

# PfaffModule3L09

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## SUMMARY KEYWORDS

pi, property, polynomial, limit, direct substitution, hold, function, plug, squared, variable, part, parentheses, rational function, apply, generalizable, stick, satisfied, domain, divided, orange

## SPEAKERS

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Welcome. In this lecture we're going to talk about the direct substitution property for limits of polynomials and rational functions, which is really kind of a generalization of what we did last time. So let's start by kind of talking about what we did last time. So last time, last time, we had that the limit as  $X$  goes to  $\pi$ . So we had a limit as  $X$  was going to  $\pi$ . And then we had, right, and I'm going to kind of write it like this to make it clear what's going on. So we had  $X$  cubed plus  $4X$ , oops, that should be, sorry,  $4X$  squared, and then we had minus 3 divided by, and then we had on the bottom, we had  $X$  squared. So I'm just replacing my variable, or making my variable orange to kind of indicate that that's what, what's going to  $\pi$  here. So we have  $X$  squared plus 5, right? And then what we actually got out through all of like the limit laws, in the end, we could have just substituted  $\pi$  into there. Okay? So I'm just going to kind of write the same thing with the parentheses everywhere. So we have, right, this cubed, plus 4, and then we have parentheses because there's an  $X$ , and then squared minus 3, and then we're going to divide this by, and then we have so this squared plus 5, okay? and all we need to do is to plug our  $\pi$  in every time where we have parentheses. So at first I replace  $X$  with parentheses, then I just put in  $\pi$  everywhere, and now we've evaluated our limit. So we no longer write limit there. Okay, it's good to know when you no longer need to write that there, like when you're done. Okay? So in other words, what happened here, so i.e., like applying the limit laws, or plugging in  $\pi$  gave the same thing. So i.e., maybe I'll just write this down here. So applying the limit laws. So applying the limit laws, or plugging in  $\pi$  gave exactly the same thing, right. Gave exactly the same thing. And this is kind of this is, this is a powerful thing going on, exactly the same thing.

Okay, so let's see, what is this an example of? So this is all an example of a particular property. So this is an example of this property. So this is a direct substitution property. And I'm going to have an abbreviation for this because it's kind of a long name. So this is the direct substitution property, direct substitution property, and we're going to abbreviate that by DSP. Okay, underline this, it's a direct substitution property. And what does it say? So here's kind of the set up, which is that if, right, we have that, what is everything we need? So we need that  $F$  is a polynomial rational function. So  $F$  is a polynomial or rational function, polynomial or rational function. Right, which is just a polynomial divided by a polynomial. Okay, so this is the first property that has to hold. And then we have another property, which is that this kind of  $A$  is in the domain of  $F$ . So  $A$ , right, which is what we're kind of taking the limit going to. So  $A$  is in the domain of  $F$ . So  $A$  is in the domain of this function  $F$ . Okay, so

when these two things are satisfied, then we have, so once if two things are satisfied, then we know that we have the limit as  $X$  approaches  $A$  of  $F$  of  $X$ , make this orange here so you can see, this  $X$  matches up with that  $X$ , that this is actually going to equal we're just going to stick  $A$  in for  $X$ , okay, like we did in the other circumstance.

Okay, so this is the property, I'll make a few comments on it. Okay, so the first part of it, right tells us, so what is this first part tell us? The first part, and this is kind of important to know, you see people talking about theorems, and it can be a little confusing as to what the two pieces tell you. So, the if part tells me that these two properties must hold in order to apply the then statement. Okay, so these two properties, and this will be something to check, so these two properties must hold. Okay, and I'm going to put in parentheses that you should check them. Okay, um, must hold to apply the then statements, the then statement. Right, the second part, okay? And then kind of what's going on in the second part here, is that we can assume this holds as long as the two properties hold, okay. So we can assume this holds, that we can just stick the  $\pi$  in as long as we're in the situation of the first two. So can assume holds, this holds as long as we're in the situation of the first two properties holding. As long as in the situation the first two properties hold. So of the first two properties holding.

Okay, so this is when we have an if then, the first part, you check that it's satisfied, and then you can apply the second part. But let's also just kind of stick this in other words. So how do we write this in other words? So in other words, we can just plug  $A$  into such such functions as we did with  $\pi$ . Okay, so we can, and it's like a lot of symbols, so it's good to kind of always parse that. So we can plug  $A$  into the functions like right for  $X$ , so can plug  $A$  into, oops that should be red.

Okay, so we can plug  $A$  into such functions like polynomials, rational functions, and we're plugging it in for  $X$ . So this is for  $X$ , or whatever your variable is, right? So for  $X$ , as we did with  $\pi$ , this can be a very generalizable setting, or at least as generalizable as when you have the first two conditions holding, okay? So it's just kind of another interpretation of what's going on here. So what's going on here, if your rational function which is a polynomial divided by a polynomial, or a polynomial, and we know that this  $A$  right, this  $A$  here, this  $A$  that you're taking the limit towards, if you know that that's in your domain, then you can just plug that in for your variable in your function everywhere in for this  $X$ . Okay? so I hope that made some sense, and I'll see you in the next lecture.