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So the previous clip, I introduced you to this idea of the cumulative distribution function, CDF. And we derived that for a particular example. So let's continue with that. So in that example, the random variable set could take these five, these five values minus one minus 100, minus 50, zero 50 150, right. And what we had is the probabilities of it taking the value 100 was zero, the probability of taking the value minus 50 was point two, the probability of it taking the value zero was point four. So and using that we derive the CDF at these points that F of F of minus 100 is point one, F of minus 50 is point three. Now the question one could ask is, what's the probability of in between those? Right? So now we know that if we ask the question, what's the probability that the profits are? What's the probability that the profit is 25? This, this is not a possible profit for this firm, right? Because this restaurant only can make this possible levels of problem. So the probability of it taking the value 25. That's zero. So the PMF at 25, zero, but what about the CDF? Right? So the CDF, let's say F of 22? is the probability that it takes profit makes profits less than or equal to. Now this is not zero, because see, how can it make profits less than or equal to 22, it can make her the profit zero, or it can make profit minus 50. Or it can make profit minus 100. Right, so the probability of this is of making profits less than or equal to 22, which is F 22. So the CDF 22 will be the probability of minus 100 plus the probability of minus 50 plus the probability of zero. So which is going to be if you add this, this and this up, that's going to be point seven. So it's the same as the as the probability of making profits less than or equal to C . So C because it doesn't make anything from any profit in between zero and fifth, right. So, the CDF at F 22 or at any points between zero and 50, it will remain as point seven right. So nothing else will get added to it until it reaches the point F of 50 Right. So this is the probability that Z tilde is less than or equal to 50, then it will be all these three values plus the probability that it makes profits of 50, which is point two. So that will become the jump up 2.2. Similarly, if you look at the CDF and in between points, right, so if you look at in between this 101 50, right, F of 124. That means the what's the probability that the profits are less than or equal to 124? It's the sum of it's the sum of these these profits, right? And which is going to be point. So if you plot this CDF, right, for this particular function, you'll see that the bit set between between minus 100 and minus 50, it will remain at point one, then as minus 50. This will jump up 2.3. There will remain point three up to all of this and then at zero, it will jump up to point seven, right? And again, between zero and the next highest which is 50. It's going to

remain at point nine. And then it's going to jump up to point nine, five at the point 100 and so on. So the CDF here, and this is particularly true for as a discrete random variable, and it will not be true for a continuous case will involve what's called a step function. Because see, this is going up in steps. One other thing that I want to point out, like if I asked the question, what's the probability that the profits are going to be less than or equal to negative two? So see here for this restaurant, the minimum profit we'll make is minus 100. So what's the probability that we'll make profits less than or equal to minus 200? That's going to be zero, because that's not possible. Or the other extreme verifiers, the probability was that it's the the restaurant makes profits less than or equal to 200. See, this is always going to be true because the maximum profit that makes us 150. So anything beyond that, right, if I asked, What's the profit, what's the probability that the profits are going to be less than or equal to 200? Right? That's always going to be true, because its profits are my either minus 100 minus 50, zero, and so on up to 150. Right? So this, the probability of this event is so again, emphasizing that this is less than or equal. I'm not asking the question that the profits are equal to, I'm saying, what's the probability that its profits are at most? So again, coming back to this graph, you see from from the minimum point below that it's going to be zero, the CDF and, and wherever is the maximum point that 150 above that this is going to be so this is how the CDF looks like as a function, right? It's a step function, it jumps up at every point at every of these. Okay, so this gives us also, this is although I did this as an example, right, many of these problems with all of these properties are, are true of CDFs, in general, for discrete functions, and you will see that they all will also be true for for continuous random variables. So first thing, the CDF is going to be between zero, right? So these are probabilities, so they will always be zero. Secondly, f of x , right, the CDF is non decreasing, because every time either the probability is remaining the same or going. Right, so it can never come down. Because at every stage, either you're adding a probability, write it so this can never come down. Third thing, if you look at the maximum possible value of the random variable, suppose that's $x_{\max}$, I call it $x_{\max}$. In the earlier case, it was 150. So that means if I look at what's the, what's the CDF for values of X beyond that $X_{\max}$, that CDF is going to be? Because again, if I look at, what's the probability that X must be less than or equal to $X_{\max}$. So what's the probability that it takes all the values it takes is less than or equal to $X_{\max}$? That's going to be one because $X_{\max}$ is the largest possible value of X , right? So it's still the always take values less than or equal to X . So this property is good. And this is also true for if I take any X which is beyond $X_{\max}$, right? Like in the previous example, they did f of 200. Now, on the other side, right, if I look at the minimum possible value of X $x_{\min}$, right, so X still takes values between $X_{\min}$ $X_{\max}$. So what if I asked what's the probability that it takes values less than here? Right? That's not possible because the minimum value that takes us $X_{\min}$. So f of x must be equal to zero for whenever X is less than. And again, same thing for us the question, what's the probability that it takes values less than or equal to here? So that's always going to happen. Therefore, f of x is equal to one for X greater than X . So let me, let me stop here for a second. And ask you to take an attempt at the clicker question, which is based on these properties. So hopefully, you were able to turn that clicker question, right. So what it asks is that here's a random variable to the following probability mass function. And what I'm asking for is what is f of f of six? Now, see 60, is the maximum possible value of X f of 60? is the probability that this random variable takes values less than or equal to 60. And that's always going to be true, because all the all the vowels that it takes are all less than or equal to 60. So this is equal. So the correct answer to that? So in this particular, so so far, right, so if we go back to the sticker question, I gave you this as the PMF. And in general, if I asked you to figure out the CDF. So what you will do is, this is the lowest possible 10, right? So the probability of X f being less than or equal to 10 is just this point four, then less than or equal to 20. That's point four and point two, that's point six, and this will be seven. At each stage, I'm just adding up the next right, so this will be point seven, five, then I'm just going to add this date. And then I'm going to add this. Right, so this is

my CD. And anything if I want to look at f of x for below 10. Right, so for X below 10. That's the minimum. So f of x plus b equal to zero. If I want to look at f of x above 60, right for x greater than equal to 60. That's the maximum. So that will always be an in between, right? So if we look at something at F in between 10 and 20. So if we want to look at something like what's the F of 15? C , F of 15, is the probability that x takes values be less than or equal to 15. Now, the only possibility is that is 10. Right? Because 20 is two is above it, right? So this probability is going to be the same as this 0.4. Similarly, if we look at what's the probability of F of 32, right, so I'm looking at something in between this and this. So this is the probability that X tilde is less than or equal to 32. In this case, it's the same as looking at the probability that X is built is equal to 30 or less. Because there's nothing in between 30 and 40 is a possibility. So it will be the same as f 30, which is. So if I were to plot, again, the PDF for this particular question, it's so below, it takes the value zero. At 10. It jumps up to point four and remains there up to 20. 20 jumps up 2.6 and remains there to 30 and so on. One other thing I want to point out is see all of these right? Putting these as unfilled circles, right? So what it means is that this, this function takes this value up to this point, but at that particular point when T , right? It's not this, it jumps up to this, similarly takes the value all the way up to just below 30. And, and then at 30, it jumps up to this, right? So that's the, that's the meaning of these unfilled circles. But what we have plotted here is is the cdf of this row. So let me stop here. And in the next group, I'm going to show you how you can use the CDF to go back to the pier. And it's going to be pretty simple. But we're just going to do it just to show that you can go back and forth between the CDF and the problem as function