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SUMMARY KEYWORDS

derivative, equal, du dx, function, dy, 3x, replace, multiply, respect, write, variable, identify, du, sine, cosine, carefully, seventh, chain rule, stick, squared

SPEAKERS

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Welcome. In this lecture, we're going to go through the chain rule. We're going to go through it with an example. The chain rule can be a little tricky at first, but then you get the hang of it. I think one of the most important things is to kind of go carefully and slowly. Because what ends up happening is that students, you know, when they don't actually go through and carefully identify the inside function, and maybe even change variables and so on, they kind of miss a little bit, some of the steps of what's going on. So it's kind of one of these things that you need to be very careful about it, but then you get the hang of it. Okay, so let me show you. So what, how does the chain rule work? So we are kind of in a setup, where we're going to have, so we're going to be wanting to take the derivative. And I'm going to take the derivative with respect to, I'm going to kind of color code this all the way through, I'm going to hope that that helps a little bit. So the derivative with respect to X , and we're going to have an outside function and an inside function. Okay, so what do we mean by outside function inside function? We're doing a composition of two functions. We'll kind of go through this, so you'll see what I mean, hopefully. So we have this, right, first, you apply G , and then you apply F . So G is on the inside, and F is on the outside. But let me just also kind of write this in a little bit different notation, that might help a little bit, is that we're looking at a situation where we have this F of, and I'm going to replace G with U . Even though it's a G here, I feel like it helps, and then an X . Okay, so inside function outside function, and so how do we take the derivative of this? So we're going to do this kind of setup here, we're going to have, as I said, we're going to kind of want to do this notational thing. So we're going to have that we're going to have on the outside, I'm going to write for Y equals F of U .

Okay, so this is like my outer function. Then I'll show you an example, and then I'm going to have the my u is going to equal G of X . Okay, so this is in this here is going to be my inner function. Okay, and so what we're going to look at here, so this is going to equal, so let's kind of write this here is I have, so this is D , and then I have my outside, so DY of DX , because I'm taking the derivative with respect to X . So the derivative of the whole thing, this is the same thing got going on here. And this is going to equal, so first I take the derivative of the outside with respect to the inside, that's going to sound a little weird, but this is why I change notation, so that I could write this as DY and then DU . And then I'm going to multiply that by, so I took the derivative of the outside with respect to the inside, and then I'm going to take the derivative of the inside with respect to the inside variable. Okay? So

derivative of the outside with respect to inside, derivative of the inside with respect to its variable. Okay? So this is the same thing as this, and I'm going to show you in an example. Okay? So here's my example.

Is, I want to look at, so I'm going to take the derivative with respect to X of cosine of, and, you know, this is color coded now, because in the first example, I want to make it really clear, $3X$ squared plus 1. Okay, and the first thing I check is that I am taking the derivative of the correct variable. Okay, great. Now, what I want to do is the first thing you want to do is kind of identify the inside. So here's my inside, because it's inside of the outside function. Okay? So this is kind of like the outside. So I'm going to want to set that to be my U . So here's my inner, or my outside in this particular circumstance, is going to be, so I have U of, and that's of the variable X , is going to equal and I have $3X$ squared plus 1. Okay, so that's my inner function. Okay, so I know that I'm going to have to take the derivative of that. So let's just kind of do that over here. So that's like, I want the derivative with respect to X , right? Because I know I need DU/DX . So here's my U . So let's go ahead and write so this is my D , my DU/DX is going to equal, and then I've got here, so this is the derivative with respect to X of this function here. So we have $3X$ squared plus 1. And this is a polynomial. So we know how to do this. And so what we're going to get here is that this is going to equal $21X$. Two.

Oh, I want this to be to the seventh. I was wondering why I was getting something funny over there. I don't want this to be, I want this actually to be to the seventh power. Okay, let's do that. I know I squared it, I instead want to be the seventh power. So that this is $21X$ to the sixth power, sixth power. Okay. Okay, so that's what I get here. Now, let's do for, so once we've identified the outer, you can look at this and replace that with U . Okay, so my outside function is then cosine of U . So I mean, once we have the inner, we can do this. So now we can do the outer. Right? Because all I do is, I replaced this inside with U . Okay? So I'm going to get my outer is going to be, so this is my F of U is going to equal my cosine of U .

I'm going to put a little line here to make sure that things don't kind of melt too much into each other. Okay, so now we also know that we're going to need to multiply so we took this, this is our DU/DX , we know what that is, we know we're going to have to find DY/DU . So we have to take this derivative here. Okay. So that's going to give me so my DY/DU , right, because this is also my mine, this is also my Y . So my DY/DU . So I'm taking the derivative, this is why I replaced this with a U so that when I took this derivative, it was something I knew how to do. Okay, and so this is going to equal the derivative with respect to U of, and then we have cosine of U .

It's good at this point to check that these actually do indeed match up, which they do. But it's good to check this because when you're going through this whole thing, it may get a little bit confusing, okay, you know that are you changing all the variables to how you need to and so on, okay? So make sure that these actually match up, which they do. Okay? So then I can just take this derivative, which is minus sine of U . Okay, I leave this green because it's going to be really important that that is in fact something that needs to be replaced. Okay? So now I'm ready to actually do this because I know that this is just equal to well it's DY/DU times DU/DX , so this is DY/DU times DU/DX .

Okay, but I've already done these over there so I can just kind of fill those in from over here. So I'm

Okay, but I've already done these over there so I can just kind of fill those in from over there. So I'm going to get that this is going to equal, do this on this line. Okay, so I have this minus sine of U times and then I have this $21X$ to the sixth. Okay, I'm not done yet. Why am I not done yet? So this is something to think about for a moment. Okay, and I'm not done yet because I still have a U, and I need for my final answer to be just in terms of X. Okay, so this is really key. So at this step I'm not done yet, so not until we have that this U disappears, so not until U disappears and everything right is in terms of X, and everything's in terms of X.

So not until everything's in terms of X. Okay, and we still have a U there, right? We still have this U here, but all we need to do is just replace it with our U from there. Okay, so this is just going to equal, so we have a minus sign of, and now we just stick this in. So I know what it is, it's $3X$ to the seventh plus 1. And then I want to multiply that by $21X$ to the sixth. Okay, and then you can just kind of clean this up, as I've said, kind of before, we usually like to move stuff to the front of sine or cosine, so that it's clear what's inside the sine function and what's not. So this is going to equal minus $21X$ to the six times sine of, and then we are $3X$ to the seventh plus 1.

Okay, so, right, so what do we do? So this is this procedure, you need this when you have an outside and an inside function. And we'll do some examples to help us identify the outside and the inside function. But in this kind of circumstance, the first thing that you want to do, right, and you're going to get the derivative of the outside with respect to the inside times the derivative of the inside with respect to your variable. Okay? So the first thing usually, it's easiest to identify the inside function first. So once you know the inside function, then you can replace that with U everywhere you see it, and you have your outside function. Okay? And then I find it helpful to actually kind of very carefully do this with all the variables so that then you can just stick these in afterwards, right? If I know what DU is, I know what DU/DX is, I can just replace those like that, and then have the final step of sticking in U to make sure that everything's in terms of X, okay? So what did I do? So I had to take the derivative of the outside with respect to inside, once I've written it as a function like this. So this is the inside, that's my U, then I can just take this derivative straight out like I'm used to, keeping that variable the same. And I also wanna take the derivative of the inside with respect to its variable. And that one's kind of the easiest one for us to see. And as I said, you just multiply those by each other, and you don't forget to sub in for your U. And we'll do some practice with identifying outside inside functions because I know it's a little tricky. Okay, so, hope that made some sense, and I'll see you in the next lecture.