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SPEAKERS

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Hello, and welcome. My name is Professor Robert J McKeown. And I'm really glad that you're here with us, beginning our journey through applied differential calculus. Now, let me begin by congratulating you for completing the earlier modules on calculus, you learned about how to measure a slope with limits, you learned about the different shortcut rules of differential calculus, things like the chain rule and the product rule, you definitely need those rules, you're going to be using those rules all the time in this module. So make sure that you've got a firm grasp and you've done practice so that it's become something of second nature to you. What you can expect in this module, is that you're going to be applying what you've learned to more complicated expressions, more complicated math problems. Now, the reason why you're doing that is because these more complicated math expressions are more like the complicated real world problems that you're going to be confronted with and trying to solve as a economics major business major, and being a student and practitioner in the social sciences, and more broadly.

A useful analogy is to think of Sir Isaac Newton who invented calculus. He invented calculus not for the purposes of simply inventing an interesting mathematical tool, he invented calculus in order to better understand and develop his theory of gravity, which we still use to this day. And similarly, you are learning calculus, not to say that you know calculus, but so that you can apply it to problems in the real world and have better insights into your field of study, such as business in the social sciences in economics, which are really all connected together. So I hope you're ready, we're going to start, and we're going to start by understanding the difference between an explicit function and what we call an implicit function.

Now to illustrate the difference between an implicit function and an explicit function, let's look at an example. And the example is on the screen in front of you. And the question says is a circle a function? And then we've got a graph of a circle, just in case you forgot what one looks like. But you may have forgotten what the equation for a circle is, and that's right here. A simple test for a function is called the vertical line test. And the vertical line test says that if at any point on the graph, we can draw a vertical line that intersects with the expression more than once, then the expression is not a

function. So to illustrate the vertical line test, let's consider X is equal to three. So here's X to equal to three on the diagram. If I draw a vertical line starting at X is equal to three all the way up, then we can see that it's going to intersect this expression when Y is equal to negative four. And it's also good across the expression when Y is equal to four. A circle $X^2 + Y^2 = 25$ fails the vertical line test.

So a circle is not a function. Now specifically, why is a circle not a function? A circle is not a function because for some X values for example, X is equal to three for some X values, there are more than one Y value. To illustrate, we could take $Y^2 + X^2 = 25$. And let's try and isolate for Y . We could rewrite this as $Y^2 = 25 - X^2$. Now let's factor the right hand side. So we'd have $Y^2 = (5 + X)(5 - X)$. Taking the square root of both sides of the expression, we end up with $Y = \pm \sqrt{(5 + X)(5 - X)}$. Of course, the square root can never be negative, that will give us an imaginary number. And you can see that X is never going to be greater than five, and it's never going to be less than negative five. Because we're taking the square root, we want to remember that this expression being squared, if it's negative, or if it's possible to get the same answer, so we can't be sure which one it's going to be. So we have Y is equal to plus or minus the square root of five plus X times five minus X .

Now we can write Y as an explicit, as an explicit function of X . Yet, we can see here that the value of Y depends on the value of X , according to this expression here, even when we rearrange for Y , it's still not explicit. And so we say an expression like this one is such that Y is any implicit or is defined implicitly as a function of X . Implicit functions are quite common, and allows us to have more complicated expressions. So we can model more complicated real world phenomena. And in the next series of videos, we're going to look at the implicit derivative, which is going to help us find the slopes of these more complicated expressions without having to solve for Y explicitly and take the derivative. So we'll be able to find the slopes of these more complicated expressions without explicitly solving for, say, the Y variable and then taking the derivative of it because, as we'll see in future videos, some expressions are too complicated. It's not possible to do that.