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SUMMARY KEYWORDS

derivative, chain rule, multiply, squared, sine, put, respect, intermediary step, parentheses, rule, product, cosine, simplifying, part, write, leave, x times cosine, equal, function, carefully

SPEAKERS

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Welcome. In this lecture again, I'm going to kind of go through a combined product and chain rule. Again, really, the moral of the story is kind of taking things one step at a time and making sure you're using the right rules where they need to be used. So this is example two. And we're going to take, so we want to take the derivative with respect to X . So the derivative with respect to, and then this is kind of a variable X of, and then our function is going to be, so we have the square root of X squared plus 1, square root of X squared plus 1, and then we're going to multiply that by E to the sine of X . Right? So you can see how we have to combine both right? There's a product, we're multiplying this time this. But for each of these parts, we have to use a chain rule, okay. So the first thing that we'll do is to apply the product rule, and then we'll see all the places that we're going to need to use the chain rule. Okay, so plan of attack, maybe I'll actually kind of write that. So plan of attack, our plan of attack will be because the chain rule is inside of each of these things that we're going to be multiplying, kind of a nice plan of attack is to one, apply the product rule, but not actually kind of work out the derivatives. So we're going to apply the product rule. And then two, we're going to, in taking the derivatives, right, be careful to use the chain rule. Okay, so in taking the derivatives, careful to use the chain rule, and maybe I'll do those on the side. In taking derivatives, carefully use the chain rule on the side. So carefully use the chain rule. And we're going to kind of do that on the side over here.

Okay, so here we go, we know what we need to do. So remember we can take, so I can take this part and leave this here. In fact I'm just going to leave that color to indicate that I didn't, I'm not taking any derivative, so X squared plus 1. And then I need to take the derivative with respect to X of this part. So I know I'm going to need to multiply that by $D DX$. So I haven't taken this derivative yet, so it's good to keep track of this part over here. So I have E to the sine of X , I haven't taken any derivatives yet. So we'll just kind of keep that green for now to kind of indicate this is still a function that we haven't taken the derivative of. And then we need to add that to, and now we just write this on the outside. So E to the sine of X . We didn't take any derivative there, so we don't have to worry about the chain rule there, just like any other one. And then we're going to take the derivative with respect to X of, and then in parentheses, again, we're going to have, so now we have this 1. So this is the square root of X squared plus 1. Okay, so looking at this, we have two derivatives to take. And each one of them is going to require that we use the chain rule, okay? And in using the chain rule for

this one, we know that this is going to be so much easier to work with, if I write this as $X^2 + 1$ to the $1/2$. Okay? So let's just kind of go ahead and do this. So the chain rule here, so let's, let's work out this derivative.

So I have the derivative with respect to X . Let's put a line here. Let's get ourselves organized a little bit. Okay. So when I'm taking the derivative with respect to X of, and I have E to the sine X , right, then I can keep track of this. So my outside not going to use all the colors that we would normally use, but my outside is going to be E to the U . And then my inside is going to be sine of X . Okay, so what happens when I take the derivative of the outside I still get E to the U . Okay, so I take the derivative of the outside and I'm going to get E to the U . And then, right, I need to multiply that by the derivative with respect to X , right? So I've got the derivative with respect to U , actually, maybe let's just kind of carefully write this out, so we don't make any mistakes. So we want to take the derivative with respect to, add in a color here, got a lot of things in my hand, just put them down. You might probably know by now when I have a lot of things in my hand, I tend to drop them. So we have the derivative with respect to U of E to the U . And then I want to multiply that by the derivative with respect to X of this sine of X .

Okay, the derivative with respect to U of E to the U , this is just E to the U . But we'll leave it red so that we remember that we're going to have to put back in our, our inside. So that's E to the U . And then we're going to multiply that by, and then this is going to be cosine of X . So that's cosine of X . Okay, but we have to handle this U . This U is actually this part right here. So this is just going to equal E to the, and then I have a sine of X times and then times my cosine of X . Okay, so I took this derivative, so that's the first thing I'm going to put in over here. I, this is, right so this is the square root of $X^2 + 1$ times, and then I'm going to put this whole thing, right, into here. So I have E to the sine of X times cosine of X . Okay, so I took this derivative. So I'm done with that one, now I have to do this one. I'll just write the plus E to the sine of X . But now I just have this whole derivative I'm going to have to put in, so I'm going to put some big parentheses here, that will be for that derivative.

Okay, so let's go ahead and use the chain rule on this one. So if I want to take the derivative with respect to X , so the derivative with respect to X of and then my function here is going to be this. So I have $X^2 + 1$ to the $1/2$. So $X^2 + 1$ to the $1/2$. And then, so my outside, right, is just going to be, so my outside here is going to be, so I have it's, it's U to the $1/2$. And then my inside is this U , right, so my inside is going to be, the inside is $X^2 + 1$. Okay, and so the derivative of my outside, right, is going to be $1/2 U$ to the minus $1/2$. Okay, so I'm going to get right, so I need my derivative with respect to U of, and then this is actually this red U , so this is U to the $1/2$. And then I need to multiply that by my derivative with respect to X of, and then we have $X^2 + 1$. Okay, so this is going to equal, well, what's that derivative? So that derivative is going to give me $1/2$ and then I'm going to keep that U there, so that I don't forget that I haven't yet plugged that in, and this is to the minus $1/2$. And then I'm going to multiply that by and the derivative of this is just $2X$. And now we have to put back in our U . So this is going to equal $1/2$, and then our U is going to be $X^2 + 1$. And that has to then go to the minus $1/2$ and then that needs to be multiplied by $2X$. And this could definitely be simplified because I have a 2 and a half and so this is just going to equal X times and then I have $X^2 + 1$ to the minus $1/2$. Okay, so now I'm able to put this here so I have X times $X^2 + 1$ to the minus $1/2$, and I gave myself a little bit too much room there. Okay. And there could be some like cleaning up or simplifying of this, but we're actually done right? So we can just kind of box that.

Okay, so to talk this through a little bit, right, in the first part, we're just using the product rule, but we're doing it very carefully. And this is why to kind of do that intermediary step where I'm saying, you know, I'm leaving this alone, I'm taking the derivative of this plus leaving that alone taking the derivative of this, because then you're like, very consciously thinking about the fact that I need to take this derivative, and I need to take this derivative, and you have it there so that when you find it, when you do it in your side work, you know exactly where it goes. In fact, it could help then to do this, leave myself with some parentheses, plus this, leave myself with some parentheses, so when I'm done, I remember exactly where I'm supposed to put these. So when I do these, I just have to use the chain rule. I have my outside function in my inside function. Again, just like with the product rule, it's good to kind of do these intermediary steps so that we don't make any mistakes and we realize things like oh, I need to put back in my sine of X , and where I'm taking the derivative or not and so on. Okay? So, I hope that made some sense and I will see you in the next lecture.