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SPEAKERS

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So so far, I've introduced you to the idea of random variables and talked about its probabilities, both in terms of the probability mass function, which is the probability of particular points, and the cumulative distribution function of the CDF, which is the probability of taking values less than. So what are all the distributions and examples that are presented to you, the number of tables fill that restaurant or ice cream sales, depending on temperature, these are fictitious examples, right? Mostly that I have made up. But there are certain distributions which encounter in the real world, and which are very useful in the real world. And so what I'll be talking about is some common distributions of discrete. So let me begin with the first one, which is called the Bernoulli distribution. So the Bernoulli distribution. So this was first discovered, or first proposed by a Swiss mathematician Jakob Bernal. And the idea is very simple that suppose a simple experiment is conducted. And the outcome from this experiment can either be a success, or a failure. So let's say Z tilde is the random variable. And it when it's a success, we say that Z tilde takes the value one. And when it's the experiment as a failure, we say that Z tilde takes the value of Z . So this is what's called a Bernoulli. Random. So it just takes two values one, and zero. And let's say p is the probability of success. So it takes the value one with probability p , and zero with probability one minus p . So this is the simplest possible random variable. So this is this is a Bernoulli random variable. And this distribution is called the Bernoulli. Distribution. Very, very simple. And if you wanted to compute the mean of this, right, so the expectation of Z tilde would be just one times p , and zero times one minus p . So that would turn. So that's the mean of this distribution. What's the variance? So if you recall, the formula for the variance is expectation of Z squared, minus the expectation of said whole thing squared. So this part, we know the expectation of Z is p . So this part is minus p squared. So what about the first one the expectation of Z squared. Now, if you look at that squared, it can either be one squared, which is one, or it can be zero, and it takes one with the probability p . So expectation of Z squared will be p times one, and one minus p times zero, right? So this, so the variance will just be p times. So that's the variance of this Bernoulli random variable. And one other thing we talked about the standard deviation, which is just the square root of the variance will be squared. Now I've presented to you this as the outcome from an experiment, but it may not be an experiment, it can be anything, anything we basically tool. So for example, suppose you go on a blind date, right? And what's, and you meet a person

there, and whether you like the person or you don't like it, so that's a two outcome. And maybe you do know this variable, that $\tilde{x}$ is equal to one if you like the person, and that $\tilde{x}$ is equal to zero if you don't. So that's another example of Bernoulli. So this is this is called an indicator variable, because it stands it's indicate something right? So one stands for like zero 10 cents. And there are lots of similar examples in the real world. It's basically any outcome. So So for example, suppose you pick a random person, what's the probability that he or she is wearing an orange shirt? So in this case, it will be one if orange. And it'll be zero, if anything else. You could ask something else, right? What's the what's your pick this random person? What's the probability that he or she is university? So again, you took the random variable, that $\tilde{x}$ equal to one if university educated zero. This literally would be a person, it could be random tree in a forest. Right? And then the same thing that is it taller than 20 meters, if height is greater than equal to 20, and zero, otherwise. Is the tree diseased or not? Right? So any two outcome even can be it is basically around Bernoulli. So this is the simplest possible distribution. But it's also not very interesting for us because it's just a single experiment or a single purse. So more interesting case, is what's the next distribution that we'll be talking about, which will be the binomial distribution, which is also was first discovered by Jakob Bernoulli. Same person with the Bernoulli distribution. So what's the binomial distribution? So the binomial distribution is that suppose you do n independent and identical? So what does it mean? So you run that same experiment, let's say n times n independent. And each trial can either be a success, or it can be a failure? Right? Each trial has in two outcomes success or failure. But instead of just one, now you're conducting n of them. And let's say you are now interested in the total number of successes out of so you ask the question, what's the probability that let's say you have four successes out of, or you have two successes out of, or no successes. So your random variable here now counts, the total number of successes of this random variable is set to follow the binomial and P distribution. So it's the variable which gives the total number of successes out of n independent and identical shots. So what are examples of it? Right? We can use the same examples that we had in Bernoulli. But instead of think of, instead of sampling one tree, now, suppose you sample 10 trees in the forest? What's the probability that three of them are deceased? Or what's the probability that all 10 of them are deceased? So that would be an example of a binomial distribution? Or you pick a random sample of eight? Students? Okay, what's the probability that one of them is wearing an orange shirt, or five of them are wearing an orange shirt? So that variable, which which counts, the total number of orange shirts be worn out of a sample of eight? That's an example of a binomial distribution. Now of course, what's important is what are the probabilities? Right? But even before we can get started with that, right, so let's think about what are the possible out trade what are the possible values it can take? Right? So it can be no successes out of n , or one out of n ? Two out of n , so on, and what's the maximum? It's n out. So the possible values that this extender can take are these values zero through so in our language that we had used before, this is called the sample space. So these are the possible outcomes. But the more important bit will be, what are the strengths? What's the probability that it takes particular. So let me introduce you to that. And the probability is given by this thing here. So the probability that the random variable takes the value x is given by what's called n choose x times p to the power x times one minus p to the power n . Now let me explain what this n choose x is. So this is n factorial, divided by x factorial by n minus x . So that's the meaning of this n choose x . And if you don't know what a factorial is, so a factorial of a number like x tilde is multiplying x times x minus one times x minus 2x Minus three all the way down to one. So that's the definition of the factorial. So this is the binomial distributions probabilities. Let's do an example. And maybe that will show you how to use this form. So let's look at this example that suppose you pick five people at random. And the probability of anyone being university educated is point three, and the several, because if you look at the Canadian population, but 30% of the population have university degrees. So now the question is out of this five, what's the probability that exactly three in the sample or

university? So we are exactly in the realm of binomial distribution, we are conducting five trials, right? So that's the five people at random, the probability of success of each trial, right the p , the probability of anyone being university educated this point three. And the question that we are asking is, what's the probability that X tilde is equal to three? What's the probability that exactly three in the sample or university so this is binomial, n equals five P equal to three. So let's look at the formula. So remember, this was n choose X times P to the power X , and one minus P , six. So here, we have five is n . So five choose three times P , that's point three to the power three point, because we're trying to find x equal to three, right? And one minus P which is point seven to the power of the rest and minus X . So first, let's review this, the five choose three. So that's five factorial, divided by three factorial, and five minus three factorial that stupid. So what is this? So five factorial is five times four times three times two times one was three factorial, so that's three times two times one. And what's two factorial, so that's two times you see this three, two, and one, this cancels out with this. So what you have left is five times four is 20 divided by two. So this comes out to be 10 times so this is point three cubed. So that's 0.027. And this is point seven squared. So that's point. And if you use a calculator to figure so this is going to be point three. So what this this what this tells you is that out of the sample of five universe five random people, the probability that exactly three have university degrees is 13.2%. So this is how you use binomial distribution to figure out. So let me stop this clip here. And in the next clip I'll pick up from here and talk more