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SPEAKERS

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Welcome everyone, today we're going to be looking at a model that is near and dear to my heart. It's a model from economics. And we call it the two period model of consumption. So we're going to have a consumer, and they're going to have only two periods, they're going to have a period today. And they're going to have a period in the future, the consumer has to choose how much to consume today, or how much they're going to eat today, and how much they're going to consume in the future period tomorrow, how much they're going to consume tomorrow, or how much they're going to eat tomorrow. And we're going to let consumption today be equal to C_1 , and we're going to let consumption tomorrow be equal to C_2 . Now these consumers, like many of us, they are impatient. They would prefer to eat today, rather than tomorrow, all else being equal. When I say that, I mean, if you offer the consumer, the choice between eating today and eating tomorrow, they would choose to eat today. So think of yourself being offered a cookie, do you want a cookie today? Or do you want a cookie tomorrow? Many people would choose to have that cookie today, all else being equal.

This impatience factor is going to be captured by the Greek letter beta, β . So β is going to be a measure of people's impatience. Now, there is an implicit relationship here between consuming in the future, consuming tomorrow, and consuming today. If you reduce how much you consume today, that means you're going to be saving more of your fixed income, that you have, your endowment that we say in economics, how much you have. And therefore, you're going to end up consuming more tomorrow, because at the end of tomorrow, at the end of this future period, everything ends the game is over. And so you're going to eat everything that you have left over, or that's available to in that future period. So reducing consumption today leads to you having more consumption tomorrow. And this leads us to that implicit relationship, where we're going to say that consumption in the future period is an implicit function of consumption today.

Here's our expression. Now, the consumer is going to get utility, utility is a measure of happiness. So the more utility the consumer gets, the happier they are. The consumer gets utility, they get happiness from consuming C_1 . And notice that we have a function here, this is a function, U of C_1 , plus they get utility from consuming tomorrow, but not quite as much. Remember that β was the discount. It's their impatience. And $U(C_2)$ is some function. $U(C_1)$ is also a function. Now we

want to take the derivative of both sides of this expression. In order to find how C_2 changes, if we change C_1 by an infinitesimally small amount, which is really this question is saying, what is the change in C_2 with respect to C_1 , what is the derivative of C_2 .

Now before we take the derivative, I just want to note that $\bar{U}$ has a little bar on the top of it because it is a constant. We want to make sure that it's just some constant number, it doesn't vary over time. It's sort of the aggregate happiness, and it's just equal to some number. So we're looking at how this expression is going to change holding the consumer's happiness constant.

Now it's important that we hold utility constant, because what we really want to know from this question is, we want to see that if we change C_1 a little bit, if we ask the consumer to decrease their C_1 , if we asked them to save a little bit more today, how much more C_2 does the consumer have to be given in order to be satisfied with a trade off, to be no worse off, to have no decrease in their happiness. So we want to see how much C_2 we have to reward this consumer with. If we ask them to decrease their C_1 we can answer this question by taking the implicit derivative of this expression right here. And when we do that, I'm going to start by writing D_{C_1} over DC_1 , U_{C_1} plus βU_{C_2} .

Now, taking the actual derivative, we're going to have, remember, U_{C_1} , this is a function right here. So we're going to get U'_{C_1} , the change in C_1 with a change in C_1 is just equal to one. So I could put a one there, but usually you won't see the one. And moving on to the next term, the β is a multiplicative constant, I can leave that just the way it is. And then we want to take, we recognize that a change in C_1 will have a change on C_2 . The value of C_1 affects the value of C_2 . And so I've got U'_{C_2} , multiplied by DC_2 over DC_1 . I'll use the Leibniz notation. And that is equal to zero because we want to maintain their happiness as a constant. So we're just setting it equal to a fixed amount. And therefore it's not changing when we change the one because we want C_2 to change.

Our next order of business is, remember we're looking at this is the thing that we want to understand. And so we're going to isolate it and solve for it. And I can subtract U'_{C_1} off both sides of the expression. And we have $\beta U'_{C_2}$, the change in C_2 over the change in C_1 is equal to negative U'_{C_1} . And now I can divide both sides of the expression by $\beta U'_{C_2}$. And we're going to have the change in C_2 with respect to the change in C_1 is equal to negative U'_{C_1} divided by $\beta U'_{C_2}$.

What is our answer? How do we interpret this? Well, I could write it as negative the gain in happiness from a little more consumption today, divided by, divided by the gain in a little happiness, the gain in happiness from a little more consumption in the future divided by our impatience. And this is more commonly known in economics as the marginal rate of substitution. And here's our marginal rate of substitution. So how much if we take away some of your consumption today? How much more consumption tomorrow do you need to be rewarded with in order for you to be no less happy than you are with the current order of things with the current amount that you're consuming today and the current amount that you're consuming tomorrow.

Hi, everyone. So we've got a lot of information on the screen here. You can see our first thing we have is our definition of the marginal rate of substitution. And that's at indifferent point. If we take away some of your consumption today, how much more consumption do you have to have tomorrow? So if I take a cookie away from you today, how much cookie am I gonna have to give you tomorrow for you to be just as happy as you would be if you just kept that cookie initially? Now another interesting fact of what we've done here is you've actually derived an indifference curve. Now an indifference curve illustrates different bundles of consumption today and consumption in the future, at which point a consumer is equally happy. So for example, if a consumer had eight units of consumption, eight cookies today, but only four in the future, they would have the same utility from that bundle, those values, as they would from, say, having two cookies, two units of consumption today, and 15 in future period.

Now, what's neat about this formula right here is that you've actually derived the slope of this indifference curve, so that any point along this slope, the indifference curve, is going to have a, or any point on this indifference curve, the slope is going to be negative, the marginal rate of substitution. Now, sometimes, you'll see this in textbooks where they don't show you the beta, they just define the marginal rate of substitution as the marginal utility from consuming today divided by the marginal utility from consuming in the future period. And just in case you're wondering about that impatience factor, economists estimate that beta is equal to about 0.95 annually. Sometimes, it could be more, sometimes it could be less, but that's typically the value that economists give to human beings impatience. So approximately, if you had a beta of 0.95, you're looking at approximately requiring 1.05 cookies in the future period to be satisfied with giving up one cookie today.