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SPEAKERS

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Hello, everyone, I'm glad you're checking out our video, we're going to continue where we left off in our previous video, we're going to continue with example one, Part B, looking at bond prices and their relationship with interest rate risk. We're gonna work with some numbers here. And we're going to use the relative rate of change that we calculated with a little help from differential calculus, we're going to use that to estimate the change in, in the bond price from a 1% increase and a 1% decrease in the market interest rate.

In Part A, we showed that the relative rate of change in the bond price with respect to a change in the interest rate could be written like so, where the numerator represents the first derivative of the Braun pricing equation with respect to a change in the market interest rate, and the denominator represents the price of the bond. You can think of that as the initial starting price of the bond, before market interest rates change. And this was equal to negative T , where T represented the maturity of the bond at time until the zero coupon bond actually pays out money. We can use this equation to estimate or approximate the change in price when interest rates change by multiplying both sides of the expression by the change in R . It's not, it's one, an intuitive way to remember what we're doing. And so if I wrote this like so, we would, they're going to divide an equal one. And we get the relative change in the bond price is equal to negative T multiplied by the change in the market interest rate R .

Now we can use this expression right here to answer question Parts B, and C. So if we have ΔR is equal to an increase of 1%, we can write that as 0.01, write it as a rate. And that means that our relative rate of change in the bond price should be equal to negative T times 0.01, or negative $0.01 T$. So if we have a 1% increase in the interest rate, the market interest rate, that's going to decrease the bond price by 1% times T . So if T was equal to five, this was a five year zero coupon bond, this would tell us that the price of the bond would go down by 5%. Looking at part C, we have the change in the interest rate is equal to negative 1% or negative 0.01. The relative rate of change and the bond price is then going to be equal to negative T times negative 0.01. And we expect the bond price to increase by $0.01 T$, or 0.01 times T . So if we had a zero coupon bond that paid off in five years, paid off some amount of money five years from today, we would expect a 1% increase in the market interest rate to increase the value of that bond by 5%.

Remember that these are approximations. And we are using the derivative at a specific point, namely point P, to try to predict or approximate the relative rate of change in the price given a change in the market interest rate. So these are going to be approximations, hopefully good approximations, but they're not going to give us the actual change in the bond price as well be demonstrated in our next example. For the remaining parts of the question, we're given more information about a bond. So we have an example bond to work with some more numbers. We have a bond that will pay \$1,000 in 10 years and the current current market interest rate is 2%. So for part D, we want to calculate the bond price, the bond pricing formula was P is equal to the payoff in the future F , multiplied by E negative R times T , or R is going to be the interest rate as a rate, the market interest rate, not the percentage but the rate, and T is equal to time to maturity. Now let's calculate this bond price when the market interest rate is 2%. So I'll write P point 02 is equal to \$1,000 pay off in the future. E is the natural exponent, that one doesn't change. And we have to the power of negative point 02 times 10. So we have 1000, E to the negative point two. And this is going to be equal to, using a calculator, the price is equal to \$818.73.

Now for Part E, we want to calculate the actual relative rate of change in the bond price if the market rate of interest increases 1%. And so if we want to do that, we're going to need the price at, well 3% minus the price at 2%, divided by the starting price at 2%. And we're going to need the price of this bond when the market interest rate is at 3%. So to find that, we just do the same, we use the same formula we used in Part D. We'll take 1000 multiplied by the natural exponent to the negative point 03 times time to maturity of 10. Again, using a financial calculator, we get the price of the bond when market interest rates rise from 2% to 3% is going to be equal to \$740.82. And so the relative rate of change and the bond price, given the market rate of interest increases 1% is going to be equal to 740.82 minus 818.73 divided by the starting price, which is 818.73 And this is going to be equal to negative 0.095. Or nine point negative 9.5%.

Now what happens if the market interest rate decreases by 1%? Let's go ahead and calculate the relative rate of change in the bond price. To get started, we need the price of the bond when interest rates fall and are now equal to 1%. So we're going to have \$1,000 multiplied by the natural exponent times negative point 01 times 10. Using a, in my case a financial calculator, but using any calculator that has the natural exponent on it, we find that the bond price with market interest rates are 1% is \$904.84. I added in the original price of the bond so we can calculate the relative rate of change in the bond price. We're gonna have \$904.84 minus \$818.73, all that divided by the original bond price of \$818.73. And using a calculator we find that the price is going to increase by 0.105 or by 10.5%.

And so here we have the answer to F. What about Part G? Well, we found that when the bond price, well I should say when the market interest rate increase to 3%, the relative rate of change was equal to negative 9.5%. And when the market interest rate fell the bond price increased by 10.5%. We have the actual rate of change here, what would our approximations look like? Well, our approximation, the change in P , 2% with a change in R of point 01, that was going to be equal to, that should be just equal to negative 10 times point 01, which is going to be a decrease of 10%.

This is our estimate or approximation. And if we were looking at the change in the bond price, when the change in r was negative 1%, we would have had negative 10 times negative point 01, which

would have been a increase in the bond price of 10%. So, we have our estimates rate here, but we have our actual rate here. And they're not identical. Remember, previously, I said that as the change in the variable of interest increases, in this case, it's the market interest rate as it as the change in that market interest rate becomes larger, our approximation using derivatives is not quite as accurate. Taking a look at the diagram in front of you on the screen, you can see a curvy line called the actual bond price. And that's the actual bond price with a payoff of \$1,000, and constant time to maturity of 10 years at various R market interest rates. And you can see clearly that there's an inverse relationship between the bond price and the market interest rate. The straight line represents our duration approximation. It's a linear approximation. And it's centered on the starting value, our starting bond price that we saw in our example. So looking at the diagram, you see the point of tangency is where where duration is coming from, it's the slope of the actual curve at a single point. And we can use that tangent line to estimate the bond price or the relative rate of change in the bond price for different changes in our and that gap between the curve and the straight line represents the air and our approximation. Now you can see where the error in approximation has been labeled on the diagram. You could even add in, say, 3% here, remember, we looked at the relative, that percentage change, and that would give us an estimate of the new price of the bond. And that our estimate would not be quite the same as actually using the values and and recalculating the bond prices and then calculating the relative rate of change, or just in this diagram, we're just looking at the bond prices themselves. And you could have 1% over here, and you could see where that would line up to and you can kind of have an, imagine that error term being represented in the price of the bond.

Now, how in the real world do we deal with these approximation errors? Well, there's many techniques that we can use is. A straightforward one would be to, we use the first derivative, we could look at the second derivative and use that to help us and that's what's called a quadratic approximation. Or we could go even further and use a use higher order approximations. And there's also something called convexity, which is really exploiting the second derivative taking advantage of the second derivative to get a more accurate estimate of bond prices after a change in interest rates.