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equal, derivative, squared, y prime, expression, implicit, function, write, prime, slope, steps, sides, negative, dy, rewrite, cubed, right hand side, change, ready, derivative operator

SPEAKERS

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Hello, everyone. In this module, we're continuing our look at implicit differentiation. In the previous two modules, we learned about what an implicit function was. And we learned about how to find the implicit derivative. Here, you can see the recipe that we came up with before. It's been rewritten and presented to you in a slightly different way. Once you have identified that Y is an implicit function of X, for example, now those variable, variable names might change. But those are the variables we're working with here. In general, you want to let Y be equal to F of X, or at least you can, can make things easier, especially when you're just getting started. And you're learning how to find the implicit derivative. You want to then, number two, apply the derivative operator to both sides of the expression. And that's something we might not have seen before where we're applying the derivative operator, not just to the left hand side or just to the right hand side, but to both sides.

Next, rearrange the expression to isolate and solve for the change in Y with respect to change in X, which you may have relabeled F prime of X as you did in step one. This step one is a method to help you as you're learning implicit differentiation. But typically in practice, we don't do steps one, or we don't do step one, I'll say. And step four is to sort of undo what we did. So you're going to replace F prime of X equal to Y prime. And you're also going to replace Y equal to F of X, if it still remains somewhere in your answer. So remember, the last line says steps one and four are optional. As you see going forward, I'm usually going to skip steps one and four. And I'm just going to do steps two and three. And so as you become more comfortable with the procedure, you too can stop using steps one and four, and just do steps two and three.

Let's take a look at our example. We're being asked to find Y prime, the change in Y with respect to the change in X, for a expression, which is Y cubed, plus 3X squared times Y, and that whole thing has to be equal to 13. Once we find the slope, we're being asked to compute the slope when X is equal to two. So to get started, we'll start with part A. We're going to take the implicit derivative of this expression. And to do that, we're going to take the derivative with respect to X for both sides of the expression. So we can write DY over DX, Y cubed plus 3X squared Y is equal to DY over DX 13. Now we're ready to take the derivative, we're going to need to use the chain rule on this first term right

here, because when X changes, Y is going to change. We know that, but we also know that Y is cubed. So if the Y cube represents the outside of the function, we're going to take that derivative first, so we get three Y squared. And then we're going to take the derivative of the inside, which is just Y prime.

Moving on to the next term, notice that we're going to have to use the product rule. So I'll factor out the three. Three is a multiplicative constant, so we can leave it on the outside. Taking the derivative, the first X squared, we get $2X$ and we multiply it by the other function Y , plus the change in Y with respect to XY prime times the first function, which is going to be X squared. Now to proceed, I'm going to open up this bracket on the right hand side here. And to do that, I'm going to write $3Y$ squared, Y prime plus $6XY$ plus $3Y$ prime X squared, and this whole thing is equal to zero. I could factor out a three here, and then divide both sides of the expression by three. So that's going to disappear and I could rewrite this as Y squared Y prime plus $2XY$ plus Y prime X squared is equal to, now to give ourselves a little more room I'm going to rewrite this expression right here at the bottom, going to rewrite it up here at the top.

And now, I'm going to continue to rearrange and I want to isolate the Y prime's, remember, this is step three, we want to solve for Y prime, and I'm going to have Y prime Y squared plus X squared is equal to, and I'll move the plus, the plus $2XY$ to the left hand sides, we have negative $2XY$ like so. And now all I have to do is divide both sides of this expression by Y squared plus X squared. When I do that, I have Y prime is equal to negative $2XY$ divided by Y squared plus X squared. And there's our answer for Part A.

Now how about Part B? In part B, we're told that X is equal to two. And we want to find Y prime. So if X is equal to two, what's Y going to be? Well, we've got Y cubed, plus three times two squared, that's our X , which is now two times Y has to be equal to 13. Well, if I keep going a little bit, I could rewrite this Y cubed plus two squared is four, four times three is 12. And we've got Y cubed plus 12 times Y is equal to 13. What value of Y is going to satisfy this equality? If Y is equal to one then this equality holds, we have 12 plus one is equal to 13. So Y is going to be equal to one. And we're ready to solve for Y prime, we've got Y prime, and we're using this expression up here. Y prime is going to be equal to negative two times two times Y , or we should say Y . Y is equal to one times one divided by one squared plus two squared. And the answer is negative four, divided by five.

Now when X is equal to two, right here, Y is equal to one at that point right there. And if I were to draw a tangent line, drawing the tangent line, we can see that the slope looks pretty close to negative four over five, it's pretty close to negative one. It's definitely downward sloping. So that negative sign is definitely true. It's downward sloping. And maybe I can just write that this is the tangent. And the slope of the tangent looks like it's, well, it looks like it's equal to negative four over five. So the mathematical operation we performed matches, what we see when we look at the graph.

Let's try another example. We've got Y to the power seven minus $3X$ is equal to some constant A multiplied by X squared. So it's going to be well, it's kind of a challenge here. If we wanted to try and explicitly solve this, the instructions tell us to find the derivative of Y with respect to X . And we'll just assume Y is an implicit function of X . Let's do the same procedure we did before. We're going to take

DY over DX, I'm going to take the derivative of this whole expression and I'm going to take the derivative of both sides and we're going to keep in mind that when the X value changes, so does the Y value. And so we're going to have $7Y$ to the power of six, that's the, we're applying the chain rule here. We took the derivative of the outside, what about the inside? Well, the inside is just a, just a Y. But when X changes, so does Y, so we're going to write Y prime to represent that change. And now we're ready for the next term negative $3X$. Well the change of that, when X changes, it's just going to be negative three.

And the right hand side we're going to have A is a multiplicative constant it remains. So we're going to have $2AX$. Now we're ready to isolate and solve for Y prime. Here's our Y prime right there. And I can start us off by rewriting this expression like so. And now we can divide both sides of the expression by this, these factors there, we get Y prime is equal to $2AX$ plus three divided by $7Y$ to the power of six. And there's no real way to simplify that. But we've taken the implicit derivative of this expression, and we found the change in Y with respect to the change in X.