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derivative, constant, pi, squared, respect, multiplying, rule, parentheses, function, cubed, product, computed, power, πx , polynomial, linearity, differentiate, lecture, remember, pull

SPEAKERS

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Welcome. In this lecture, we're going to go through how to take the derivative of a polynomial. It's one of the most useful basic skills to have. And it's going to kind of, we're going to use in combination, you've already learned the power rule, but that was only like one term. But we're lucky because when it comes to derivatives, there's something called linear, like the derivative operator is linear, which really means that you can pull out constants. And if you're taking, like, you're adding two functions, you can take the derivative of the of those one at a time, unlike when you're multiplying, and you had to use a product rule, you really can just do these one at a time. So we're going to kind of combine those tools, and I'll show you an example. So here are the key tools that we're going to need. So the first one is the one that you've already seen. So this is just going to be this power rule that you know, but it's good to have it here, just so that you can kind of reference what we're actually using. And the way that this one works is if I have the derivative with respect to X , I'll keep the variable X for now, of, and then I'm taking, right, I'm taking X , so this had to be, this had to be the same X as this X here, we have to remember that, okay? And then we take this to any power, it doesn't matter what it is, as long as it's a power, okay, that this is going to equal and then we have NX to the N minus 1. Okay, so the second thing that we have is kind of, kind of the, the linearity of the derivative. So this one is new, and I'll indicate that in a second. So this is the linearity of differentiating.

Okay, so let's kind of indicate here that this, this is new, this linearity. So this one here is new. Okay, so this is a new one, it has two parts to it. The first one is that if I take the derivative with respect to X of a constant times, if it's just a constant, we've learned because we actually computed that, that you will get zero. But if it's just in front of a function, okay, so I have that this is going to sit in front of this function, I'm going to switch that from orange to red. I think I can do that, and you can see that this is actually red. Let's do that little enthusiastically to make sure. So this is going to be this. And then if I take the derivative of this, sorry, the constant multiplied by the function, notice that this has to be a constant, this doesn't work if that's a function, non constant function, okay? Then I can actually pull the constant out.

Okay, and then I have the derivative with respect to X of that function F . So this is the first part of linearity, there's two parts to linearity, this is the first part, okay? And the part, right, this has to be a

constant. So has to be a constant or need to use product rule. Right, if it's a function, that's a non constant function, you must use the product rule, constant or must use product rule. But you can just pull it out. If it is a constant, you can just pull it out and you're going to see us doing this. Okay, there's the second part to linearity, which is, this is like our dream come true. That doesn't work with when we're taking products. So the derivative with respect to X of, and notice these are two, I'm adding together two functions here.

This is why I decided to save orange, because I want to use it for the second function okay? What happens when I do this? I'm going to get, I really can just do these one at a time. I can take the derivative with respect to X of F , and then add that to the derivative with respect to X of G . So F and then I have G . Okay, just like this. And, what these two together allow us to do, okay, and again, maybe I'll emphasize this here, this must be addition or, you know, need product rule. If it's a product, right, you can't, you can't do that with multiplication. So this must be addition. Okay, there's not an option to use this with multiplication, this is a very specific rule. But we can define, we can combine these to differentiate term by term. Okay, so let me write that down, as we can combine to differentiate what we would say would be term by term. Differentiate polynomials specifically.

Term turn by turn. Okay, so let's do this in a particular example. I'll start with a basic example. And then in a separate lecture, I'll do one that has kind of funny powers. But it's good to kind of work your way up to more and more difficult examples. Okay, so I'm going to take the derivative with respect to X . And then I'm going to have something like $5X$ cubed, and then I'm going to add it to πX squared, and then I'm going to subtract off E . Okay? This is the constant E . Okay? And maybe I'll kind of emphasize that this is the constant E , or the constant function E .

Okay, so this is going to equal what do we do? So I take the derivative with respect to X . Oops, yeah, let's, let's break this, let's first do the additivity part. So I can take this and break it apart using kind of our rule, or we can use this rule here to kind of break this apart, of πX squared minus the derivative with respect to X of E . Okay, I just broke this part using the additivity, like over here, and then I, now I can actually pull out my constants. Okay? And so, right, this constant, now we can pull out our constants, this comes out. What happens with this, right? This, right equals zero, because the derivative of any constant is zero. Because the derivative of every constant is, every constant is zero. Okay, so now we can go ahead and do this. So yeah, I said, I pulled this out. Yeah, I did make that right over there, so that's nice and convenient. So I can go 5 times the derivative with respect to X of X cubed plus, and then I want to pull out that constant, so I get π times, now it's just going to sit there in front the derivative with respect to X , of x squared. And then I have minus zero, right? Because I computed that there, it's a constant, so I took care of that. Now I can actually compute these, okay? So, and I'll just keep making these red, so this is 5. And then we remember how to do these. Right? This is actually the situation. So now we'll go through to actually kind of help myself here as to color coding kind of conjunction with that. So you have an X here and an X here. And we have a cubed here, and a squared there. Okay, so now these colors kind of match up a little bit. That should be helpful. So remember, we bring down the 3 and then the X and then I go 2 minus 1 here, right? So we have sorry, 3 minus 1, right? So that's going to be 2.

And then, so then we're done with that one. And then we have plus, we keep that π out front. I know it looks like I'm excessively using parentheses and so on, trust me, students do massively better when they see parentheses used parentheses as I am. And then we have X and now we need to take 2

when they as obsessively use parentheses as I am. And then we have x , and now we need to take 2 minus 1 , which is just going to be 1 . Right, so we have 2 minus 1 , which is just going to be 1 . we close up our parentheses, and then we're going to get, in the end, so I have 3 times 5 . Right? This is, I'm just multiplying 3 things, so I can do this 3 times 5 is 15 . I have this X squared there, right? And then I want to add, and now I have 2π , like this, and then I have X to the one. So that's just X . Okay, so I have $15X$ squared, plus $2\pi X$, I checked my work from before, it's just always a good idea, got the same thing. Okay. And so, what do we do here? Right? When I'm differentiating a polynomial, the first thing is I can use this kind of additivity that comes from here, okay? And this additivity from here allows me to do these one term at a time.

But we know that the derivative of this constant E , E 's a constant, the derivative of that is zero. If there was, if it was like E to the X , that wouldn't be a constant, but it's just E , so that's a constant, so it's zero. Then I can pull out my constant. So I do that here and pulled out the 5 , but I still have to take the derivative this. I pulled out the π , but I still have to take the derivative of that. Then we remember how to actually do the derivative of a , of one of these powers. And you just kind of drop down the power. And then you subtract one off from the power. Drop it down, subtract one off, but we know that X to the one is X . And these are just products, so you can just kind of pretend the parentheses almost aren't there and we can multiply 3 and 5 and 2 and π , and that's what we get. Okay, so I hope that made some sense, and I'll see you in the next lecture.