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derivative, differentiating, cosine, squared, respect, write, function, rule, product, cosine function, sine, multiply, variable, subtract, compute, formula, front, limit, confused, equal

SPEAKERS

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Welcome. In this lecture, particularly after you've seen how we can compute the derivative using the definition, which is a limit, and you could imagine this could get really horrible, and so you're going to become grateful that some mathematicians realized there are kind of ways, there are like tools that we have to do this without that. But I say these are tools, there are, there are tools that are optional. So they're like, kind of like a bag of tricks, and you can try the different ones. There are tools that are in some sense necessary. And these are not necessary in the sense that you could compute them from the, you could compute the derivative from the limit. On the other hand, these are tools that in these particular circumstances are necessary, if you want to be able to compute it any other way. You can't do, there are certain things that you're going to be inclined to want to do that you cannot do. And I'll try to emphasize that many times, because it can get tempting. But unless you want to compute things all the way from the limit, you're going to have to actually kind of follow these rules. But you can think of them as nice tools, so that you can still actually tackle these things without having to do the limit. Okay, so the first one that I want to show you, and for each of these, I'll give an example is the power rule.

So here's how the power rule goes. Suppose I have a situation like, so I had the derivative with respect to X of, and I'm going to take it of, and I'm going to go X to the power N , okay. Very, very important, these have to match up, okay? I'm taking the derivative with respect to X , and this is an X . Okay, the power can be any number, but kind of the base, this, this variable here needs to be the same as what I'm differentiating with respect to okay? But in that circumstance, I know how to take this derivative. So what do I do? The first thing that I do is I do, you can kind of think of it as dropping this down in front. Okay, I still stick my variable X . And now in the exponent, I'm going to subtract off 1, so I still have an exponent, but it's 1 less. So I dropped down the exponent to the front, and I subtracted 1 from the exponent up there. So let's do an example. So I'm going to look at, here's my example. And maybe I'll even emphasize this here. This X here is the same as I'm taking the derivative with respect to X . Okay? So I'm going to take the derivative with respect to this X of, and then let's go ahead and do so I'm going to do X cubed.

Okay, so the first thing I do, and I'm actually going to put this as kind of a warning here. So these have to match up as I was just saying, okay? So this is the one way to say that I'm asking you, please

be careful. Okay? Caution, okay. So please check carefully that this, so check carefully, right, that this matches the variable we're differentiating with respect to you. So carefully, that this, and by this, I mean, this matches the variable I'm differentiating with respect to, which is this 1, right? So matches the variable I'm just, I'm differentiating with respect to.

Differentiating with respect to, okay. So the variable differentiating with respect to is what I mean, that's this here, right? Okay, so this is kind of the caution. We check that this kind of first step, we're good to go. So we're going to keep going. Okay, and then this is going to equal, so what's the process? Again, I take this 3, I drop it down to the front, so I go ahead and do that first. I write this variable X, should say what I'm differentiating with respect to, and then I subtract off 1 from there. So we'll do this kind of the first time around, and the first few times around, we'll do that kind of carefully, and put in the stack where we write it like that first, so that we can, can keep track of what we're doing. And then we have $3X$, 3 minus 1, right, I know that's 2, and I'm done. Okay? So that would be my answer. Okay. So in this example, when I'm differentiating X^3 , what did I do? I dropped down the 3 in front, it kept my X, I subtracted 1 off of the power, and this would be my answer. Okay, so that's how you use this power rule. So let's go ahead and do another one. So now we're going to do the product rule.

Okay, so in this one, I'm going to be differentiating with respect to, you know what, let's go ahead and change up the variable, I don't want you to get stuck in thinking that these only work with a particular variable, so let's just mix it up a little bit. So I've got the derivative with respect to, let's say T now. And I have two functions. Okay, so I have a function F, and I have a function G. And what do I get when I differentiate this, I'll write it out kind of in symbols, and then we'll do an example. So hopefully, that will clarify what we're writing. So I'm just going to take this function F without differentiating it. And then I'm going to multiply that by when I take the derivative with respect to so I'm taking the derivative with respect to T of G. And then I'm going to add to this, I'm going to take, I'm writing out G in front, and then I'm going to multiply that by, so it didn't, right, so I didn't take the derivative of F multiplied it by the derivative of G, I don't take the derivative of G, I multiply it by the derivative of F. And of course, because addition is commutative, it doesn't matter which order I write these in, as long as in one of them, I'm taking the derivative of F and one of them and take the derivative and G. And in like, the other one is just there without taking the derivative of it by the other one, I mean of F and G, okay? So let's go ahead and do an example here. I think things always make more sense when we do an example. So what I want to do is I want to take the derivative with respect to T of, and now I have here, so my first function is going to be T^2 , and then my second function is going to be cosine of T.

Okay, so first, I leave the first function alone. So I go ahead and do that my first function is T^2 . Go ahead and write that there. And then I'm just, you know, I'm going to do this one step at a time, it's good to do that just to not make any mistakes. So I'm just going to tell myself, I'm going to take the derivative of cosine of T and actually take the derivative in the next step. But just keep track of what I'm doing carefully. Okay, and then I remember, so G here is cosine of T. And then I want to take, right, this derivative of T^2 with respect to T of, and then right, this is my T^2 . Okay, so let's see, did I write this all out correctly? First, I left my first function alone, I take the derivative of my second function, I left my second function alone and took the derivative of my first

function. Okay, now let's actually do these computations. So this one is going to equal, so I'm going to have, and now I'm just going to kind of leave this so we've maybe used enough colors, and then the derivative of cosine of T is minus sine of T . So I can just write that here.

Alright, and then I'm going to add, let's just bring down that cosine of T . And then I want to take the derivative of this with respect to T , so I should get $2T$. Why is it $2T$? Because it's $2T$ times 2 minus 1 , which is 1 , right? So that's $2T$ to the one power, or $2T$, okay? And then this is just going to equal, so minus T squared, I brought the minus to the front, that's okay because this is multiplication. It would not be okay if this was addition. But because it's multiplication, the minus sign just comes out to the front, and I'm multiplying T squared times minus sine of T . It's not like subtracting. Okay? And then I go ahead and add this. And then there's something we like to do, which is we like to bring the $2T$ to the front and I'll explain to you why. Which is that otherwise sometimes our brains get a little confused and we think that maybe this is, goes inside of the cosine function. It doesn't go inside of the cosine function, so it helps our brain if we stick it out in front to separate that it's not inside, okay?

So that's my final answer. Okay, to recap what we did with this one, we'll kind of recap these one at a time. Is that, right, left my first function alone took the derivative of my second function, left my second function alone took the derivative of my first function, right? And we're going to add those together. The derivative of cosine of T is minus sine of T , notice that this is a product here. Okay? This is not, I'm not subtracting that this is a product. And then I have cosine of T times the derivative of this, right? And maybe I can kind of, in case this was a little tricky for you, how did I end up getting this was I got $2T$, 2 minus 1 equals $2T$ to the 1 , but we know that that's just $2T$, okay? And then in the last step, we actually kind of want to bring that to the front, with the reason just being so that we don't get confused and think that is something we're inserting into the cosine function. Okay? Okay, so that's that one. And I'm going to show you one more of these rules. So if you have a product, you actually need to use this. And I said, I would kind of highlight this. Unless you want to take the limit. If you have a product, you can't just take the derivative of this and then the derivative of this, this is what people like to do sometimes, it's wrong. You actually need to use this rule, but it's a nice easy rule. The trickiest part is just in the beginning remembering it. But once you get used to remembering it, it's a nice easy rule, but you have to use it when you have a product. Okay? It's kind of non-optional. So now let's do the quotient rule.

Okay, so, oh, let's use a , we're going to take the derivative with respect to, I'm going to try to mix things up. To emphasize again, what I'm taking the derivative with respect to doesn't matter. So let's go ahead and do it with respect to Z , why not? Okay, let's do it with respect to Z , okay? And I'm going to take this of, this is another one, these ones you have to use this rule, I have the function F on top, I'm dividing that by, I have the function G on the bottom. Okay, and so what is this equal? This one's a little complicated, but you just kind of learn it. And well, there is a little rhyme thing. So I want to go low, right below. And then I want to take D high, so the derivative with respect to Z of, or whatever variable I'm differentiating with respect to, of high, so low D high, minus high D low.

So low is the bottom, all over low squared. Low there, okay. So this is the formula. It's a little complicated, but it's okay. So let's go ahead and use it. So I'm going to take, to go ahead and try to now mix things up even a little bit more to emphasize that the variable doesn't matter is that I'm

going to have, so if I want to take the derivative with respect to, let's just go ahead with respect to X this time. So the derivative with respect to X of, and I have X squared divided by cosine of X.

Okay, so I'm going to get that this is going to be, so I have cosine of X times the derivative with respect to X, and then I have X squared minus X squared, and then the derivative with respect to X of, and then we have cosine of X. And then we're going to go ahead and divide that by, and then we have cosine of X squared, okay?

So I'm squaring the function, which is actually kind of like squaring the outputs. Squaring all the outputs, okay? So, now we can just go ahead and take these derivatives. We already did them over here. I kind of did that on purpose, so that it was clear. You know, we're, what we're doing that you weren't kind of puzzling over what the derivatives were. Okay, I'm going to actually put parentheses here so that it's clear that this is this product here. And then I want to subtract off, so go ahead and use parentheses. It's good to be careful about this when you have, particularly when you have something like cosine of X, where it's like a function, and you can like see the input there, and you're going to want to get tempted to multiply things. It can get really, it can be helpful to put in the extra parentheses, even though it seems a bit ridiculous, okay? And then we have minus sine of X squared, sorry, minus sine of X, because that's the derivative of cosine of X. And then this is all over, there's no derivative there. It's just cosine of X.

Okay, and then we can do this like simplifying, again, we kind of like to take this minus sign, pull it to the front, which actually turns that into a plus, and bring this to the front so that we don't get it confused with what's going on inside of cosine or the input of cosine. So I just get $2X \cos(X) + \sin(X)$. Be careful, because I pulled that minus out, it's no longer there, I almost got tripped up myself. And then this is cosine of X squared. If you've ever seen, sometimes people put that 2 here, I'm not going to do that, because it's just an added thing to get confused about at the moment. But it is kind of the shorthand, sometimes, it sometimes makes life a little easier. Okay. So it's just another formula, I could kind of go through what I'm doing at each step is taking derivatives, I wanted to emphasize in this one, in particular, that when you're taking, you know, and so throughout, I tried to emphasize, you really just need to make sure you're very, what you're taking the derivative with respect to matches up with your input variable. But once that's true, the formula kind of works. You don't have to have even though I wrote this formula with a Z, it's okay for differentiate with respect to X, as long as your functions are functions of X, and then you just need to change everything to X there. So I did that on purpose. Hopefully, if it confuses you, now, then you can kind of sort it out and it won't be something that will confuse you in the future. Okay, so I hope that makes sense, and I'll see you in the next lecture.