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SPEAKERS

Robert McKeown

Hello, everyone, Robert J McKeown here. Welcome back to another video. I'm really glad to have you with us. Today we're going to be looking at implicit differentiation. This time, we're going to make sure you know how to do it with Leibniz notation, as well as the prime notation. Leibniz notation can be really useful, because it gives us more detail about what we're taking the derivative of, while prime notation kind of hides that. So here in front of you is an example where we're looking at finding the implicit derivative or the derivative, Y prime derivative of Y. But we want to write it in Leibniz notation. So let's take a closer look at this problem. So we've got an expression, here, we want to take the derivative of both sides of the expression per usual and write it like I've written it before.

Now when I take the derivative, same situation, we've got a product of two functions right there. So I can factor out the two. And the derivative with respect to X is just one, so we're going to have two, Y plus, I want to take the derivative of Y with respect to X. It's an implicit function of X, right? DY over DX, instead of that Y prime that we're used to here. And this thing is, whoops, forgot a part, of course, DY over DX, that's going to be multiplied by 2X, the whole thing is going to be equal to zero, we can factor out a two. And since we've got a zero on this right hand side, that two is just going to disappear, so we'll cross those out. And now we've got Y plus DY over DX, X is equal to zero. Now I want to solve for DY DX. And I have DY DX is equal to negative Y over X. And the correct answer is D.

Now in previous examples, we always used the Y and the X notation. And this is useful because we're used to writing Y on the Y axis of our diagrams and X on the X axis of our diagrams, like so. But we don't have to restrict ourselves to using Y and X values. Often in the social sciences, we want to rename things with other values than X and Y. For example, in economics, we'd like to let C be equal to consumption. See that in an example shortly, we'd like to let I be equal to investment. And we don't always want to choose Y and X as our variables of interest. So here's an example to get you used to using other letters than just Y and X, we've got W and V. And we're being asked to find the derivative of Y with respect to V, using implicit differentiation. So why don't we do that as another example.

I'm going to get started by taking the derivative of both sides of the expression. So I'm going to write

I'm going to get started by taking the derivative of both sides of the expression. So I'm going to write D over DV , we've got the DV in the denominator. And I'm going to take the derivative of the left hand side, so I'll write it like this. And we're going to take the derivative of the right hand side with respect to the, I'll write it like so. Now, remember that W , the value of W depends on the value of V . So if we change V , we're also going to change W . Now we can get started and take the implicit derivative of this thing. So we're going to get $4W, DW$ over DV . I'll add some brackets in there just to make it maybe a little easier to look at. Moving on to the next term, we just have negative two. The change in V is just equal to, with the change in V , with respect to the change in V is just equal to one. And this is equal to, we have a W over here so we're gonna have to change in W with respect to a change in V .

Now we'd like to collect terms. So I'm going to move DW over DB to the left hand side, and I'm going to move the negative two to the right hand side. So I'm going to add to to this equation, both sides of the equation I'm going to add two, and I'm going to subtract DW over DV for both sides of the expression. $4W, DW$ over DV minus DW over DV is equal to two. We can factor out DW over DV . So we have DW over DV , multiplied by $4W$ minus one, is equal to two. And now I can divide both sides of the expression by $4W$ minus one. We have DW over DB is equal to two, divided by $4W$ minus one. So if I want to answer this expression, if I want to know what W prime is, if I want to know the change in W with respect to V is, then I just need to know what my W value is, and this expression will give it to me.