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SPEAKERS

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So let's now complete review of Bayes rule by extending it somewhat. Something that which we haven't covered in the previous module. So if you recall all your examples that we did so far, but in all of those examples to only two possibilities, right, either someone was infected or not infected, or in the accident example, either one was drunk or not drunk. But in the real world, there are they can be lots of different possibilities, lots of different types. So for example, if you think about voters, they could be conservatives, liberals, moderates, extreme conservatives, extreme liberals song, right. Similarly, among buyers, they could be someone who is enthusiastic buyers, some who are value buyers, some who are not very interested buyers, and so on. So the question is, how can we extend Bayes rule to contacts where they can be more than two types. And what I'll show you is that we can do that quite naturally. And we, and it's useful because this idea of of using information to update our belief about what type of a voter one is what type of buyer one is, right? So this is a very useful idea. So how do we do an extension of Bayes rule in a case like this? So if you recall, Bayes rule is this idea that what we're looking at is the probability of B given A and and it's given by probability of A intersection B divided by the probability of a right, and then we can ride this probability of A intersection B as probability of A given B times probability. And we can break up the probability of A which is in the denominator into either A occurring due to B, B happening, or A occurring due to B complement. Now, suppose we have three possibilities for B, B1, B2, B3. So how we're going to figure out probability of say B1, given that A has occurred, but how do we figure this out. So what we're going to do is exactly the same way that we did here, it's going to be the probability of B1 intersection A divided by probability of A. So we are still going to be using the same formula here, the only thing is now when we're looking at probability of A right here, it was only either B or B complement, now he can occur either due to be one awkward, or due to be two happening, or due to be three happen. And that's what we're going to incorporate in this denominator order. So let's do that. Now, since I said, right, so it's the denominator, which is going to change and that's the part C, you remember the law of total probability, if a can occur either through B one, B two, or B three, the way we can write P of A is P of A given B one times probability of b one plus probability of A given B two times probability of B two plus probability of A given B three times probability. Right. So this is now your Bayes rule, now extended where there are these three possibilities for B, B, one, B two, and B three. And you can see that like, it doesn't have to stop at three, right? So there could have been five possibilities for B, all you

have to do is then instead of these three terms, there would be five terms here, right? B of A given B one, times B one, and so on up to probability of A given B five times P of before. So this is a natural way to extend the base. So let's do an application using this. So and this is the type of problem and this example is the type of problem that banks and people who make loans they always face. So you know that the different types of borrowers, right? And what you're always afraid of is when you bought lend money to someone is what if that person doesn't pay you back? What if that person defaults? Right? So let's say there are three types of borrowers. So there are good diligent borrowers. For these people, the probability of default is only 10%. So some of them, maybe they run into financial problems, but otherwise the good, diligent borrowers, but their probability of default is 10%. And what's your prior about, about the proportion of good diligent borrowers in the in the economy? Let's say that's point four. There's another class of borrowers who are good, but were negligent, let's say they sometimes forget, not because intentions are bad, but just because they may get busy, they may forget, right? So they have a higher probability of default, their probability of default is 40%. And what's the prior about what fraction of borrowers are good, but negligent? Let's say that's also point four. And what you're really worried about, is this group, who are the bad boards, right? So these people, their probability of default is one. So they're 100% likely to default? And what's your prior about? What's the proportion of bad borrowers in the economy? Let's say that's point two. And so what banks do is that they often look at your credit history, how many defaults whether any default in the past, and so on, and to make an judgment of which of these three types of borrowers one is, right? And typically, this is kind of captured by what's called credit score. Right? So but the basic idea is that given that you see a default, what's the probability that this borrower is one of the one of these bad boys, because the default could have been happening, like if it's a bad borrower, for sure, he will default that even like good borrowers, they could be negligent and make default, or they could be diligent. And still, sometimes they make defaults, right? So just the fact that someone has defaulted doesn't automatically mean that he's a bad boy. So what we're going to use is again, Bayes rule here. So it's a probability that this is a bad given that you observe a default. And we're going to make use of Bayes rule. So on the numerator is is probability of sorry, this should be the other way around. It's the probability of default, given bad, right? Times the probability of that. And in the denominator, is is the probability of default. And it could be because it's a bad borrower. Or it could be because it's a good diligent borrower, or it could be because it's a good negligence. Right? So we're now going to put in all the values here. So on the top is probability of default given bad, that's one times the prior probability that someone is bad is point two. At the bottom is, is this right? probability of default, given bad is one times probability about that's point two. Then the second part, which is probability of default, given that one is a good diligent, right, so that's point one. What's the prior that one is a good diligent? That's point four. And the last one is the probability that defaulting by a good negligent, right, so that's point four, what's the probability that one has a good negligent, that's powerful. And so this is point two. The bottom is point two, plus 0.04 plus point six. So this turns out to be two over for synthesis point. So what that means is that if you observe a default, right, now you're updated probability that this is actually a bad borrower that goes up, it goes up to 50%. Right? So remember, you started out with a prior that this could be a bad borrower as 20%. Right. But now that you've observed the default, you see that this goes up 2.5. And what we are making use of here is the extended Bayes rule. So what I would like you to do is try this out as an exercise to figure out what's the probability that one is a good diligent borrower, given that you see a default? Notice, again, make use of Bayes rule to calculate that, and you do if you do it, your answer should be point one. So please try this out as an exercise on Bayes rule. So this sort of updating is always done by banks, financial institutions, etc. and how they relate this probabilities that they follow that they're updated beliefs about whether the given borrower with a given credit history. What probability that this is a bad borrower are a good, negligent or a good diligent type. So that

determines what sort of loans whether they are at all willing to extend loans to this bar to this borrower, what sort of terms do they set in terms of the interest rates, the down payments, things like that? Right, so So Bayes rule plays a central role in determining those decisions. Right? So what So what hopefully I've been able to convince you is that Bayes rule is a is a very useful tool. And it can be extended to cover where cases where they're not just two possible types, but there could be multiple types. So let me stop this clip here. And that's the end of my review of Bayes rule and material from the previous module. And in the next clip, we'll start off with a fresh topic that of looking at random variables.