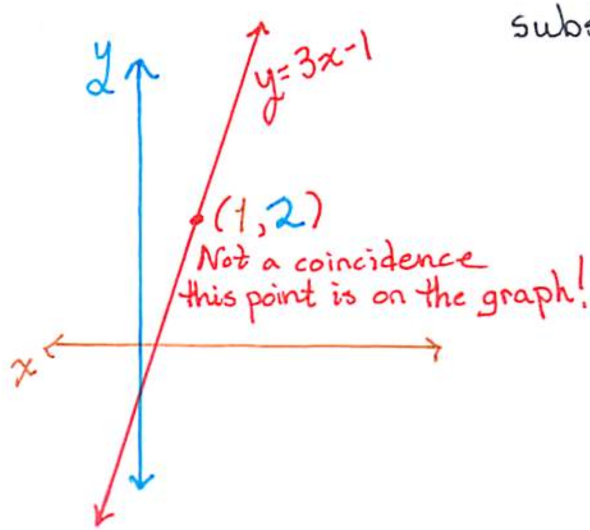
Divide both sides by 3

$1 = x$,

i.e. $\boxed{x = 1\text{ hr}}$

Intuitive Answer:

Travelling 3 miles at $3\ \text{mi}/\text{hr}$ takes 1 hour.



Changing an Equation to a Function and What Happens When Change Variables

The example represented a situation with $\boxed{y = 3x - 1}$ where

$$\begin{cases} x = \text{time input (time that's passed)} \\ y = \text{distance past Orangeville} \end{cases}$$

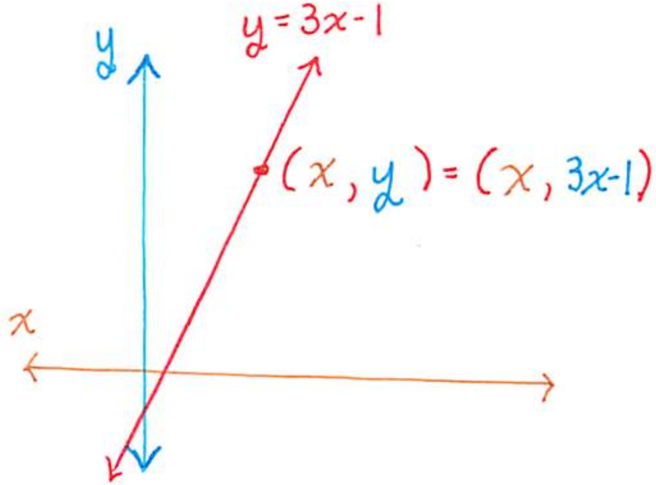
Can write as a function $\underbrace{f}_{\text{Distance past Orangeville}}(\underbrace{x}_{\text{Time input}}) = 3x - 1$

Often like notation hinting at meaning:

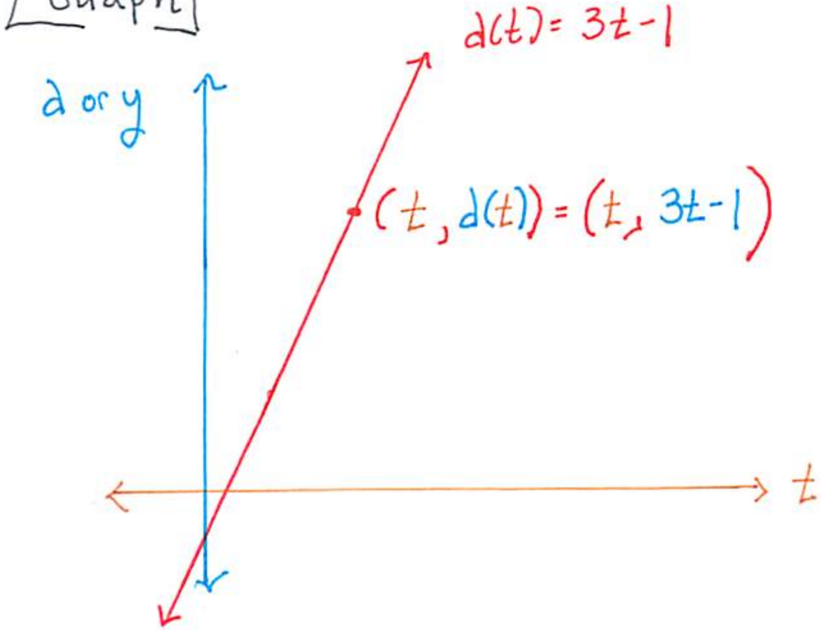
t = time input (time passed)

$d(t) = 3t - 1$ distance past Orangeville (after t hours)

Graph Before



New Graph



Exactly the same except $x \leftrightarrow t$, $y \leftrightarrow d(t)$

Graph of Function Second Example

Q: What does the graph of the function

$$\boxed{g(t) = 3t^2 + 1}$$

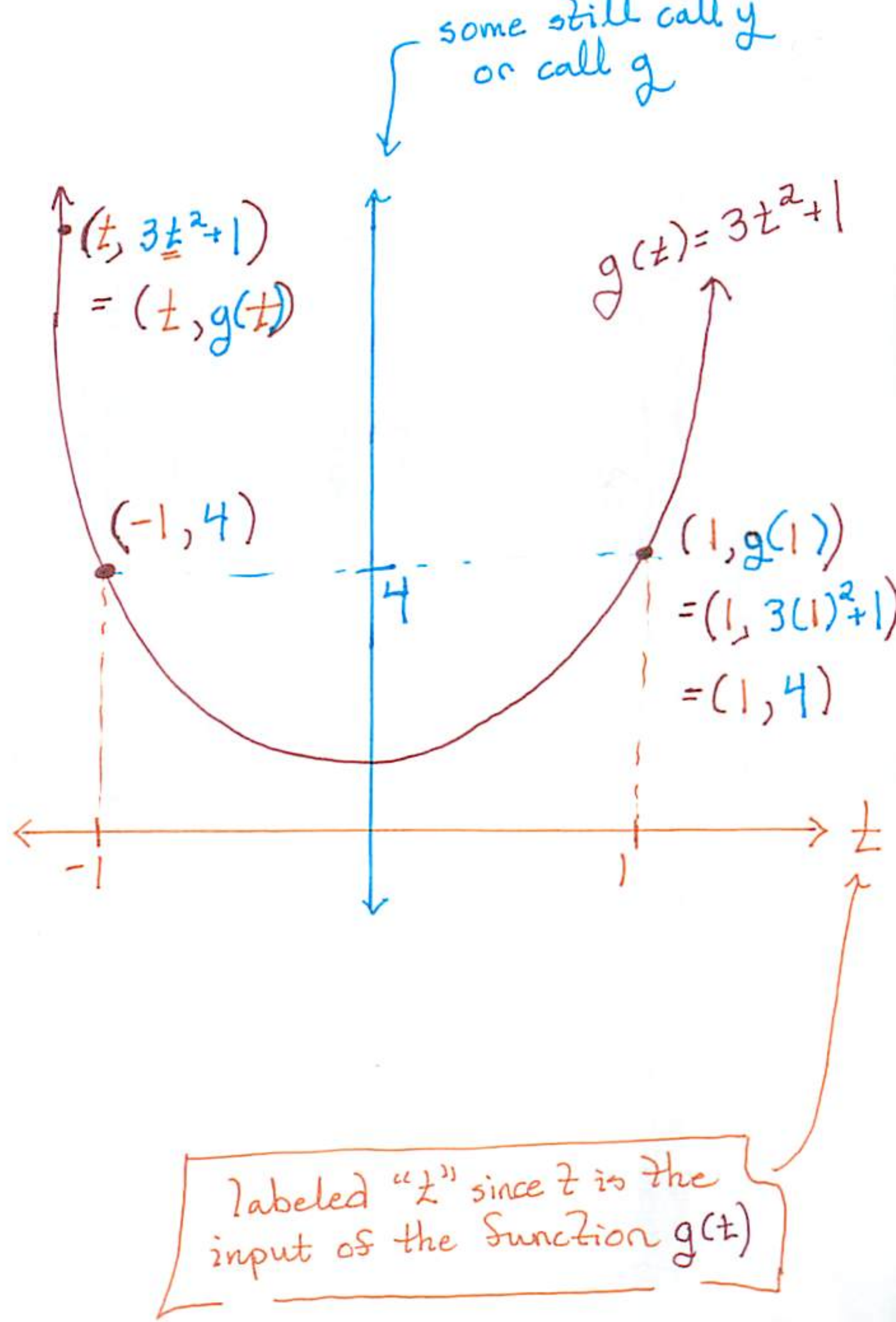
Breaking It Down

① Horizontal axis labeled by the input variable (here "t")

② Can't always work backward from output to find a single input

(4 came from both 1 and -1 as the inputs, i.e. t-values)

This is because g is not injective, i.e. fails the horizontal line test



Example Connecting Word Problems, Functions, Graphs

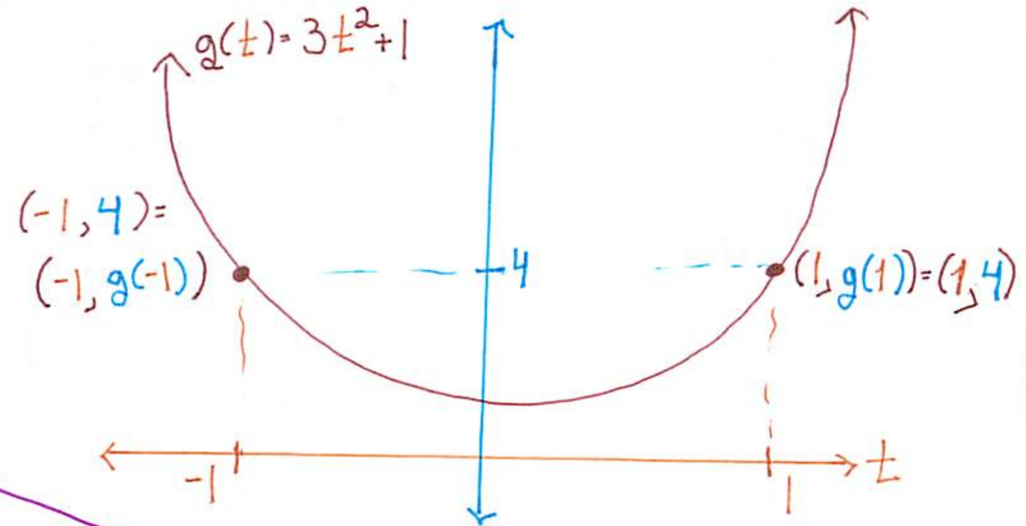
Task: In the land of Pfaffian lives a blue-tailed rodent whose tail can't survive extreme temperatures. The dependence of the number of tails falling off per day on the temperature is given by $g(t) = 3t^2 + 1$, where t is the temperature in celcius.

temperature in celcius

① How many tails will fall off per day when the temperature is $1°C$? $t = 1°C$ $g(1) = ?$

② Is there another temperature at which that same number of tails will fall off in a day?

Another temperature t with same Tails/day off?



$g(t) = \underbrace{g(1)}_{4}$

$g(t) = 4$

$\boxed{3t^2 + 1 = 4}$

① $g(1) = \underbrace{3(\overbrace{\ \ }^{1})^2}_{3} + 1$

$= \boxed{4 \text{ tails/day}}$

② $3t^2 + 1 = 4$ Subtract 1 from both sides

$3t^2 = 3$ divide by 3

$t^2 = 1$

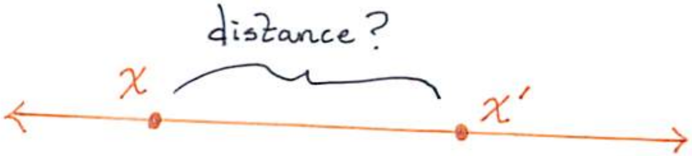
$t = \pm 1$

$\boxed{\text{Answer: } t = -1°C}$

$\boxed{\text{Distance in } \mathbb{R}^2}$

Q: Given 2 points (x, y) and (x', y') in $\mathbb{R}^2$, how do we find the distance between them?

Warm-Up: In $\mathbb{R}$?

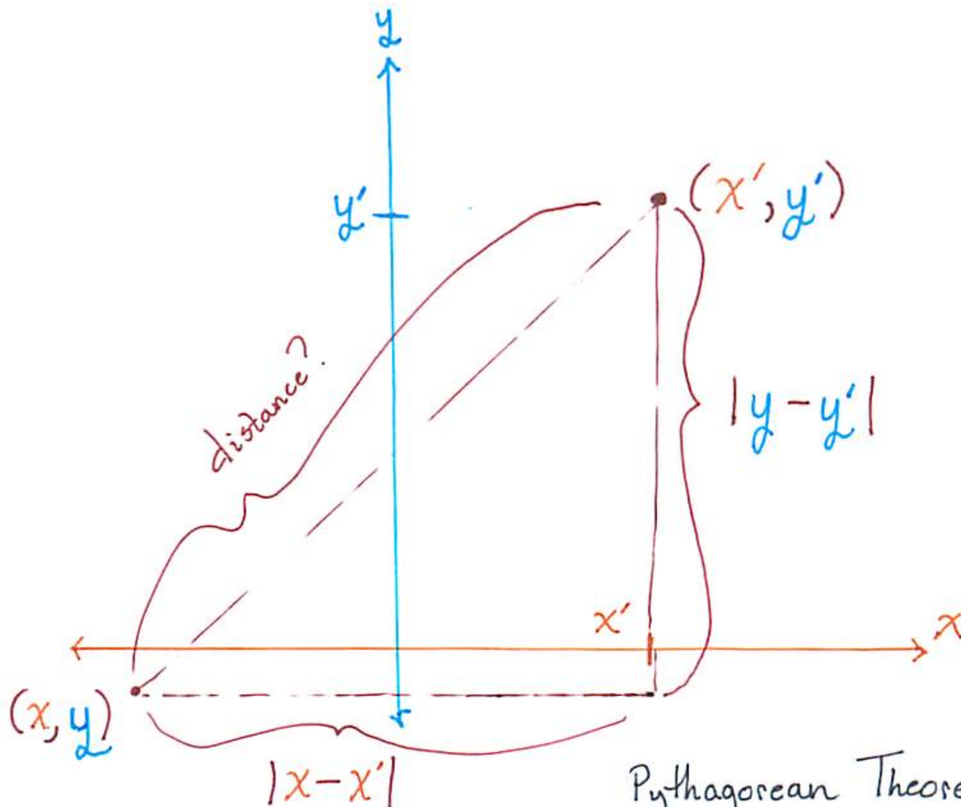


A: Distance between x and x' is

$$|x - x'| = |x' - x|$$

(order doesn't matter,
but we do need to take the absolute value
to make the distance positive)

In $\mathbb{R}^2$:

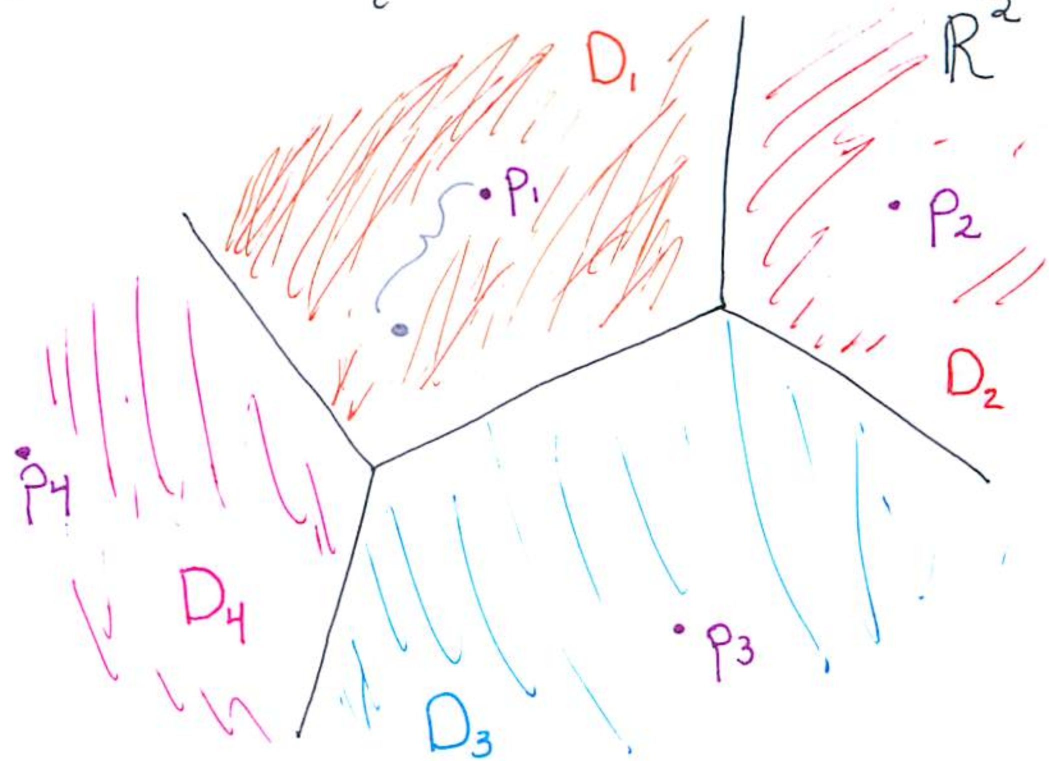


Pythagorean Theorem!

$$(\text{Distance})^2 = (|x - x'|)^2 + (|y - y'|)^2$$

$$\boxed{\text{Distance} = \left((x - x')^2 + (y - y')^2\right)^{1/2}}$$

Fun Example: Voronoi Diagrams



Some Applications:

- Areas served by businesses/services
- Geography examples
- Sport analytics (area controlled by a player)

What's a Voronoi diagram?

Input: Set of points in $\{p_1, \ldots, p_k\}$ in $\mathbb{R}^2$

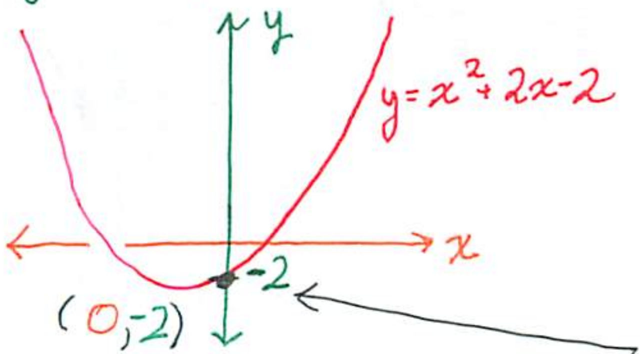
Output: Voronoi region D_j is all points in $\mathbb{R}^2$ closer to p_j than to any other p_i

x- Intercepts and y-Intercepts

y-intercept:

where graph intersects y-axis
(find by subbing $x=0$ into $f(x)$)

Ex: y-intercept for $f(x)=x^2+2x-2$



Sub $x=0$ into $f(x)=x^2+2x-2$:

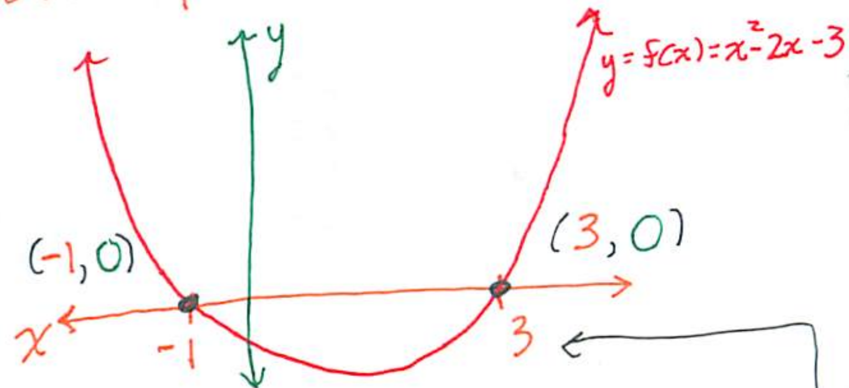
$f(0)=(0)^2+2(0)-2=-2$

$\boxed{\text{y-intercept is } y=-2}$

x-intercepts

where graph intersects x-axis
(find by setting $f(x)=0$ & solving for x)

Ex: x-intercept(s) for $f(x)=x^2-2x-3$



Solve for x with $f(x)=0$ by factoring:

$x^2-2x-3=0$
$(x+1)(x-3)=0$

$\boxed{\text{x-intercepts are } x=-1,3}$