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SPEAKERS

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Hello, everyone, welcome back. Here today we're going to be looking at bonds. We're going to apply what we've learned about derivatives to pricing bonds, or more specifically, looking at how the price of a bond is going to change when interest rates change. This is going to be helpful for you, because one, it's going to let you apply what we learned in a previous video to a practical example. It's a finance example. If you're not planning to go into business, that's okay, it's still going to be useful for you. Because in the future, hopefully, you will have maybe a mortgage one day, and you'll buy a property or you need to think about how to invest your money, and this is going to help you think about investing in bonds. And conversely, a mortgage is kind of like the opposite of a bond, you're borrowing at a fixed rate, usually at a fixed rate, it could be variable. So but let's focus on bonds. And we can kind of push mortgages aside, and perhaps we'll talk about them a little bit later.

Now, you're probably aware of the stock market, everybody kind of knows, at some degree, they've heard of the stock market, even if they don't really understand exactly what a stock or a share is. For our purposes, what I want you to understand is that a stock rewards investors with dividends. So occasionally management, they might say that every year, they like to pay out a certain dollar figure in dividends, but it's not guaranteed, they, business managers can choose to pay dividends or not. On the other hand, bonds promise to pay a fixed amount. And this promise to pay a fixed amount actually makes them very sensitive to changes in the interest rate. There's a few ways you can think about how they're vulnerable to changes in interest rates. One, if you're, if you've taken a first year course in economics, you might call it, there's kind of an opportunity cost when the interest rate goes up, you could have invested your money and got a higher return somewhere else, but you're locked into this bond that's paying you a fixed rate, that money it's paying you isn't quite as appealing as it used to be. Another way to think about it is a bond promises to pay you a fixed amount. And that market interest rate is very vulnerable to inflation. So if inflation is going up, the value of the money that you get in the future from these bonds, isn't going to be worth as much, it's not going to allow you to have as much purchasing power as you would otherwise.

Practitioners in the field of finance have a variety of tools that they can use to quantify and measure the sensitivity of bond prices to a change in interest rates, we are going to look at one which is called duration. And there are actually many various forms of duration, we're just going to do kind of a very

simple example together of duration. Another thing to keep in mind and just a practical bit of advice is that this interest rate that we're going to be looking at is influenced by central banks, like the Bank of Canada or the US Federal Reserve. And so part of what you're learning here is why the business news is always going on about interest rates and whether interest rates are going up and going down. What you're learning here is part of that story. Because the Bank of Canada and the US Federal Reserve, they change the interest rates to try and change the business decisions of businesses, but also the investment decisions of investors. We need a little bit of information to get started. So we're going to use what's called the time value of money, or TVM. And we're gonna use this time value of money to price a zero coupon bond. What is a zero coupon bond? Well, essentially, it's a bond that has only one payment to it. So it may pay out in one year's time, I'd say \$1,000 in one year, or could pay out \$1,000 in 10 years, or it could pay out \$1,000 in three months. It doesn't pay interest, say every, every year or every quarter or something like that. So we call that a zero coupon bond. It's just got one payment, one payment. And the reason why is because this is a very simple bond, and it's going to make our example much more straightforward than would be otherwise. A zero coupon bond pays a lump sum face value just that one payment F in T years. So we call this T here this is the maturity or the time to maturity of a bond.

And the bond can be bought and sold at price P . And this P is going to depend on our face value, the time to maturity and the market interest rate R , which is continuously compounded. This is our pricing formula. So we can find the price of a bond, the price of a bond, by taking the face value and multiplying it by the natural exponent to the power of negative R times T . Our first question is asking us to find the relative rate of change in the bond price with respect to the market interest rate R . To get started, we have to remember, well what is the relative rate of change? it's going to be the derivative of P with respect to R divided by that initial price. The initial price, which we're just going to call P , so the relevant relative rate of change in a variable is going to be its derivative divided by its value on the level before you took the derivative. So this is our relative rate of change.

To solve this expression here, let's start with numerator, we've got the change in P with respect to a change in R . We've got our pricing formula down here. And we're going to take the derivative of this expression with respect to R . And we get F which is just a multiplicative constant, e to the negative RT , right, it's just the derivative of the natural exponents itself. But we also want to take the derivative of the exponent, which is going to leave us with negative T . What is the denominator equal to? Well, it's just this formula right here, we already have an expression for the denominator. Combine them together, we get the change in P with respect to R over P is going to be equal to negative T times F , e negative RT divided by F , e negative RT . And you can see, F divided by F is equal to one, e to the negative RT divided by itself, it's also equal to one, and we're just left with negative T . And so we found what we were looking for. And it's equal to negative T , negative the time to maturity.

So now let's look at our last part of the question, which is that we want to interpret our result. We've got two useful results here. The first one, if there is an increase in the market interest rate, this is going to lead to a decrease in the bond price. Similarly, if we have a decrease in R , that's going to lead to an increase in the bond price. So if we think about that for a moment, and you're at home, and you're watching the news, and you're a bond holder, and you hear that the US Federal Reserve is increasing rates, you just made money, the value of the bonds that you bought have gone up, and so you can sell them for more money today. On the other hand, if you hear that the Bank of Canada has

raised interest rates, bond prices are going to decrease. So if you're a bond holder, if you've been buying bonds, if you've been saving your money and putting them into bonds, they're going to be worth less and if you sell them today, you're not going to get as much money. The second, the size of the change, the magnitude of the change is going to depend on the time to maturity. So the size of the change in the price of the bond depends on the time to maturity. So, if you have a bond with a time to maturity of 30 years, it's going to be more impacted, it's going to have, it's going to have a bigger change in the price of that bond with the long maturity than if you had a bond with a one year maturity, or a six month maturity. The reason being, you're getting your cash sooner, and you can reinvest it at this now higher rate. But if you're locked in for 10, 20, 30 years, then the value of the bond that you have is going to be worth quite a bit less. And one last thing to observe on this slide is that this is our duration. The duration of a zero coupon bond is equal to the time to maturity, we don't worry about the signage here, we take the absolute value of duration, and we say that the duration of zero coupon bond is equal to its time to maturity. In the next parts of the question, we're going to work with some numbers. So you can see what this looks like when we actually have values.

To illustrate the results that we saw on the previous slide, I put together this diagram, you can see a couple of things on this diagram. One, we have two bonds, both bonds have the same face value. And they're both facing the same market interest rate. But they have different times to maturity, a different time to maturity. That's the only thing that differentiates these two bonds, one has a time to maturity of 10, we can see that's the top of the purple colored bond on the top, and one has a time to maturity of 30. And you can see that is in blue and a little bit lower. Now notice that both bonds are negatively or their price is negatively impacted by an increase in the interest rate R as we move from left to right on the diagram, R is increasing and the bond price is decreasing. That was result number one that we saw. Result number two is also illustrated on this diagram, you can see that interest rate changes, the bond with a time to maturity has a greater decrease in its price than the bond with a time to maturity of 10 years. We can see that as we're moving from left to right. The value of the bond with a 30 year maturity is always less than the value of the bond with a 10 year maturity.