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SUMMARY KEYWORDS

limit, squared, pi, property, plugging, converging, divide, equality, direct substitution, write, cubed, lecture, subtraction, rational function, emphasize, happening, polynomial, quotients, talk, break

SPEAKERS

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Welcome. In this lecture, I'm going to go through an example of how one actually applies limit properties, specifically to a rational function that is one that's a polynomial divided by a polynomial. And this is going to, the principles that are going to show up in this are actually going to turn out to be, their kind of their own property, which we'll talk about in the next lecture. But right now, we're really just focusing on what does it look like to apply the limit properties to get something substantial. So let's go ahead and do that. So here's our example. Example is, and we're going to look at, we're going to take the limit as, and then we're going to have as X , so as our input variable goes to π , of and then the function that we're going to look at it is going to be so on top, we're going to have X cubed, we're going to have plus $4X$ squared, and then we're going to have minus 3, all divided by, and then we have X squared plus 5. Okay? And what do we get here. So this is going to equal the limit as, so the first property that I want to apply here, is going to be property four. So this equality that we're getting here is coming from property four. And what is this going to tell me? So we're taking the limit as X goes to π , we're going to kind of emphasize a lot of times that you should keep writing the limit every single time. First off, because the only thing that's correct when you have the equality there, but also it helps you to keep track of when you have actually evaluated the limit or not. Okay, this is just kind of good practice. So we have on top, so first, we can do our function that's on top, which is X cubed plus $4X$ squared, minus 3, okay? And then actually, let's move this over a little bit to emphasize that the equality that's happening is for the entire thing, because then we're going to divide it by the other limit. So we have the limit as X goes to π , so it's the same, right, our input variable is converging, or is going to the same value there, right, and then we're going to divide by this limit. So we have X squared plus 5.

Okay, so that works with quotients, right? So now let's do the next one. So the next equality is going to kind of allow us to break apart the summations inside. So that's going to be property one. So now we have equality again, and then this is property one that's going to take us down here. So this property, and now we get the limit as X approaches π . And then we can we can, right, we're going to do these one at a time. So this is X cubed, I'm kind of writing a lot, but it's, it's good to very carefully go through this once. So now we have the limit as X approaches π . And then we can, right, so, right, this addition moved to the outside, so now we're at $4X$ squared. And now we have subtraction here. So that's actually property. That's not just property one that we have here. This is also property two, so I should actually say this, so one, and two, right, because we're going to do the subtraction one for

this one. So this is the minus because we have a minus there. And then we have the limit as X goes to π of, and then we just have this 3, that's all that's left to do. Okay, on top, but we still have to do the bottom. Okay, so let's go ahead and do that. So we go, so we divide that. And then we can do the same thing with the bottom, there's only two terms. So we'll do each of those. So we have a limit, and a limit. And in each of these X is going to π , so we have, for each of them our X is going to be approaching π . Okay, and then we're going to have, so we have X squared and then 5. So we have X squared and then we have 5, right? And that addition just becomes adding together the limits. Okay, and then we're going to keep on going. So I think we'll keep on going to up here. So the next step, I'll kind of write it like this, to indicate we're kind of going all the way to up here.

So now I'm going to take the limit, right, so what I'm going to do in this one is that I am going to be allowed to pull out this 4. So let's go ahead and do this. So on top, I still have the 3 limit. So this is all equal to, and I still have 3 limits on top and 2 on bottom. So, I kind of saved myself a little bit of work by writing these limits. And then they're very, well okay. So we have limit and then limit, and this is still addition on the bottom of those limits. This one is still subtraction. And this one I pulled out my 4, okay? So that 4 is now in front here. And we have addition, okay? And in each of these, so what's happening? Each of these is still the variable X is converging to something. So we'll just write all those in at the same time to save ourselves, swapping around markers in the hands. Okay, they're all converging to the same thing, which is π . So we can write that in. And now let's write in what we get. So the first thing is from there, so that's X cubed, right? From this one, we pulled out the 4, so we're just left with X squared, this is X squared. Then we subtract off and then we have a 3 here, right? Because that's what's inside of that limit. And then we have an X squared and a 5, write those in. So this is an X squared and a 5. Okay?

Now, for each of these, each of these is, there's nothing wrong with just plugging π into each of those. Okay, these are just like X squared, and so on. So we can go ahead and do, so this is going to equal, and so we're just doing these one at a time, and we're plugging in π for X . Okay, so I'm going to get, I'm just going to actually write them as, to kind of emphasize what I'm doing, so we're doing this cubed. And so we just put π into there. Okay, and that's this limit here. Now, let's do this limit here. So I have plus, and then I have a 4. And then I have this as an X squared. So I have like this, and then I want again, plug the π in. And now I have the subtraction. And this is just a 3 and that just kind of stays a 3. Nothing happens when you take the limit of that, because it never changes. And then I have, on the bottom I have, right, this X squared that I'm going to put π into, then I'm going to add here the 5.

Okay, and so what I did in this one, so getting from here to here, I used two properties, I used seven and nine. So I used seven and nine. Okay, so here's kind of an important observation in what's going on. So I'll go through that. So important observation. Okay, so our important observation is that we could get the same answer if we just plugged in π . Okay, so we could get the same answer, right, by plugging in π . Right, so this process of plugging in, right, so plugging in π for X , right, every time we see X . So for X in okay, where I have a, right, so I have here, this function here. I could look at that, I'm just going to write all the X 's in orange, to kind of emphasize that this is where things get plugged in. Okay, so in where I have X , so cubed plus 4, then I have an X again. That's going to get, got replaced right, and this was $4X$ squared, and then I had minus 3 all divided by, right, I had this X , right, which gets replaced, X squared. And then this was plus, 5. Okay. And this, this, this fact that I could have just plugged in this π , which was what we were taking X was going to limit there like that,

um, this is, actually has a name. And we're going to talk about this more explicitly in the next lecture, or one of the next lectures. And this will be what's going to be called the direct substitution property. So this will be the direct substitution property. Okay, so that's going to be kind of this important property that when we're talking about continuous and rational functions that are defined at this value here, that we can actually just plug it in. Okay? But getting up to that what we saw happening was that we could use our limit rules to kind of break this apart so that we actually are just down to dealing with kind of the X squared 's and, and so on, and you could actually use then the product kind of rule at that point. Okay. So that made some sense, and I'll see you in the next lecture.