

module_prob3_lecture10

Mon, 2/7 10:36AM 15:45

SUMMARY KEYWORDS

variance, squared, expectation, calculate, multiplied, random variable, similarly, measure, square root, values, x minus μ , formula, standard deviation, probability, called, μ , stock, riskiness, spread, cake

SPEAKERS

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So the previous two clips, I talked about the expected value of random variable. And in this clip, I'm going to talk about another important measure. This is called the variance. And what the variance tries to do the strange to measure how dispersed or spread out the possible outcomes of random variable. So let me try to give you the basic idea using an example. So consider random variable X . And see these are the possible values of x . And here's the mean of this random variable, so called center on the district variable. And now suppose you want to ask the question, how spread out how far apart are these points. So one way to do it is that if you look at x one, and look at its distance from the mean, x one. And similarly, look at x two to smooth. Similarly, look at, let's say, x seven, and look at its distance from the mean, and so on. The problem is that see, if you just add up all of these, right, so some of these are like this is negative because x one is below the mean, x two is below the mean. So x two minus μ is negative, and this $1x$, seven minus μ , because x seven is bigger than the mean, that's positive. Right? So some of these accounts will cancel each other. But if you want to just measure the spread, you don't want that happen. So the way to do it, is that you take the squared, because if you do take the square of a negative number that becomes a positive. Similarly, if you look at x two minus mean, this is negative, but if you take square root that becomes a positive. And to maintain consistency, you do that for all of these distances. The second thing that you do is that, look at this. So this is the distance of x one from the mean, now highlight how likely is x one to come up? The probability of that is p of x one. Similarly, how likely is x two likely to come up? It's p of x two. So if you know multiply each of this by the relative weights, and then add all these up do this for x three, and so on. And you do this for all the possible values, yes, you get a measure of how spread out the random variable is. And this is called the variance. So so that, so the definition of variance said, is given by x minus μ squared multiplied by the probability, and you add it up over all the possible. Now if you recall, your expect the, the lecture from expectation, see each of these values multiplied by the probability, that's the expectation of this. So the variance of x is nothing but the expectation of x minus the mew whole thing squared. So the definition of the variance of X is that it's the expectation of X minus μ X . And the variance is denoted usually by σ^2 . So this is a Greek letter sigma. And it's denoted σ^2 , and the squared is to reflect the square. So this is the definition of variance. Once again, it's expectation of X minus μ X squared. And to calculate it, all you have to do is look at every possible subtracted

from the mean, square that and then multiply it by the crosswalk. And you add this up across all this idea of variance, right? So that's the definition. But this idea of variance is super important anytime you're looking for, to measure variability. So for example, you look at the stock. And you care not only about the returns the the expected return from the stock, but you also probably care about, what's the, is it possible that the stock will really go down? Or will really go up? How much jumping around is there in the stock? So how would you measure that? That's variance, so that gives an idea of the riskiness of the stock. Similarly, these days, there's a lot of talk about how weather has become a lot more. So how would you measure that? What do you do is measure the variance of the rainfall, snowfall temperature, and the sun there's one other thing which is connected, right? This is called the standard deviation facts. So that's just the variance, the square root of the variance. So it's the square root of sigma squared. And so that is sigma x. So this is called the standard deviation. And this is often the short form. So now that I've introduced you to this concept, so let's do an example where we'll calculate the variance. So this is our old example of cake sales. And I've just looked at one location, let's say location one. So the possible values are zero 10, 20, 30. And we've calculated the mean of this before, so that zero times point one plus 10 times point six, zero, this is 10, times point six, that's 6. 20 times point two, that's 4. 30 times three, so this is now, in calculating the variance, it's the expectation of $X - \mu$ squared. Right? So it's $x - \mu$ squared, multiplied by the. So let's calculate those things here. So let's look at $s - \text{the mean Holtec}$ squared. So this is zero, minus the mean 30. So this is squared. Here, it's 10 minus the mean 13. So that's three squared. This is 20 minus 30. That's seven squared. And this is 30, minus 1370. And how do you get the variance, all we'll have to do is take this 13 squared, multiplied by the corresponding probability does. Same thing, look at this, which is three squared, multiplied by the corresponding six. Same here, seven squared, multiplied by two, and the last one is 17 squared, multiply. And if you do this calculation, the μ 61. So the variance of cake sales in this location is 60. And you can compare it with the variance of cake sales in the second location, location two, you can calculate that yourself and check if one is more variable. And now that we have the variance, which was 61. Now the standard deviation will just be the square root of that. So that's great. So let me give you another an alternate formula for variance, just deriving it from this basic definition. And sometimes this alternate formula is easier to calculate. So how do we get this alternate formula since the same variance right, but I'm just going to present it to you in a different form. So this number is expectation of $X - \mu$ squared. So let's use the quadratic formula. So if we open up the brackets here, so this will become x^2 , right? plus μx squared minus two times the first term terms the second. Now remember the rules of expectation, right? So this is this is plus this is minus, so we can break this up into expectation of x^2 . This here, see this is a constant, this is a mean, right? So like in the previous example, it was 30. So this would be 13 squared. So in general, so this would just be μx squared minus expectation of two times x times meal effects. So let's do this. Let's keep this the same. This is new X squared minus c , this is a constant, two is a constant, new x is a constant. Let me take that outside. And what's left is expectation. I've just taken the constant themselves. So now see, this will be expectation of x^2 , of keeping this the same. And I'll see what is e of x , e of x is just the mean of x . So that's new X . So what I get here is two times new X times another new X , so that's minus two squared. So what this comes out to be is expectation of X^2 minus c , there's a new X squared here, there's a minus $2x$ squared. So this becomes minus square. So this here is an alternate formula for the variance of x . It's the same thing, just rearranged in a different way. And I'll show you why. In some cases, it's easier to calculate. So the ultimate formula is the variance of X is expectation of X^2 minus μx squared. And if new X is just the expectation of x , so if I put that in, so it's expectation of x^2 , minus the expectation of X whole things. So another formula for variance, and you'll often see this is given by this, that the variance is going to be the expectation of X^2 minus expectation of X whole things. So

let's use it in our in our cake formula. So remember, we have the expectation of sales is 30.
 Right? So what we need in order to get the variance is expectation of S^2 minus
 expectation of S whole thing squared. So this part we know is just going to be 13 squared. The
 other part we still have to calculate, which is expectation of S^2 . So how are we going to
 calculate this, so C , this is S is equal to zero, s^2 will also be zero times point one. This is
 10 squared will be 100 times its probability point six. This will be 20 squared is 400. Multiply
 that by point two. And this will be 30 squared, that's 900 times point one. So if you if you
 calculate this, together, these will add up to two. So this is expectation of S^2 . So this will
 just put into 30 minus this. That's 230 Minus 169 So it's the same value that we had got before,
 which shouldn't be any surprise, right? Because it's the same variance formula. But it's the
 same variance. What we are just using is a different reformulation of that same form. So do try
 and remember this because this is a useful formula. Sometimes it's easier to calculate. So let
 me stop