

7

Sat, 11/27 10:57AM 9:07

SUMMARY KEYWORDS

derivative, squared, prime, g prime, quotient rule, expression, second derivative, implicit, y prime, functions, simplification, multiply, denominator, understand, recipe, equal, constant, double, textbook, divided

SPEAKERS

Robert McKeown

Hello everyone and welcome. In this video we're going to be looking at functions within functions, and continuing with this idea of implicit differentiation. Now why do we want to look at functions within functions or functions within an expression like you can see in front of you with example one? We have G of X , where G of X is a function. This is just to help you practice and get comfortable with the idea of taking derivatives in a general form. So G of X is a general form of a function that hasn't been specifically defined. Why would you want to practice this skill and get comfortable doing these kinds of operations? Well, one, it shows up in economics and business quite frequently. Two, it's very helpful for you to understand how to read a textbook, and to follow a recipe or instructions that you would find in the textbook. And lastly, it'll also help you when it comes to reading academic papers and other resources that are going to be helpful to you in your future career as a student and as a practitioner. So that said, let's turn our attention back to example one, and let's see if we can take the implicit differential of this expression and find Y prime.

As previously mentioned, G of X is a function in a general form, and so we're going to take the derivative accordingly. Now we want to find the Y prime, the implicit derivative, so we're going to take the derivative of both sides of this expression. So I'll write DX $4GX$ plus Y cubed equals D DX two. Now looking at $4G$ of X , four is a multiplicative constant, and the derivative of G of X is just G prime of X . So we have $4G$ prime, X plus Y changes when X changes, so a change in X is going to affect Y . We're going to use the chain rule. So we have the three coming down, and we have Y squared multiplied by Y prime, and two is a constant, so it's just going to be equal to zero. Now we want to solve for Y prime, so we can write this as Y prime is equal to $4G$ prime X divided by three Y squared.

Now suppose we wanted to find the second derivative, we could do that. We'll take the derivative with respect to X of Y prime. And then I'll just write the this expression right here. Now Y prime, what's the change in Y prime with respect to X ? That's just going to be Y double prime. And over here, I have a quotient. So we're going to use the quotient rule. And we'll have for the second derivative of G of X , or the derivative of G prime of X is equal to G double prime X . The multiplicative for constant stays there. And then, following the quotient rule, I multiply the derivative of the numerator, which is

this thing right here. I just multiply that by the denominator, and then I'm going to subtract the derivative of the denominator multiplied by the numerator. And this whole expression is going to be divided by the denominator squared, $9Y$ to the power of four.

Now I can simplify this expression just a little bit. So I'm going to have $12Y^2 G''$ of X minus $24Y G' X$, and I want to replace Y' with what we came up with earlier in our first step when we found the first derivative. So I want to replace Y' with the identity that we have over here, $\frac{4G' X}{3Y^2}$ and the denominator here. Now as usual with the quotient rule, it leaves us with a pretty messy answer, we can do a fair bit of simplification here, mostly for this session I want you to know how to take the derivative, how to take the second derivative. But let's see if we can simplify this. So I can write Y'' , I'm going to separate the two terms. So we're going to have $12Y^2 G''$ divided by nine, Y to the power of four minus $96 \cdot 24$ times four, and it's $96 G' X^2$ divided by $9Y$ to the power of four, multiply by $3Y^2$. And I almost forgot a Y right there in the numerator on the right hand side term.

We can keep going. Now I'll rewrite Y'' . And if we look at the right hand term first, 96 divided by three is 32 . $G' X^2$, and those two, the Y , Y divided by Y^2 is one over Y . So we would have $9Y$ to the power of five in the denominator, Y to the power of four times Y , Y to the power of five. And I'll just rewrite our left hand side term. And it looks like we could have Y'' is equal to $12Y^2 G''$ minus $32 G' X^2$ over our Y to the power of five. I wouldn't worry too much about simplifying it this far. The important thing is whether you can take the derivative, the first derivative and the second derivative. Simplification is kind of the icing on the cake, which is to say that it's nice to have, but not necessary to understand what's going on.

Now there was an additional step of simplification, notice there's a 12 in the left hand term, and a 32 in the right hand term. And so since they're both even numbers, they must have a common factor and that common factor is four. If we factor out the four, we get this expression over here. You know, simplification here is really not necessary for understanding what's going on in the video. When we take the second implicit derivative, we get a complicated kind of messy answer. It's nice to simplify it but not really necessary to understand what's going on. For you. The most important thing is to practice taking the first implicit derivative and the second implicit derivative know how to do that when you have a function and its general form. And that's really gonna set you up for success in the future. It's going to help you understand the textbook. It's going to help you understand lectures. It's going to help you read papers online, that give you instructions and recipes for performing different operations.