

PfaffModule5L05

Wed, 2/2 4:34PM 7:53

SUMMARY KEYWORDS

delta, derivative, product, rectangle, change, small, bigger, function, delta f , tiny amounts, length, multiply, $f g$, area, rule, bit, width, completion, height, equal

SPEAKERS

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Welcome. In this lecture, we're going to go through where the product rule comes from. The reason why it's not as much the you won't be able to use it unless you understand this. The reason I really do this, in some sense is because I wish that somebody had shown me this, I feel like for completion, it's good to have seen this. Okay, so let's go ahead and do this. So I have what's going on here. So the derivative measures how fast a function changes. So this is kind of the first piece of this. So the derivative measures, so the derivative measures how fast a function changes.

How fast a function is going to change, so changes. Okay? Well, so for rectangles, so the next thing that we know is that for a rectangle with side lengths F and G , so for a rectangle with side lengths. So we're going to have F and G , this is our two functions here. So we have these functions that are, so we have F is one function, and the next function is G . Okay, then we're going to have, okay, that the area is, the area is going to equal just the product of these, right, F times G . So let's actually just start out with this. I have, right, I have a rectangle, right, and this length is F and this length is G , and then the area is F times G .

So let's start with that. Now, if we make F and G , so if we make this F and G , so if we make F , and G , just a little bit, so a little, and this is here, I mean, a delta, like a change, you know, this is like, change in, oh maybe I'll make that green too, a change in F and change in G . So these are the amounts that we're going to make these bigger, so just a little bit bigger. Then the question would become, how much bigger is the new area? So how much bigger is the new area? Right, which we know to be, so what is this new area? Well, it's going to be F plus this change, we made it just a little bit bigger, like a change in F amount bigger. So this is what it newly is once we've added that small amount bigger, but we're still, then we're going to multiply that by G plus, we made G just a little bit bigger. So now it's G plus delta G . Okay, so let's do this bigger in our picture, too. So I'm going to make F a little bigger. So we're going to make F bigger by delta F . And then I'm going to make, that was supposed to be a straight line up, but it's going to be okay. And then we're going to make G bigger by delta G . Okay, and now we get a new rectangle.

Okay and let's make some observations on here. Right? So we have what is the area of this one un

Okay, and let's make some observations on here, right? So we have what is the area of this one up here? The area of this one is G , right, it's this length times this width, the height. So this is this big. And this one is this height and this width, so we get F times ΔG . And what about this little part up here? Well, this little part up here is actually ΔF times ΔG . And then the point is going to be that this is, is like second order small. Okay? It's much, much smaller when you multiply two really small things, that's another degree of smaller. So our new area, so adding up all these areas is, so we have $G \Delta F$ plus, and then we have, right, so we have $G \Delta F$ is what's coming from here. And then we're going to add to that this $\Delta F \Delta G$. So let's add that. So we have the ΔF , ΔG . And this here, right, we've already done this area. So I'm just looking at the change in area. So we did this one and this one. So we have this one left. so we need to add in $F \Delta G$.

And then we kind of notice that we can ignore this because as I was saying, this is like second order small. Okay? So ignore since super small. It's like super small, right? This is like second order small. Okay, so what this ends up being approximately equal to, is we get G , right, we just so this one we kind of ignore because it's way too small. So we end up getting G times our change in F plus F times our change in G .

Okay, so to kind of recap what's going on is if I wanted to see, right, so I like, the product rule is going to be telling me, right, how does a product change like $F G$, and these are functions. So we make F a little bit bigger, right, so it's F plus ΔF , we make G a little bit bigger, so that's G plus ΔG , and then we see how much the area changed, okay? So this is how much the area changed. And you can just kind of figure out these areas by looking at right, this length here is G , because this length is G . And G times ΔF , and then this length, this height here is F , because that's the same height as that, times ΔG . So that's what this is, this is the product of the two tiny amounts. But when you multiply two tiny amounts together, in comparison, that's really small. Okay? So you end up getting something that looks, should look a lot like the product rule if this was like a derivative with respect to X , and this is sorry, a derivative of F , and this was the derivative of G . Okay, so that's kind of where it comes from, which is cool. So I hope that made some sense, and I'll see you in the next lecture.