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So in the last clip, I introduced you to this important idea of finding the expected value for a variable. So in this clip, I'm going to extend the idea of a little bit more to talk about some of the properties of the expected value function. So the basic idea of finding expected value is that if you're finding the expected value of a random variable X , take each of the values of the variable multiplied by the probabilities and add all of these up. So this extends this idea that instead of just the direct random variable X itself, if you're looking at trying to find the expectation of a function of the random variable, a function g of x for a variable, and I'm going to talk about examples of that in a minute. You do exactly the same. So what you look at is g of x and multiply it by the corresponding probability effects. And you add this up over all possible values of x . Right? So now, instead of for example, this is the end variable, that instead if I want to figure out the expectation of a function of this random variable, so instead of these random variables are put in the function values. And then multiplied by the corresponding probabilities that add up across. So what sort of examples are these? Now recall? Earlier, we talked about how you could have a random variable apart of tables in a restaurant? And from that derive another random variable, which is the profits so that profits were a function of the number of tables occupied in the restaurant? Or similarly, the ice cream parlor? You had? The profits depend on sales dependent on the random variable temperature, right? So sales were a function of the random variable temperature. So let's do a similar example, in particular context here of tree trunks in a forest. So maybe one is interested in the lumber yield from a tree. Right? So how much lumber can you hope to get from a tree of diameter D . And let's see, this is given by a formula like this $10 \pi D^2$. So I'm not exactly telling you how that formula is derived. But what it is related to is that if you have a tree trunk of a particular diameter, what you're interested in is the area of that circle, and then multiply that by the length of the tree, right? And that gives you some idea of the total amount of wood that you can expect to get from. So this here is a function of D . And what you could be interested in is what's the expected lumber? So how do we do this? Right? So we have here, the D can take values 12, 3, and 4. And what we are interested in is figuring out the mean yield. So you have why. So first thing, let me write down the various values that this function can take. So when D is equal to one, when t is equal to one, what's the value of this? There will be 10π times one squared, so that will be 10π . What about when it is two, so when d squared is two, it'll be 10π times two squared, that's four times 10π . That's 40π . When

D is equal to three, so this d squared will be nine, so this will be 90π . And when D is equal to four, right, so four squared will be 16 times standard 160. And what we're interested again, is finding out the expectation of this yield y , right? And how do we find it? It will be this value multiplied by its corresponding probabilities. So that'll be 10π , multiplied by point three plus, take this value, which is 40π , multiplied by its corresponding probability, which is four, then 90π . This multiplying by its corresponding probability 160π multiply by its corresponding probability point one, add all these up. Right. So this is then three π . This is what is the 16 by this is 18 by, and this is going to be 16. So, if you add all these up, so they add up to 16 and 16 is 32 plus 18, that's 50. And so 53π , and now, we can put in the value of π , which is 22, or seven. And if you calculate this out, this will be 166. Point fives. Right. So, the expected yield is going to be 166.57. But let me emphasize the idea once more. So, what do you have looked at is the various values that this function can take and multiply it by the corresponding probabilities, right, so, each term multiplied by the corresponding probability and add up. That gives you the expected value of this function of. And you can do this with any function, it need not just be this, it could be $10\pi DQ$ square root of D , so on, but you will follow exactly the same. Now, let me talk about some properties of expected value. And these are very useful. And not just for expected value, we'll see that they will turn out to be useful when we go forward into variance, which is the next topic of the scope of this module. So, first thing, if you taking the expectation of a constant, it's going to be a const, it will going to be that constant. So, for example, if you take, what's the mean of three, right? That's not even a random variable, it always takes the value three. So what's the expected outcome there is going to be three? Or what's the expectation of π ? Right? That's always going to be okay, that's something very, very simple, right? Now, we know that the expectation of meaning the mean of variable is just given by X times P of X . What about if we do A plus B times X ? Now see, this part is a constant. So the expectation of that constant is just a . And this is a constant b , which is multiplying two random variable X tilde. Right? So if we were to find expectation of b times x tilde, right, so what that would be is every time you're multiplying B times X , multiplying it by P of X , right? So you can take that be outside. That's common to all of those terms, you can take that outside, what's left is XP offense. And see this here is just the expectation of x . So if you're multiplying a random variable by a constant, you can bring that constant outside and what's left is just going to be the expectation. Right? So overall, expectation of a plus Bx tilde is going to be a plus b times expectation first. Okay, now let's consider one other property. This is if we have two random variables, x tilde and y tilde. And we want to look at the sum of B of X tilde plus C of Y tilde. And we want to find the expectation of this sum. What it is, will be it will be the sum of the two Separate expectations. So this part will be b times expectation of X tilde plus C times expectation. So let's, let's do an example where we make use of this. So consider a cake shop, which has two locations. And here are the sales in location one, this is given by S one tilde. So that zero, 1020 and 30. And in s two, which is the bigger locations, the sales can either be zero 2050 Or half, right? And you have the probabilities there. So first thing if you're going to look at the expected sales, so in store one, right? So again, you do zero times point one, that's zero, plus 10 times point six, that's 6, 20 times point two, that's four plus 30 times point one, that's three. Right? So the expected sales in Location One is, what's the expected sales in location two? It's, again, zero times point one. So that's zero, or 20 times point two, that's 4. 50 times point five that's 25. 100 times point two that's 20. So that's 49. Right? So the expected sales in the second location are high. Now what about profits? Right? So let's see, the baker, the shop, it has a fixed cost of \$25, for location one, and \$100 for location two. For ice cream, it makes it location one it makes \$1 profit per ice cream and location two, it makes \$1.50. So now, if we want to look at the total profits, right, so if this is the sales in Location One, it makes \$1 profit per ice cream minus the cost 25. And in location two, it says $R S$ two. And for ice cream, it makes a profit of dollar 50 minus \$200. So if you're going to be looking at expected profits, you're looking at the expectation of this, right, so this is the expected profits. Now see,

there's one random variable here, there is another random variable here, and there's this constants. Right? So we know that the expectation of the constants are just the constant, so there is minus 25 and minus 100. Let me take that outside that minus five. And what we have left is expectation of S_1 plus 1.5 times S_2 . So, so we're going to make use of the rule that we just learned this expectation of S_1 plus 1.5 times expectation of S_2 minus 125. So we already figured out, remember, we figured this was 13. And this is 49. So we're just going to put these things here, this is going to be 13. This one is 1.5 times nine minus 1.5. And if you do your calculation of this, so it makes a loss of 38 point. So the expected profits for the skate shop is minus 38.5. But here is where we are making use of the rule that we just learned. So the expectation of the sum of two random variables is just the sum of their expectations so the X But the profit we caught was minus 38. So before ending this clip, I'd like you to attempt a clicker question. So you may wish to stop the video at this point, and then the clicker question. So hope you got a chance to do it. Right. So what the trigger question asks is that a random variable Y has expected value of \$20? What's expectation of $40 + 5y$. So this is just going to be 40 plus five times expectation of y. So that's 40 plus five times 20. So this is 100. This is 40. So this will be 100. So the correct answer to this question. So let me stop the clip here. So here we learned all about the properties of the expected value function. In the next clip, I'm going to talk about another important measure called variance.