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SPEAKERS

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Now that we know what an implicit function is, we can start looking at implicit differentiation. That's going to be the topic of today's video. So if you look at the problem of the circle that we had before, now we're going to go through a recipe to finding the slope of this circle, without having to go to the bother of explicitly solving for Y and then taking the derivative. Now we're looking at an expression like this one the circle, we want to recognize that the value of Y is going to depend on X. So Y squared plus X squared has to be equal to 25. So if X is increasing, that means we're going to have to decrease Y to make sure that they still sum up to 25. And that's going to be useful as well, because we can almost say that Y is kind of a function of X. Y is going to change when X changes. We'll come back to that a little bit later. The next thing we can do is, while the next step to this recipe is to take the derivative with respect to X, and we're going to want to apply the chain rule, the chain rule is going to be important here, and it's going to affect how we take the derivative of this term here. Finally, we want to rearrange the expression to isolate and solve for the derivative, which we will usually denominate with a Y prime using prime notation. But we could also use Leibniz notation, which would be DY over DX. And sometimes using that Leibniz notation is easier and clearer.

So let's take the implicit differential here. And let's find the slope when X is equal to three and Y is equal to negative four. So let's use the recipe we had before. Now, one kind of trick that we could use is we could say that, especially when you're getting started, let's let Y be equal to F of X. And I'm going to rewrite this expression as X squared plus F of X squared is equal to 25. And now let me take the derivative with respect to X on both sides of this expression. And when I do that, I'm going to have 2X, just the power rule there, then I'm going to have, and here's where I want to use the chain rule, have to F of X times the derivative of the inside F prime X is equal to what's the derivative of 25 with respect to X? Well, 25 is just a constant, so it's equal to zero. Now let's solve for half prime of X. And we could do that by subtracting 2X off both sides of the expression, oops. And I'm still solving for this thing here. So I'm going to divide both sides of the equation by two F of X.

And that leaves us with negative X over F of X. Now remember, we let Y be equal to F of X. So we want to undo that substitution. We did that substitution to make it clear how we were using this method, this implicit differentiation. Now let me replace Y is equal to F of X and let's say Y prime is equal to F prime of X. And if we make that transformation, we get Y prime is equal to negative X

divided by Y . I adjusted the size of the screen to give myself a little more room. Remember, the question is asking us to find the slope when X is equal to three, and when Y is equal to negative four, so let's do that. We've got Y' is equal to negative X over Y , which is equal to negative three over negative four, which tells us that the slope is going to be equal to three over four.

Our last step is to illustrate the slope that we found right here, this three quarters, we're going to illustrate with a tangent line on a graph. And in the slides, we've got this nice diagram prepared for us. And so we found that we're looking at the slope, at three negative four. And this slope is supposed to be equal to three over four, a tangent line that just touches, that just touches this expression. At the point right here, it does look like the slope is three over four, it's very close. And so we found the slope of this expression, when X is equal to three and Y is equal to negative four, it's three over four, you can see it right here on our diagram. And just to be clear, this line here, that's our tangent line.

So there, we did it the first time. Let's try it again. We've got a new example here, we're going to look at this one $2XY$ is equal to 12. And we're being asked to find the derivative and we're being asked to use the implicit differentiation method. So let's start off with A. And if we want to use the method, first thing, we recognize that Y is the value of Y , it depends on the value of X it's, so it's, Y is an implicit function of X here. Let's jump right into another example. We've got $2XY$ is equal to 12. And we're being asked to use implicit differentiation. So let's do that right away. We've got $2XY$ equals 12. Let's take the derivative of both sides of the expression with respect to X .

Now let's make sure we use the product rule on the left hand side, so we have two, and we're going to have the derivative of the first times the second. So we've got one times Y , plus the derivative of the second times the first Y' times X , and this is just going to be equal to zero because the derivative of a constant with respect to anything is just going to be equal to zero, the constant doesn't change. Now we can divide both sides of the expression by two, zero divided by two is still zero. And I can write Y plus $Y'X$ is equal to zero. Now we want to solve for that Y' . And so we could rewrite this as Y' is equal to negative Y over X . Now notice the question told me to use implicit differentiation. And I've got this expression here. If X is equal to six, maybe I will, maybe I'll do this next. So if X is equal to six, then this implies that Y is equal to one. Right? If we've got two times six times Y is equal to 12 and Y must be equal to one and if that is the case, then this implies that the slope is going to be negative one over six. And that's part B.

There you have the implicit differentiation. But let's take another closer look at our example up here. Notice that we could solve this for Y , we could rewrite this as Y is equal to 12 divided by $2X$, which is like saying Y is equal to $6X$ to the negative one. And if we take the derivative of Y with respect to X , we're going to get negative six over X squared looks different, it doesn't look like what we had when we used implicit differentiation. But let's try and plug in our value of X , X is equal to six in here and see what we get, we get dy/dx , which is just a way of writing Y' is going to be equal to negative six over six squared, which is $-1/6$. And so we end up with the same slope using either method.

So we used implicit differentiation because that's what the problem told us to do, right here. But we could have solved for Y explicitly and then taken the derivative. We would have gotten an expression

could have solved for Y explicitly and then taken the derivative. We would have gotten an expression like this, where we've just got an X on the right hand side and no Y . But if we do implicit differentiation, we had a, we had a Y on the right hand side and an X right down here. But they both got us to the same place, the answer using either method is going to be the same. We can check our results on a graph on a diagram, you can see there's a diagram in front of you on the slides. The expression here that we were evaluating was essentially Y is equal to six divided by X . You can see it graphed here. Where were we analyzing the result? Well, we were analyzing the result when X was equal to six, and we showed that when X was equal to six, Y was equal to one. And at that point, it's possible to draw a tangent line just touching the, in this case it's a function, this expression when X is equal to six Y is equal to one, tangent line just touches at this point. And it looks pretty clear that the slope is indeed negative one over six. It's downward sloping, so it's negative. And it's, you know, just a little bit below zero. So it looks like we did a good job, our calculus, our implicit differentiation is matching what we would get with Graphical Analysis.