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SPEAKERS

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Hello, and welcome back. In this video, we're going to be looking at other applications of derivatives of differential calculus. And we're going to be specifically looking at how we can approximate change in, say, the Y value using the derivative. And what we're going to do is we're going to fix the derivative at a certain point use the value of the derivative at that fixed point, to make estimates about the change in Y . Why might we want to do this? Well, computationally, it might be really difficult to calculate the value of Y or the change in Y at different values of X . This is going to be true if we have really complicated functions to work with. And it's also convenient when we're just working with theory to be able to find derivatives and then approximate the change using that derivative without necessarily worrying about what the actual rate of change might be. So with that said, let's get started. I'll give you some intuition. And then we'll look at some examples together.

Let's approximate this function using the derivative. So we'll start by identifying an X value that we're interested in. And we want to find the corresponding Y value to this X one, so I can just walk up to the line. And there is the corresponding Y with our X . Now focusing on this point here, if we want to find the slope at that point, graphically, we can do that by drawing a tangent line. So why don't we draw a tangent line that will identify the slope at this point on our diagram, and if we do that, and if I draw correctly, it'll look something like this. The tangent line at X one, Y one has slope equal to half prime X , where this function here, this is F of X , and it's equal to Y . So up till now, this should all be familiar to you.

Now let's consider another point on this diagram. Let's consider X two. And maybe we're interested in the Y value, which corresponds to X two, or maybe we're just interested, well, let's stick with that, let's stay focused on the different value of Y . What's the value of y going to be? And I'm going to call this point A. And the one on, the point on the tangent line is going to be point A, and the point on the function is going to be point B. Now, let's calculate, how would we calculate the Y value at point A?

Well, I could write Y is going to be equal to this tangent line has slope F' of X , and it's fixed, the slope is fixed along a point line, a line has the same slope at any point along that line. So it is fixed

slope is fixed along a point line, a line has the same slope at any point along that line. So it is fixed. And we can use the slope to help us here. So we're going to have F' of X , and we're going to multiply it by the change in X , which is going to be $X_2 - X_1$. So we have the change. We have the change in X . And we have the change in Y , which is the change in X multiplied by the slope of this tangent line. If we want to get the Y value, we could write F of X_1 plus the change in Y . And that's going to give us the Y value at point A . And this part here, this is our approximation. And it's our approximation using the derivative. And although I haven't told you, this is known as a linear approximation.

So we will be using the slope of the curve at X_1 to estimate the Y value at X_2 . Now why might we do this? Well, suppose we have a very complicated function, we might, we saw a function in a previous video that looked like this. It might be hard to find the correct Y value when we change X . And so you might want to use a linear approximation instead. Now what about the actual value, let me remind you about the actual value. So what is the value of Y at point B ? Well, the Y at point B is going to be equal to F of X_2 . And that is going to be equal to F of X_1 plus the change in X . Or we could just write it as F of X_2 . And we could write Y_B there, Y_A there.

But like I said, it might be really difficult to calculate this. Or you might just not want to be interested in calculating, and you might be fine with finding the derivative and using that for your discussion in your analysis. An interesting observation here is notice that as the change in X gets larger, as we move further away from our initial starting point at X_1 , the distance between the function and the tangent line is increasing. This suggests that using an approximation using the derivative to approximate the new value of Y , or the change in Y is going to be less accurate as the change in X becomes larger. In the previous example, we looked at the new value of Y . Now let's take a look at the approximate change. We're approximating the change with the derivative this time. So a little bit different, but all look very similar, because we're actually not going to be interested in the absolute, in the value of Y , we're going to be interested in the change in Y . And so we start off with our function Y is equal to F of X , we find the derivative dy/dx , which would be equal to Y' using prime notation. dy/dx is, you know, we could just write that also as F' of X . A way to remember this, and to think of it, it's kind of like we're going to multiply both sides of this expression by dx , the change in X , and when we do that, we're left with dy is equal to F' of X multiplied by dx where we have dx is equal to the change in X , and dy is equal to the approximate change in Y .

Here we have what I wrote previously written nicely for us. And we have a diagram side here. So we've got Y , we have the Y axis, we have the X axis, let's illustrate approximating the change in Y , given change in X , I'm going to do this graphically. So I'm going to draw a tangent line like so. And that gives us the slope of that tangent line has slope F' of X , call this point here X_1 . And suppose we're interested in X_2 again. So let's calculate dy . We want to calculate our approximation. And if we want to calculate the Y , it's just going to be equal to F' of X times $X_2 - X_1$. And that's going to give us the change in Y .

What about the actual change in Y ? If we want to know the actual change in Y , sometimes it's written with this delta, this triangle. It's going to be equal to, well here this is F of X_2 . This is F of X_1 . The change in Y is going to be equal to F of X_2 minus F of X_1 , which in this case is just going to be $X_2^2 - X_1^2$. And so in this situation, well

we could rewrite this as $F(x_1 + \Delta x) - F(x_1)$ where Δx is just equal to $x_2 - x_1$, the change in x . Going back to the diagram, our change in x is given, we know what the change in x is. The change in y is the distance vertically from our starting position to our new point on the function at x_2 .