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SPEAKERS

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Welcome. In this lecture, we're going to talk about one of these things that ends up causing problems for a lot of students that, you know, like not big problems, but they students end up making mistakes. This is a very common kind of set of mistakes in here. And so I wanted to spend a little bit of, you know, spend a lecture to kind of get help get this straightened out as to why we can't just decide what infinity minus infinity is, because it could actually be anything. So we're going to look at some examples. And my pen was not with me, okay, we're going to look at some examples. And in each of these circumstances, we're going to have these two functions, F of X and G of X . And it's actually going to be true that so if I took the limit, as X went to infinity of these, you know, the difference between these functions, so F of X and G of X , right? If I went through, and I computed this, like this, so if I, if I wrote, so it's going to be important that if I wrote the limit, and I took these two limits separately, like this, so each of them X is going to, whatever X is going to, maybe it's infinity, it doesn't actually matter. It's just the outputs, that would matter. Right, that, so if I took this, you know, this would be one of these, oops, grabbed the wrong marker. This would be one of these circumstances, where oh, I really don't like, really don't like that that looks green, because it kind of takes away the whole point. Okay, let's just write that again.

Right, each of these would be, each of these limits would be infinity. So this would be one of these circumstances where one might write something like this. But this is, right, and then this could actually end up being anything. So let's look at these examples. So for the first example, I want to take F of X equals X plus 2, and G of X equals X . Okay, then in this circumstance, so what happens when I take, and the, and both of these limits go to infinity, right as X goes to infinity. So I do have that each of these limits is infinity as X goes to infinity for both of these functions, so this limit equals infinity as does this limit.

But if I take, right, so now let's look at what happens if I take the difference. So the limit as X goes to infinity. So I'm going to take F of X minus G of X , that's minus G of X , and then I'll put in what the functions actually are. So this is going to equal the limit as X goes to infinity, my infinities are a little sloppy. As X goes to infinity of, and then we have, so what do we have? We have X plus 2 minus X . So we have X plus 2 minus X , right? But what is this in here, this is just 2, right? So this is 2. So this is going to just, well, I'm taking the limit as X goes to infinity of 2. So this is going to equal 2. And notice

this 2, it precisely came from right here. So I could have made that any number by just changing that 2 to whatever I wanted. It could have been π , it could have been E , it could have been minus 1000. It doesn't matter, I could have gotten any output. The output is just taken from here. Okay? So that's the first example I wanted to show you. The next one was, so if I have F of X equals X , and then I have G of X equals, oh, I wanted to have that be $2X$ as a different example. So if I have F of X equals $2X$, My eyes went down farther than I wanted. So, and then G of X equals X , okay, so now when I take the limit as X goes to infinity, I'd like to continue doing that, as X goes to infinity of, well for both of these again, this is, so for each of these I get this is infinite. So also when I take this limit of G of X , so both of these limits are infinite.

So this would indeed be one of these circumstances where it looks like I'm subtracting infinity, okay? But when I take the limit as X goes to infinity, so F of X minus G of X , I think that you'll understand the point if I keep it in the colors. So I have 2 of X minus, and then from G of X I have X , so I'm just going to get X . Okay, so what am I doing? I'm taking the limit as X goes to infinity of just X , right? That's what I'm left with. And this is going to be infinity. Okay, so so far, I've managed to get any number, right? I got 2 there, but I could just replace that 2 with anything. I could have gotten any number, I could get infinity, and now I'm sure you could guess that actually just swapping the order here, I'm going to get negative infinity. So in this circumstance, I'm going to take F of X equals X . And I'm going to take G of X equals $2X$, right, and then when I take the limit, well, first off the limit, we already have both of those limits from over here. So we know that both of those are, are infinite. And then when I take the difference, so I'm taking X minus $2X$, I am indeed, so I'm going to end up with this is the limit, I'm just, I end up with minus $2X$ in here, so this is going to equal the limit as X goes to infinity of minus $2X$, right?

Oops minus X . One of my copies of X is it's just one of my copies of X that gets cancelled, minus X . So I just get left with minus X . Right? And so this is going to equal minus infinity. So I like to go through this with students. And it usually, hopefully, it can help a little bit. These mistakes are very, very common. You can't just take infinity minus infinity and think you know what it is. Some people think it's zero, some people think it's infinity. You really actually, but one of the lessons also of this example is a lot of times you're able to kind of combine things down, so to a limit where you, which you can't actually compute. Okay, so I hope that made some sense and was helpful and I will see you in the next lecture.