Module: Introduction to Algebraic Expressions

Module Outline

- Example 1: *Economics*
- Example 2: *Politics*
- Example 3: *NBA* — Excel/Google sheets
- Example 4: *Geography*
- Example 5: *Model of Infections* — Excel/Google sheets

Example 1: Economics

- Maya's monthly expenditures:

  \$400 on rent, \$300 on food and drinks

  + 1 *GAMESTOP* stock, priced at \$170

  +10 *FORD* stocks, priced at \$12 each .

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  $+$ 1 *GAMESTOP* stock, priced at \$170

  $+$10 *FORD* stocks, priced at \$12 each .

- What is her total monthly expenditure?

- $400 + 300 + 170 + 120 = 990$

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Example 1: Economics

- **More general**: If she buys x *GAMESTOP* stocks

  $+$ y *FORD* stocks, what is her expenditure?

- \$400:rent, \$300: food, x *G_STP* @ \$170; y *FF* @ \$12.

- Expenditure $= 700 + 170x + 12y$.

  If her monthly income is \$1500, what is her savings?

Clicker question 1.1

- In this example, if Maya also buys z Air Canada stocks priced at \$30 each, what is her total expenditure?

  Recall: \$400:rent, \$300: food, x *G_STP* @ \$170; y *F* @ \$12.

  (a) $1500 - 170x - 12y - 30z$

  (b) $700 - 170x - 12y + 30z$

  (c) $700 + 170x + 12y - 30z$

  (d) $700 + 170x + 12y + 30z$

Solution to Clicker question 1.1

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Example 2: Politics

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  Valley: 800 eligible voters; 90% will vote for party **L**.

  Plain: 1200 eligible voters. Is a **swing** district.

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- *Survey*: finds fraction x of the voters in Plain say they will vote for party C. The rest said they would vote for party L.

  Question: *How many votes does party C hope to get overall?*

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  Plain: 1200; fraction x for C:
- If $x = 0.5$ (i.e. 50%) :
- More generally:

Clicker question 1.2

- In this example, how many votes does party **L** receive? Recall:

  Hilly: 1000; 80% for **C**. **Valley**: 800; 90% for **L**:
  Plain: 1200; fraction x for C:

  (a) $920 + 1200(1 - x)$

  (b) $880 + 1200(1 - x)$

  (c) $920 + 1200x$

  (d) $880 + 1200x$

Solution to Clicker question 1.2

- In this example, how many votes does party **L** receive? Recall:

  Hilly: 1000; 80% for **C**. **Valley**: 800; 90% for **L**:
  Plain: 1200; fraction x for C:

  (a) $920 + 1200(1 - x)$

  (b) $880 + 1200(1 - x)$

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- Answer: C can hope to win if the survey reveals that more than 51.67% of the voters in Plain plan to vote for C.

Example 3: Sports Analytics

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- **NBA**: How to compare players?
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- Individual player efficiency **EFF**:

$$\frac{PTS + REB + AST + STL + BLK - MissedFG - MissedFT - TO}{GP}$$

$PTS =$ points, $REB =$ rebounds, $AST =$ assists,

$STL =$ steals, $BLK =$ blocks, $MissedFG =$ missed field goals,

$MissedFT =$ missed free throws, $TO =$ turnovers,

$GP =$ games played (can be found on NBA.com)

Example 3: Sports Analytics

- **Excel/Google sheets**

  Use formula function to compute

$$EFF = (D3 + E3 + ...)/C3$$

  Copy and Paste Formula for other players

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Excel/Google sheets activity

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  Do player rankings change as a result?

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- **Think and Research:** How to measure NFL players?

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- It decides to plant some trees which require a spacing of 1m between each.
- How many trees can it plant in a row along the width of the tract?

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Tract of land 99 m wide and 200 m long to plant trees.

- How many such rows can it plant?

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Tract of land 99 m wide and 200 m long to plant trees.

- How many such rows can it plant?
- If the tract of land were x m wide and y m long, what is the total number of trees it could plant?

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- On day $\mathbf{t = 3}$, number of new infections:

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- $\mathbf{t = 1}$, number of infected people $= 1$
- $\mathbf{t = 2}$, number of new infections: : $np \equiv \mathbf{R}_0$ **[Reproduction number]**
- $\mathbf{t = 3}$, number of new infections: $(np)^2 \equiv (\mathbf{R}_0)^2$

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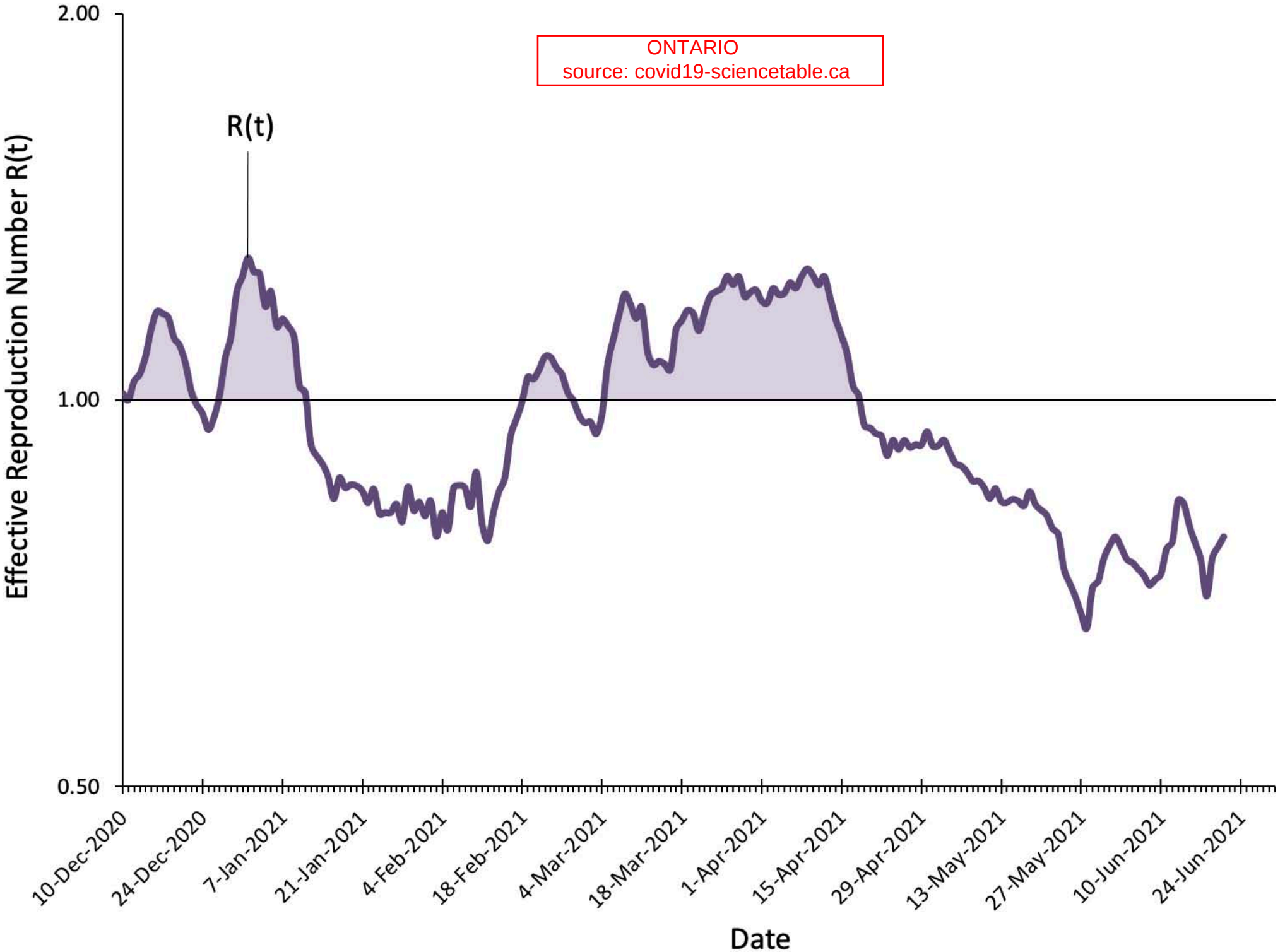
- $\mathbf{t = 3}$, number of new infections: $(np)^2 \equiv (\mathbf{R}_0)^2$
- On day $\mathbf{t}$, number of new infections:

All Variants Combined



Example 5: Spread of infections

- **Excel/Google sheets**
- On day t, number of new infections: $(np)^{t-1} = (\mathbf{R}_0)^{t-1}$

Clicker question 1.3

- Suppose day $t = 1$ begins with 10 infections.

  How many new infections will there be on day t?

  (a) $(np)^{t-1}$

  (b) $10(np)^{t-1}$

  (c) $(10np)^{t-1}$

  (d) $(np)^{t-1} + 10$

Solution to Clicker question 1.3

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Example 5: Spread of infections

Excel/Google sheets activity: Spread of a pandemic

- **Think and Explore:** In this model, how do lockdowns affect the spread? How do vaccines affect the spread?