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SPEAKERS

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Hello and welcome. Today we're going to be looking at the second implicit derivative. So just like there was a first derivative and a second derivative, there is a first implicit derivative and a second implicit derivative. And just like the second derivative you already know, it tells us something about the curvature of the original expression. It also tells us about whether the first implicit derivative is increasing or decreasing. Look at the slide in front of you, you can see we have our first question, it's a relatively straightforward one. But as you'll see, finding the second implicit derivative can be a little messy. And also there's an extra step that we want to add in that we didn't have to consider with the first implicit derivative, and I'm going to show you that in a few moments.

Here we have a problem that is similar to a problem we saw before. If we want to find the second implicit derivative, then we can start by finding the first implicit derivative. And so I can write $\frac{dy}{dx}$ over $\frac{dy}{dx}$, I want to take the derivative, both sides of this expression. Notice we have a product here, X is a function of X , obviously, and Y is also an implicit function of X . Taking the derivative, we have to Y , that's the derivative of this part here, multiplied by Y , plus we have $2XY'$. The change in Y with respect to a change in X . 10 is just a constant. So when we take the derivative of it, we find it zero. We can rearrange and solve for Y' . And we're going to be left with negative Y divided by X . And of course, Y' is our first implicit derivative.

Now to find the second implicit derivative, we have two choices, we can take the derivative of this expression right here, or we could take the derivative of the first implicit derivative or that expression, which has Y' isolated on the left hand side. In general, I recommend taking the derivative of the expression before we start rearranging things. And before any term might get cancelled out because it's equal to zero, or we're letting it interact with zero in a way such that it disappears. That's my personal preference. But in this situation, either choice will be fine. Now, let me rewrite this out. So we're going to take the derivative with respect to X of $2XY'$. And I'm going to write $\frac{dy}{dx}$ over $\frac{dy}{dx}$, negative $2Y$. I've rewritten or duplicated that expression and pasted it at the top right just to make sure that we have enough room. And now I'm going to follow through and take the derivative of both sides of this expression.

We're gonna have $2Y'$ plus XY'' is equal to negative $2Y'$. And we don't need those twos, they're just going to disappear. And I can rewrite this as Y' , we've got a Y' here and a Y' there, plus XY'' is equal to zero. Now we're going to want to replace Y' with its expression right here. So let's remove the Y' , we know that Y' is equal to negative Y/X . And if we replace this Y' right there, we're going to have 2 negative Y/X plus XY'' is equal to zero. Now we want to solve for Y'' , that's our second derivative. And we could have XY'' is equal to negative 2 negative Y/X . And we're going to have Y'' is equal to $2Y/X^2$. And there we have our second implicit derivative. Why might we need the second implicit derivative? Well, as I mentioned in the introduction, the second implicit derivatives tells us whether the first implicit derivative is increasing or decreasing.

Here we have an example that we, we saw before, we have the definition of a circle. So we're going to have something like that, where we've got Y here, X there. And we want to find the second implicit derivative. So we're going to be looking at trying to determine whether the slope is increasing or decreasing, our answer is going to tell us whether the slope is increasing or decreasing. So let's start by finding the first implicit derivative, I'm going to take D/DX , the derivative of both sides of this expression. And when I compute the derivative, we're going to have $2X$ plus $2Y'$. Y is an implicit function of X , and it's going to be equal to zero. We can simplify this a little bit the twos divide both sides of the expression by two. And we're going to have Y' , Y' is equal to negative X/Y , and Y' is equal to negative X/Y . Now, there's a couple ways to go about this. Depending on which choice we choose, our process will look different. But our end result should be the same. So we want to take the derivative of the first derivative, but we've got, well, we've got three candidates. Three candidate expressions to choose from, I'm going to choose this one up here.

And if I take the derivative of this expression I guess I'll write, I'll write the derivative of zero with respect to X is just going to be zero. We're going to have one plus Y' , Y' plus Y times Y'' is equal to zero. Now I've duplicated this expression down here and re-pasted it up here at the top right hand side of the slide. Now we want to replace Y' with what we found to be its first implicit derivative. So we've got we're going to fix this expression here so that it doesn't have any Y' 's it just has Y 's X 's and Y'' 's. Why are we doing that? Well, one, it's going to make it easier for us to interpret the expression that we solve for. And it's also going to make it clear that we have the right answer. If we don't substitute for Y' , we might not be sure that we got the same answer if we took the second derivative, derivative of one of these other two functions down here. We're gonna go ahead and make the replacements so we've got one plus Y' is negative X/Y . And that whole thing is going to be squared, because we have Y' , Y' is equal to Y' squared plus, back to the expression, we're making the substitution plus Y times Y'' is equal to zero.

Now we want to solve for Y'' . We're going to have Y , Y'' is equal to negative one, minus minus X , I'll leave it like that minus X/Y^2 . Before I go any further, I'm just going to rewrite this expression, I've got Y , Y'' is equal to negative one minus negative X/Y^2 is just equal to X/Y^2 . So I've got X/Y^2 . What's a way that I can show this? This is kind of like having negative one squared X/Y^2 . And so you can see that negative one squared is just going to be equal to one. It's a sort of an aside here, although we don't have much room. Now getting back to our main expression here, we want to solve for Y'' .

So I'm going to write Y'' is equal to negative one over Y minus X^2 divided by Y^3 . I did a little resizing to give me some more room, I want to simplify this expression right here, what are we going to do? Well, we can factor out a negative, that would give us Y'' is equal to negative one over Y plus X^2 over Y^3 . And then I can find a common denominator by multiplying the numerator and denominator right here by Y^2 . And we get Y'' is equal to negative Y^2 plus X^2 divided by Y^3 .

And there's our answer. This is telling us how the slope of the circle is changing whether the change is increasing or decreasing. And it's going to depend mostly on this value Y . Notice that if Y is a negative number, negative number cubed is a negative number. And the denominator here is always positive. So that if Y , the coordinate Y , is a negative value, then this should be increasing, then the first derivative is increasing, and that's going to coincide with this part of the circle. The slope is increasing, then when Y becomes positive, when we break the X axis intercept, and the slope is decreasing.