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SUMMARY KEYWORDS

limit, property, divided, domain, squared, equal, cancel, factor, function, direct substitution, apply, denominator, part, dsp, emphasize, putting, multiplying, satisfied, abbreviate, orange

SPEAKERS

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Welcome. In this lecture, I'm going to talk about how you can use a direct substitution property in a circumstance where first you kind of need to cancel, as one might say. So let's look at an example. So suppose we're in the circumstance of, so this is an example where we're going to take the limit. And we're taking the limit as X goes to minus 3, so X goes to minus 3 of, and then our function that we're taking the limit of, is $X^2 + 6X + 9$, and dividing this by $X^2 + 3X$. Okay, so yes, okay. So the first question is, like, can you directly apply this directed substitution property, which we abbreviate by DSP recall. So can we apply DSP? And this is where we remember that in order to apply a property, you need to have that the if part is satisfied, okay. So, so no, because the if part isn't satisfied. So why is the if part not satisfied? Which is that if we substitute, so subbing minus 3 into the denominator for X , right? Minus 3 into the denominator, right, for X is going to yield out zero, okay? And we can't divide by zero, so we're not actually in the domain, right? We had to actually have that this value A is in the domain, okay? So we can't divide by zero, and so we're failing the part of the property which says that minus 3 would have to be in the domain. Okay? So now we need to fix this. And how do we look at fixing this? Okay, so we're going to try something. I'll kind of try this over here, which is going to be to first factor and cancel. Okay, so we're going to try factoring and canceling first. So factoring, we're going to try factoring and canceling first. And let's go ahead and do that. Okay, so let's take this limit. Again, this is the limit as X goes to minus 3.

And then we have the same function here. So this is $X^2 + 6X + 9$ divided by $X^2 + 3X$. Okay, we can go ahead and factor this on the top, right, you can tell that this is a square because you have 2 times 3 here and 3 squared there. So we know this is a square on top. So we can go ahead and, to factor that let's oh, let's make sure to write our limit first. Because we haven't taken that yet, right, we have to remember, I kind of emphasize this a lot of times and it's important to always write that when you haven't taken a limit yet so that you know when you're done or not. This is one of these things you learned for many years of teaching. So I have $X + 3$ squared on top, again, because this is 2 times 3, and this is 3 squared, then we divide that by we can't have an X in each of these, so I can pull out the X . And then, so I have to take X^2 divided by X . And then I have to take $3X$ divided by X , there we go. Okay, and now we have the $X + 3$ on both the top and the bottom. So let's kind of look at that, that's going on there. Okay, so this is equal to the limit as X approaches minus 3, and let's kind of pull out a copy of $X + 3$ on both so this is very clear what we're doing. So I have $X + 3$ times $X + 3$, and then we're dividing that by X times $X + 3$. Okay?

And now we kind of realized what do we actually have going on here? We have this. So we have X plus 3 divided by X , times X plus 3 divided by X plus 3, right? What's going on kind of with that, is that this thing here, this part here that we're multiplying by is actually 1. So we can just kind of pretend that's not there because it equals 1, so we're multiplying by 1. Okay? So this kind of reduces down to taking the limit as X goes to minus 3. So we have X plus 3 over X , I'm going to emphasize that this X is what's being replaced by putting orange here. So X plus 3 divided by X , okay?

Now we can apply the property. Why can we play the property? Because minus 3, it gives zero on top but it doesn't give zero on bottom. So it's actually in the domain. Okay, so this is going to equal using, now we can apply the direct substitution property, because minus 3 is in the domain of this function. So we go ahead and do this. So this is equal, and we do this thing to be kind of careful. Where we put parentheses every time we see an X . This makes it a little less likely that we're going to make a mistake, plus 3, and then every time we have parentheses, we replace that by minus 3 using the direct substitution property like this. Okay, and now we're good to go because, right, the top is zero, which makes this zero, the bottom is minus 3, which is not zero. Okay. So this is kind of our answer. Right? And as we talked about, and I kind of talked about this as we went along, but this is important to emphasize, okay. So why was it okay to cancel factor plug in? So why was it okay to cancel factor and then plug in?

Well, the last part was okay because we then satisfied the conditions. But really kind of, in terms of limits, what's fundamentally going on, is that we don't care what happens at negative 3. So we don't care what happens, right, at exactly that value negative 3. So happens at X equals negative 3. So right, you don't care what happens at that exact limit value. That's part of the things that's true about limits. Okay, so let's kind of like the formality of the example. Let's kind of look at that. So what is this formality of this example? Formality of example? Well, what is this, what's really going on here is that if we have that F of X and G of X are the same function, as long as we're not at some, you know, the, as long as we're not at this A here. So as long as, so if these are the same function, when you know this X here does not equal our A , okay, the X being this input variable, in fact let's make these orange. Okay, so when X is not equal A , then we have that the limit as X approaches A of F of X is going to equal the limit as this X approaches A of G of X as long as this limit exists, okay? G of X if the limit exists.

Right, if these two limits exist, then you're going to get the same thing, right? As long as the two functions are the same away from this, this red A . Okay, so this is kind of the formality of what we were talking about here. Okay, so what kind of went on here, we want to do this this thing of factoring and canceling and so on, it's very useful when taking limits, but there's kind of this key thing going on here. Which is that you know, this actually works. Right? As long as once you factor in during cancel down, you can apply the direct substitution property because, right, you've no longer got your denominator as zero, you've no longer got that this A value is making the denominator zero. So it's not in the domain. Okay? But once you if you're able to by factoring and canceling, get it to where it is in the domain, then you're good to go with the direct substitution property, you can just plug that A value in, okay? And this is the real thing that's going on. And why does this work, right? Because these two functions are the same as long as I'm not at minus 3, right? Which is what we mean here. Okay, so here we're saying it means like, these two functions, they behave exactly the same, you're

getting exactly the same outputs as long as it's not minus 3 you're putting in. And that was kind of the key thing going on here, and then the limits are the same. Okay, so I hope that made some sense and I'll see you in the next lecture.