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limit, approach, equal, function, x equals, exist, values, close, axis, draw, rapidly, function f , infinitely, sine, moral, reason, abbreviation, happen, ignore, side

SPEAKERS

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Welcome. In this lecture, I'm going to go through what's going to happen with the limit as X goes to A of a particular function F of X that's not defined at X equals A . So we're going to go through two more examples, we went through one before, we'll go through two more now. Okay, so the moral of last time, before we get started we'll kind of remember that the moral of last time was to ignore what happens at X equals A . So ignore what the actual like, you know, if I'm trying to input exactly the value there, you know, that's not what I want to know. I just want to ignore what happens at X equals A , and I want to look at what happens close by, okay? And then this time, is going to have kind of a new lesson. So the lesson this time is going to be that sometimes the limit does not exist. So sometimes the limit, and we're going to write that as DNE, right, and this is, for does not exist. It's like very common, so you should kind of get used to that abbreviation. Okay? So let's look at the first example here. So this is example two.

Okay, and in this example, we're going to look at the function F of X equals, so we have F of X equals sine of 1 over X . So this is a composition of two functions. And then what we're interested in, so what we want is the limit, right, and then we want to take the limit as X goes to zero, so we want X to approach zero, sine of 1 over X , so this function here. So we're going to take that limit of sine of 1 over X , and we want to know what that's going to give us. So we're looking at the limit as X is approaching zero. So let's kind of draw this function here. So I have the X axis, like this, and I have my Y axis like this, and well, what's happening is that, so you can kind of look at, like, if you know, I start out with something that looks kind of like, just kind of like the sine function, but then what's going to happen is, as it gets closer and closer, it's going to start getting, like more and more rapidly going up and down. Okay? And the same thing happened actually on the other side, as I get closer and closer, more and more rapidly goes up and down. So starts going like super, super fast, up and down. Okay, well, let's think about that for a moment. What is that? So if I was going to say, you know, you might say, well, what is it approaching as I approach zero? Well, I do see sometimes it's one infinitely close. But also, sometimes it's zero infinitely close, and sometimes it's minus one infinitely close, right? And the same thing happens on the other side. So there's no real good answer for what the limit is. Okay? So the limit in this circumstance, so the limit here as X approaches zero, so as X approaches right, our zero is here, that's where our zero is. So as X approaches zero of this function does not exist.

Right? And the reason, okay, and I said this, but let's kind of just, kind of write this out, since it's impossible. Since it's impossible to get close enough to, right, this X equals zero. So close enough to, right, having X equal zero to make, right, these F of X values really close to the same value. Whatever, you know, whichever value we choose, there are other values that are also showing up that close. Okay? Really close to the same value, the same value, okay? So that's just kind of this example where the reason why you don't have a limit is because it keeps on moving around. No matter how close I get to here, I'm seeing all these other values, it doesn't choose just one. Okay? So that's one example. It's kind of a fun one, it's one, you know, somebody asks you for an example that does that, it's kind of a clever one, because it really goes like rapidly as you get close, I kind of like that. Okay, so this this example, so this is example three. And in this one, the moral is kind of going to be that it can depend on the direction that you approach. So the direction from which you approach X equals A from.

Okay, so let's kind of look at this one. So in this example, I'm going to look at the function, here's my function. So my function is going to be F of X is going to equal, it's kind of one of these split functions where I have, so it's going to be X if X is less than or equal to 2, and it's going to be minus X if X is greater than 2. Okay, and then the question, of course, then is the limit as X approaches 2. So we're looking at the limit as X approaches 2 of F of X . We want to know what that's going to be. So let's kind of draw this first, like, that can be helpful. So let's go ahead and do that. So I'm going to draw my axes, I have one axis here, this. Okay, and then I have another axis here like this. Let's maybe extend this axis a bit further this way, okay. Um, and what's going to happen? Is it this the graph of this function? So it's going to go up, it might actually help for me to kind of draw it and then label things. For the reason being that sometimes otherwise, you can't get things to match up correctly. Okay? So I'm looking at. Okay, so this is, right, so this is going to be 2, this is going to be minus 2. Okay, so then what's going to happen? So as I'm approaching, so what happens is I approach from this side, right? So what's going to happen on this side, so what I'm going to get, so this is actually so this is called the 2, like the 2 and then you put a little minus up here side, because I'm kind of less than 2, I don't know, it's just how we do that. So then we're going to say, so what happens as I approach from this side is my output values approach 2, right? So what this is actually kind of saying as I go from this side is that the limit as X approaches 2 from that side of F of X is going to equal 2. So that's like my left hand limit is actually 2 coming from, as I'm approaching 2 from values that are less than it, then I'm going to approach 2 there, okay? But what happens is I approach from the other side.

Which is going to be that, right, so just like as I went from this side, I approach 2, right? Now, as I'm approaching from this side, my values are getting closer and closer to minus 2, right? Like that, they're actually coming from that side, okay? Which is to tell me that, as I'm going over here, so my limit as X approaches 2, from kind of the right hand direction of F of X is going to equal minus 2, okay? But then we're looking at these and these two values are not the same. Okay? So this value and this value are not the same, those two limits are not the same, okay? And so this does not exist. Since really, we have like the limit as X approaches 2, the left hand side of F of X does not equal what we would get when we would take the limit as X approaches 2 from the right hand side of F of X . Okay? So, right, so these are kind of two different ways that the limit could not exist. There's other ways. It can go up infinitely far, but we'd start with these. And so, the point kind of being that, you know, if you're not closing in on a single value as your X values close in on, or input values closing on your special value down here, if you're not closing in on an output value, then your limit doesn't exist. Okay, so I hope that made some sense and I'll see you in the next lecture.

