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SPEAKERS

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Welcome to a new module of the course, this is the third module in the probability section of the course. And here we'll be covering discrete random variables. Let me begin by giving you an outline of the module. So we'll start with a review of some of the material that we did in the previous module, conditional probability. And in particular, we'll spend a little bit more time on Bayes rule. And then we'll be moving to the main segment of this module, which will be on random variables, which can both be discrete or continuous. So in this particular module, we'll be focusing mostly on the former, which is the discrete random variable. I'll be talking about the probability distributions. And I'll be talking about expectation and variance. And then we'll end the module by looking at some common distributions of some discrete random variables, distributions, which you're likely to encounter in real life and in your courses. So let's begin by reviewing by some of the material that we covered in the previous module and conditional proof. So we began with the idea of conditional probability, where the occurrence of one event plays a role on the probability of occurrence of another events, right. So when we're looking at B, how is that impacted by the fact that even a has occurred? So probability of B given A, that's the conditional probability, then we looked at the idea of the general multiplication rule, which is, if you're looking at the probability of the intersection of two events, A and B, you can write that as the probability of B given A time's the probability of a right, so this is that A has occurred. And then times the probability that B occurs given that is A has occurred. And the third thing we looked at was the idea of total probability or marginal probability. So if you're looking at the probability marginal probability of event B, now, even B could have occurred either due to the occurrence of a or due to the occurrence of a compliment. So so the first part will be that, okay, so he has occurred. So probability of that is P of A times now that given a has occurred, what's the probability of B occurring. So that's one possibility, plus the other possibility that the reverse happens, which is a compliment, right? And then what's the probability of B happening given that a complimentary so the total probability of occurrence of b is the combination of these two. Now, you can also break this up, suppose B can occur due to many different ways of occurrence of A right either it could be A one occurring, or A two occurring and so on up to A n. So if you want to look at the total probability of B occurring, then you have to look at all these different cases, one that he one occurs, and then probability of B given A one, or that a two occurs, and then probability of B given A two, and so on that and,

and going up to probability of A n occurs times probability of B given A. So this is the idea of total or marginal. And then we ended the module with this important Bayes rule. And here's the formula for it. So what we're trying to find out is the probability of B given that A has occurred. And that's given by probability of A given B times probability of B divided by the total probability of B of A occurring, right? And a could have occurred either because of B, right, or because of B complement. So we'll put both of these possibilities in the denominator. And on the numerator is the probability of A given B times the probability of A. And we derive this formula in the previous module. And what it also told you about this is an interpretation is couple of interpretations of it. One is that given, we know that P of A given B, how to find the reverse probability, right, the reverse conditional probability of B given A. And I gave you the example that suppose you know, that drunk driving increases the chances of accidents. So, we know the probability of accident, given that one is drunk, how do we find the reverse? That is, what's the probability that someone is drunk given that that someone had an accident? And the second interpretation is the following that we started with a prior about B. And now we got some new information. So the new information is this bit of a right, we got some new information. And based on that, how do we update our belief. So, we start with a prior. And when we update our belief due to the new information, that's the probability of B given this new information, a. And this updated probability is called posterior. So I ended the last module by introducing you to this example of testing for an infection. And I had asked you to use Bayes rule to figure out the, the updated probability. So I hope you took the chance to figure that out. What I'm going to do now is review that example and use Bayes rule to figure out that updated probability, and you can check your answers. So what was the example is that so suppose you suspect that your probability of having an infection is point three, and you do a test. And the test is not completely accurate. So if you're infected, if you're truly infected, then the test comes out to be positive with 70%. Right. But sometimes the test also comes up positive, even if you're not infected. And the probability of that happening is point one. So what we are interested in is to figure out what the what is the probability that you're infected, given that the test has come up to be positive? Right? So if the test is positive, then what's the probability that you're actually truly infected? And this is the perfect scenario for the Bayes rule. We'll see. So what we're looking at is probability of infected given positive. So on the top is probability of positive given infected times probability of infected. So let's put that in. So we know that the probability of positive given infected that's point seven, the probability of infected is point three, right? So this was my prior divided by the bottom is that same, the first term is the same as the numerator. So it's probability of positive given infected times probability of infected. And the second term is, what's the probability that the test turns are positive, given that you're not in fact? So that probability is point one times the probability that you're not infected. So what is that? So here, the probability of infection, the prior probability of infection is point three, so the probability that you're not infected is the complementary probability. So that's one minus point three, which is point seven. So this is point seven. So this turns out to be point two, one divided by point two one plus 0.07. So that's 22.2 and 2.21, divided by point two, eight, so that's 21 over 28 which is three fourths or points seven, five. So what it means is that See, initially you suspected that of being infected, that probability was point three. Now you do you've taken the test, which is an imperfect test, right? But But given that this test has come up with the positive, right, then you update your belief that you're infected 0.75. So this is now your posterior probability of being infected, given that the test has come up with a positive result. So what if you're like me? And you say, Yeah, .75 is not good enough, right? I want to be even more sure. So maybe, then you take a second just now, you know, if you can do that, right, so the question is the second, like you take the same test, but you take it a second time. And you know that, again, the test is imperfect, right? It can come up to be positive. Yeah, even if you're not infected, right. But what if the second test also comes up to because what is now your updated probability after getting two positive test results? Now we can again, use the same

Bayes formula. Right? Well, one thing to note is that see, initially, I started out with a prior of being infected Oh, .3. Right. Now, given that the first test came out to be positive, I have updated my product. Right? Now I know that given the first test is positive, I, I know that my probability of being infected is point seven, five. So when I'm taking the second test, my new prior that I'm infected, is no longer point three, it's my updated. So in this case, my new prior is going to be point seven, five, right when I'm taking the second test, now, suppose the second test also turns out to be positive. So then, then what is my what is my updated belief that I'm infected, given that the second test is also positive? Again, I'm going to use Bayes rule. Right, so probability of positive given infected, that's point seven times the probability of infected now I have the new prior, right, so my new prior that I'm infected is point seven, five. So this is point seven five, the bottom again, point seven times point seven, five, plus the remaining probability that that the test shows positive, even if one is not infected. So that's point one, times the probability that one is not infected. So now, if, if the probability of infected is point seven five, the probability of not infected is the remaining which is point two, five, this is point five. And if you know calculate this, it becomes point nine four. So what it shows you is that if you take the test a second time, and the second test also turns out to be positive, then then your updated belief that you're you're actually have an infection given that both test results are positive, that is 95%. And remember, what we're doing is we're using Bayes rule all the time, but what we're doing is at every point given the results from before I have updated by and people do this all the time, because as you new information comes along, right you update your prior right and then some more new information comes along then you use your Bayes rule using this new prior and so on. And, and you can do this, like you remember in the earlier stock example that we had done that here was a stock and Dr. Stock does you gives you a buy recommendation. And maybe another stock analyst gives you a truly a don't buy recommendation. Right. So how do you how do you take into account? The fact that one says buy the other says not buy? How do you take that into account in your in your updated belief that you do it exactly the same way that based on Doctor stocks by recommendation, you update your prior to a new product, right? And then the other analysts may be says don't buy right now starting with the new prior, you take that information on board and calculate again, you update your belief based on the recommendation of the secondary nodes. What you're doing is again, the same procedure that we did with this COVID testing that you start out with the first bit of information, update your prior and then you incorporate the second bit of information